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Fractional Derivative of Dirichlet Average of Generalization of K_4 and its approximation

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Abstract
Dirichlet average is average given by Dirichlet. The Dirichlet average of elementary function like power function, exponential function etc is given by many notable mathematician, Actually, We have convert the elementary function into the summation form after that taking Dirichlet average of those function, using fractional integral and get new results. These results will be used in future by mathematician and scientist. Thus we have find a connection Dirichlet average of a function and fractional integral.

I. INTRODUCTION

Carlson has defined Dirichlet average of functions which represents certain type of integral average with respect to Dirichlet measure. He showed that various important special functions can be derived as Dirichlet averages for the ordinary simple functions likex^t,e^x etc. He has also pointed out that the hidden symmetry of all special functions which provided their various transformations can be obtained by averaging x^n,e^x etc. Thus he established a unique process towards the unification of special functions by averaging a limited number of ordinary functions. Almost all known special functions and their well known properties have been derived by this process[1–5]. In this paper the Dirichlet average of a new Special function called as Generalized K_4 – function has been obtained [6,7].

DEFINITIONS
We give blew some of the definitions which are necessary in the preparation of this paper.

Standard Simplex in $R^n, n \geq 1$
We denote the standard simplex in R^n, $n \geq 1$ by [1].

$E = E_n = \{S(u_{1,}u_2, \ldots u_n) \,:\, u_1 \geq 0, \ldots u_n \geq 0,\, u_1 + u_2 + \cdots + u_n \leq 1\}$(2.1.1)

Dirichlet measure
Let $b \in C^k, k \geq 2$ and let $E = E_{k-1}$ be the standard simplex in R^{k-1}. The complex measure μ_b is defined by E[1].

$$d\mu_b(u) = \frac{1}{B(b)} u_1^{b_1-1} \ldots\, u_{k-1}^{b_{k-1}-1}(1 - u_1 - \cdots - u_{k-1})b_k^{-1} du_1 \ldots du_{k-1} \qquad (2.2.1)$$

Will be called a Dirichlet measure.
Here

$$B(b) = B(b1, \ldots\, bk) = \frac{\Gamma(b_1) \ldots \Gamma(b_k)}{\Gamma(b_1 + \cdots + b_k)},$$

$$C_> = \{z \in z{:}\, z \neq 0, |ph\, z| < {}^{\pi}/_{2}\},$$

Open right half plane and $C_>$k is the k^{th} Cartesian power of $C_>$

Dirichlet Average[1]
Let Ω be the convex set in $C_>$, let $z = (z_1, \ldots\, z_k) \in \Omega^{\mathrm{k}}, \mathrm{k} \geq 2$ and let $u.z$ be a convex combination of $z_1, \ldots z_k$. Let f be a measureable function on Ω and let μ_b be a Dirichlet measure on the standard simplex E in R^{k-1}.Define

$F(b, z) = \int_E^0 f(u.z) d\, \mu_b(u)$(2.3.1)

We shall call F the Dirichlet measure of f with variables
$z = (z_1, \dots z_k)$ and parameters $b = (b_1, \dots b_k)$.

Here
$u.z = \sum_{i=1}^{k} u_i z_i$ and$u_k = 1 - u_1 - \cdots - u_{k-1}$(2.3.2)

If $k = 1$, define $F(b, z) = f(z)$.

Fractional Derivative [8]
The concept of fractional derivative with respect to an arbitrary function has been used by Erdelyi[8]. The most common definition for the fractional derivative of order α found in the literature on the "Riemann-Liouville integral" is

$$D_z^{\alpha} F(z) = \frac{1}{\Gamma(-\alpha)} \int_0^z F(t)(z-t)^{-\alpha-1} dt \quad (2.4.1)$$

Where $Re(\alpha) < 0$ and $F(x)$ is the form of $x^p f(x)$, where $f(x)$ is analytic at $x = 0$.

THE NEW GENERALIZED K_4 – FUNCTION
Here , first the notation and the definition of the Generalized K_4 – function, introduced by Ahmad Faraj , Tariq Salim , Safaa Sadek, Jamal Ismail [9, 10] has been given as

$$K_{4(m,n)}^{(\alpha,\beta,\gamma),(a,c);(p;q)}(z) = \sum_{k=o}^{\infty} \frac{(a_1)_{mk} \dots (a_p)_{mk}}{(b_1)_{nk} \dots (b_q)_{nk}} \frac{(\gamma)_k \, a^k (z-c)^{(k+\gamma)\alpha-\beta-1}}{\text{K}!\Gamma\left((k+\gamma)\alpha-\beta\right)} \quad (1)$$

Here $\alpha, \beta \in C, \mathrm{Re}(\alpha) > 0,\ \mathrm{Re}(\beta) > 0$ $(a_i)_{mk}$, $(b_j)_{nk}$ are the pochammer symbols and m, n are non-negative real numbers.
When $c = 0$in equation (1), we have

$$K_{4(m,n)}^{(\alpha,\beta,\gamma),(a,0);(p;q)}(z) = \sum_{k=o}^{\infty} \frac{(a_1)_{mk} \dots (a_p)_{mk}}{(b_1)_{nk} \dots (b_q)_{nk}} \frac{(\gamma)_k \, a^k (z)^{(k+\gamma)\alpha-\beta-1}}{\text{K}!\Gamma\left((k+\gamma)\alpha-\beta\right)} \quad (2)$$

EQUIVALENCE
In this section we shall show the equivalence of single Dirichlet average of$K_{4(m,n)}^{(\alpha,\beta,\gamma),(a,0);(p;q)}(z)$ function $(k = 2)$ with the fractional derivative i.e.

$$S(\beta, \beta'; x, y) = \frac{\Gamma(\beta+\beta')}{\Gamma\beta} (x-y)^{1-\beta-\beta'} D_{x-y}^{-\beta'} K_{4(m,n)}^{(\alpha,\beta,\gamma),(a,0);(p;q)}(x)(x-y)^{\beta-1} (3.2)$$

Proof:

$$S(\beta, \beta'; x, y) = \sum_{k=o}^{\infty} \frac{(a_1)_{mk} \dots (a_p)_{mk}}{(b_1)_{nk} \dots (b_q)_{nk}} \frac{(\gamma)_k \, a^k (z)^{(k+\gamma)\alpha-\beta-1}}{\text{K}!\,\Gamma\left((k+\gamma)\alpha-\beta\right)} R_n(\beta, \beta'; x, y)$$

$$= \sum_{k=o}^{\infty} \frac{(a_1)_{mk} \dots (a_p)_{mk}}{(b_1)_{nk} \dots (b_q)_{nk}} \frac{(\gamma)_k \, a^k}{\text{K}!\,\Gamma\left((k+\gamma)\alpha-\beta\right)} \frac{\Gamma(\beta+\beta')}{\Gamma\beta\,\Gamma\beta'}$$

$$\int_0^1 [ux + (1-u)y]^{(k+\gamma)\alpha-\beta-1} u^{\beta'-1} (1-u)^{\beta'-1} du$$

Putting $u(x-y) = t$, we have,

$$= \sum_{k=o}^{\infty} \frac{(a_1)_{mk} \dots (a_p)_{mk}}{(b_1)_{nk} \dots (b_q)_{nk}} \frac{(\gamma)_k \, a^k}{\text{K}!\,\Gamma\left((k+\gamma)\alpha-\beta\right)} \frac{\Gamma(\beta+\beta')}{\Gamma\beta\,\Gamma\beta'}$$

$$\int_0^{x-y} [t+y]^{(k+\gamma)\alpha-\beta-1} \left(\frac{t}{x-y}\right)^{\beta'-1} \left(1-\frac{t}{x-y}\right)^{\beta'-1} \frac{dt}{x-y}$$

On changing the order of integration and summation, we have

$$= (x-y)^{1-\beta-\beta'} \frac{\Gamma(\beta+\beta')}{\Gamma\beta\,\Gamma\beta'} \int_0^{x-y} \sum_{k=o}^{\infty} \frac{(a_1)_{mk} \ \dots \left(a_p\right)_{mk}}{(b_1)_{nk} \ \dots \left(b_q\right)_{nk}}$$

$$\frac{(\gamma)_k\, a^k}{K!\,\Gamma\big((k+\gamma)\alpha-\beta\big)} [t+y]^{(k+\gamma)\alpha-\beta-1} (t)^{\beta'-1} (x-y-t)^{\beta'-1} dt$$

Or

$$= (x-y)^{1-\beta-\beta'} \frac{\Gamma(\beta+\beta')}{\Gamma\beta\,\Gamma\beta'} \int_0^{x-y} K_{4(m,n)}^{(\alpha,\beta,\gamma),(a,0);(p;q)}(y+t)(t)^{\beta'-1}(x-y-t)^{\beta'-1} dt$$

Hence, by the definition of fractional derivative, we get

$$S(\beta,\beta';x,y) = (x-y)^{1-\beta-\beta'} \frac{\Gamma(\beta+\beta')}{\Gamma\beta} D_{x-y}^{-\beta'} K_{4(m,n)}^{(\alpha,\beta,\gamma),(a,0);(p;q)}(x)(x-y)^{\beta-1}$$

This completes the Analysis [10–18].

II. CONCLUSION

Dirichlet average of a new Special function called as generalization of K_4–function,which is recently given by Ahmad Faraj , Tariq Salim , Safaa Sadek, Jamal Ismailhas been obtained. This function is an extension of R- function which is introduced by Lorenzo and Hartly (1999). This is the modification of K_4 – function given by Kishan Sharma and these functions have recently found essential applications in solving the various problems in the various field like as biology, physics, applied sciences and engineering.

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