

Queuing Theory: Boon for analysis of services of banking system

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ABSTRACT

The impact of lines on how long it takes for clients to receive banking services is becoming a major concern due to the possibility that making consumers wait too long may cost them money. In Delhi, queues are a common sight at numerous banks, leading to lengthy suffering and financial costs. This study aims to determine the trade-off between minimizing the total economic cost and providing a satisfactory and reasonably short time of service to customers. Data was gathered over a month at the commercial banks' Main Branch in Delhi and organized into a multi-server single line queuing model. Management should adopt a five teller model to reduce economic costs and customer satisfaction, as it is better than a four or six teller system in terms of average waiting time and total economic cost.

I. INTRODUCTION

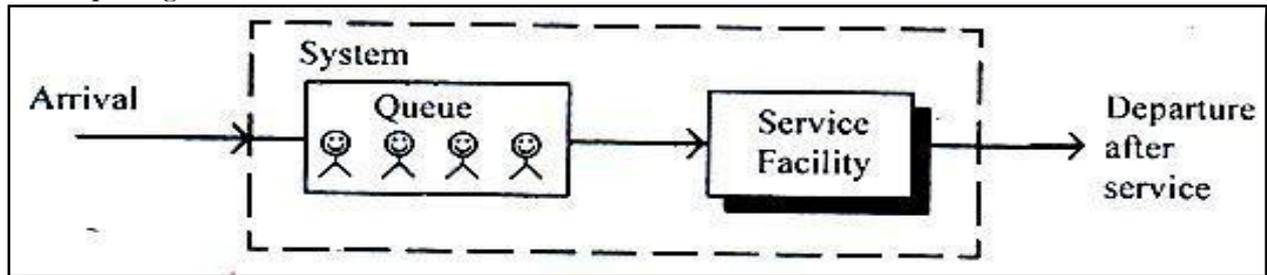
Mathematicians specialise in queuing theory study and simulate the process of standing in line. This essay examines the development of queuing theory and provides examples of models and practical applications. It also explains how to extract meaningful data from a given queuing system, such as average waiting periods. A.K. Erlang is regarded as the founder of queuing theory, having released "The Theory of Probabilities and Telephone Conversations" in 1909. His entry into the field was spurred by his employment with the Copenhagen Telephone Company, where he pondered the issue of how many telephone circuits were required to offer phone service and avoid customers having to wait an excessive amount of time for an open circuit. Erlang's solution to the switchboard problem paved the way for contemporary queuing theory. The majority of research for this work was derived from the chapters on queuing theory and its applications in the book "Operations Research: Applications and Algorithms". The fundamental switchboard problem that paved the way for modern queuing will be defined in the second portion of this essay. We will examine notation, queuing specialties, birth-death processes, steady-state probabilities, and Little's queuing formula to understand the theory. Queuing theory is a subfield of operations research that studies the flow of units for service, formation of the queue, or joining of the line. It is named after Erlang, A.K. (1878–1929), a Danish engineer who wrote articles about the analysis of telephone traffic congestion. Queuing theory is the theoretical study of standing in a line and how it applies to customer or customer satisfaction. It uses performance metrics and mathematical models to improve client flow.

Receptionists with high volume outbound customer workloads and/or those who provide multiple points of service may benefit from queuing theory to reduce long lines and other problematic queuing systems, which can have a negative impact on customer satisfaction. When there is a shortage of a good or service, a line forms. A few of the elements that contribute to lengthy wait times or service delays include: bank employees' lack of enthusiasm and dedication (Belson 1988), staff overload, bank officials serving consumers in many sections, etc. These put bank managers under stress and tension. Customer discontent frequently results from dismissing a customer without attending to their demands (Babes and Sarma, 1991).

Cash Transaction Model

The Queuing model is commonly labeled as M/M/c/K, where first M represents Markovian exponential distribution of inter-arrival times, second M represents Markovian exponential distribution of service times, c (a positive integer) represents the number of servers, and K is the specified number of customers in a queuing system.

M/M/1 queuing model:



M/M/1 queuing model means that the arrival and service time are exponentially distributed (Poisson process). For the analysis of the cash transaction counter M/M/1 queuing model, the following variables will be investigated:

λ : The mean customers arrival rate

μ : The mean service rate

$\rho = \lambda / \mu$: utilization factor

Probability of zero customers in the bank: $P_0 = 1 - \rho$

The probability of having n customers in the bank: $P_n = P_0 \rho^n$

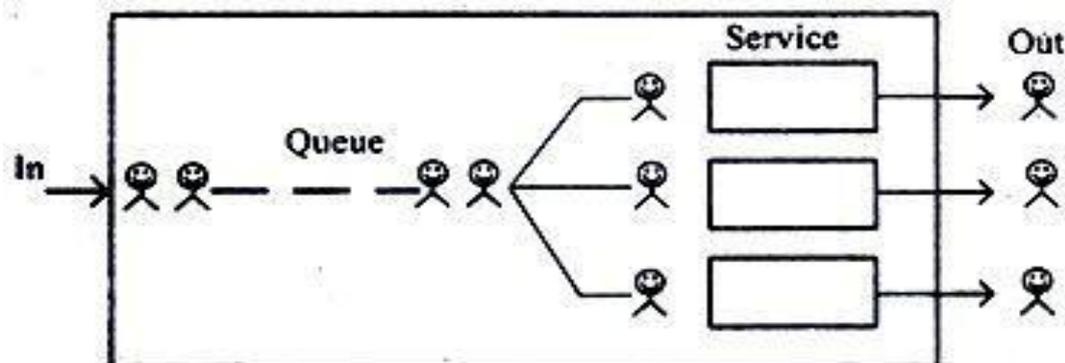
The average number of customers in the bank: $L_s = \rho / (1 - \rho) = \lambda / (\mu - \lambda)$

The average number of customers in the queue: $L_q = L_s \times \rho = \rho^2 / (1 - \rho) = \rho \lambda / (\mu - \lambda)$

Wq: The average waiting time in the queue: $W_q = L_q / \lambda = \rho / (\mu - \lambda)$

Ws: The average time spent in the bank, including the waiting time $W_s = L_s / \lambda = 1 / (\mu - \lambda)$

Now, we discuss the same for M/M/s Model



All customers arriving in the queuing system will be served approximately equally distributed service time and being served in an order of first come first serve, whereas customer choose a queue randomly, or choose or switch to the shortest length queue. There is no limit defined for number of customers in a queue or in a system. We will discuss the case for $s = 2$.

λ : The mean customers arrival rate

μ : The mean service rate

$\rho = \lambda / \mu$: utilization factor

The probability of having n customers in the bank: $P_n = P_0 / \rho^n$

The average number of customers in the bank: $L_s = L_q + \mu$

The average number of customers in the queue: $L_q = P_s \rho / (1 - \rho)^2$

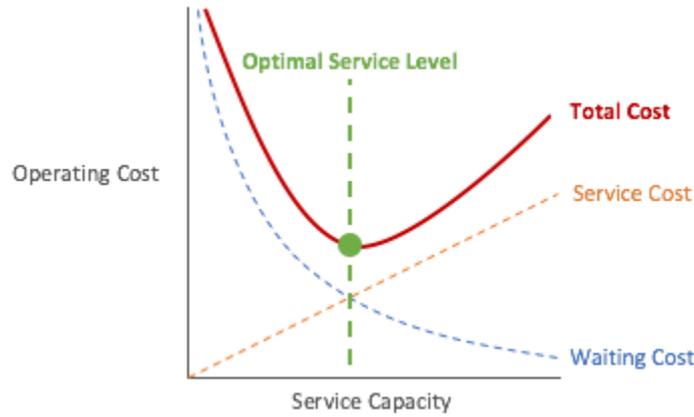
The average waiting time in the queue: $W_q = L_q / \lambda = P_s / (s \mu (1 - \rho)^2)$

The average time spent in the bank, including the waiting time: $W_s = L_s / \lambda = W_q + 1 / \mu$

Queuing theory is the mathematical study of waiting lines, or queues [1]. In queuing theory a model is constructed so that queue lengths and waiting times can be predicted [1]. Queuing theory is generally considered a branch of operations research because the results are often used when making business decisions about the resources needed to provide service.

Waiting lines or queues is the major source of difficulties to any organization or service provider institutes like hospitals, banks etc. Queuing theory basically a mathematical approach, used for the analysis of waiting lines. It deals with the problems that involve waiting (or queuing). Queuing theory is used to analyzing the congestions

and delays of waiting in line. It is used to develop more efficient queuing system that reduce customer waiting time and increase the number of customer that can be served. Customers waiting time depends on the number of customers on queue, the number of servers serving line, and the amount of service time for each individual customers. The time wasted on the queue would have been wisely utilized elsewhere (opportunity cost of time spent in queuing). In a waiting line system, when considering improvements in services to offer an optimal service level, consider the cost of providing a given level of service against the potential costs from having customers waiting.



II. RESULTS AND DISCUSSION

Table 1: Day (One) 1 Queuing System Analysis of the Servers

Monday						
	Server 1		Server 2		Server 3	
Time	Arrival Rate	Service Rate	Arrival Rate	Service Rate	Arrival Rate	Service Rate
10:00-11:00am	14	10	13	10	17	13
11:00-12:00	17	12	20	16	21	19
12:00-1:00pm	18	16	24	19	29	27
1:00-2:00pm	17	16	20	19	21	18
2:00-3:00pm	12	8	15	12	18	17
3:00-4:00pm	4	3	10	8	12	7

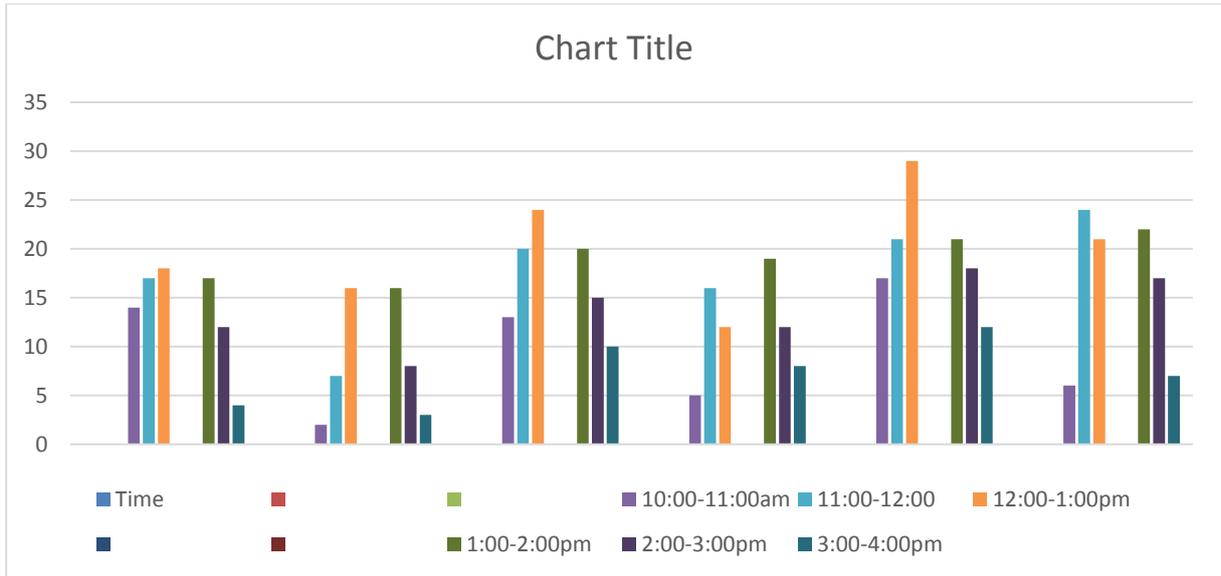


Table 2: Day (Two) 2 Queuing System Analysis of the Servers

Tuesday						
	Server 1		Server 2		Server 3	
Time	Arrival Rate	Service Rate	Arrival Rate	Service Rate	Arrival Rate	Service Rate
10:00-11:00am	15	10	17	14	15	12
11:00-12:00	20	18	29	25	25	21
12:00-1:00pm	28	25	27	24	32	28
1:00-2:00pm	25	21	21	20	24	21
2:00-3:00pm	30	27	25	23	21	18
3:00-4:00am	18	15	17	15	12	9

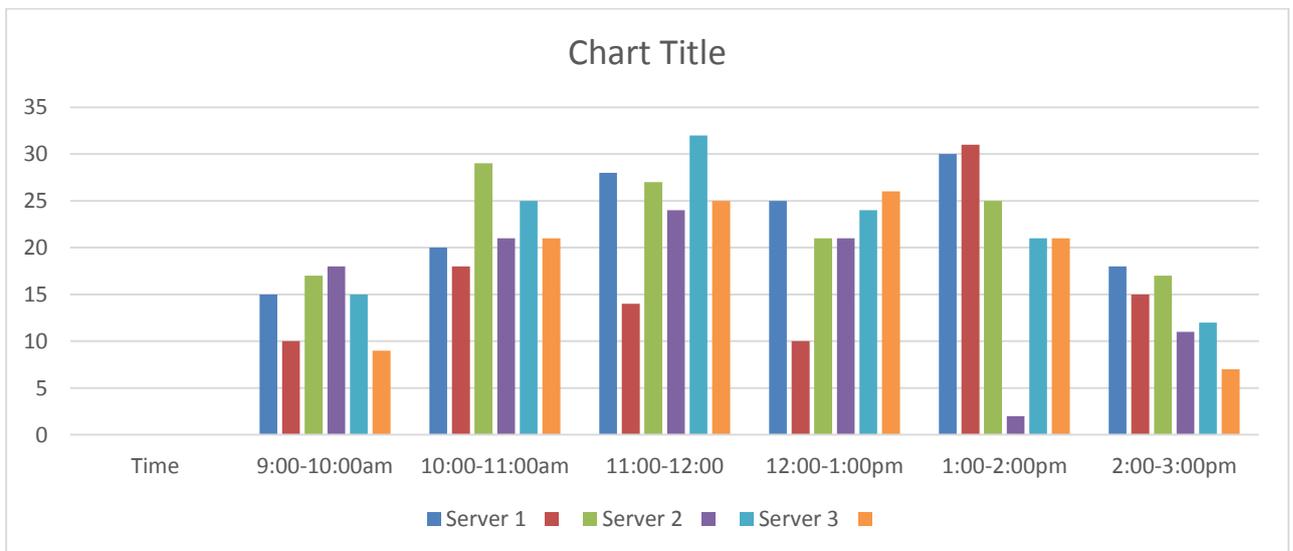


Table 3: Day (Three) 3 Queuing System Analysis of the Servers

Wednesday						
	Server 1		Server 2		Server 3	
Time	Arrival Rate	Service Rate	Arrival Rate	Service Rate	Arrival Rate	Service Rate
10:00-11:00am	10	8	12	110	14	10
11:00-12:00	15	14	24	17	21	217
12:00-1:00pm	21	18	24	21	28	24
1:00-2:00pm	24	21	19	18	21	18
2:00-3:00pm	26	27	24	20	20	17
3:00-4:00am	14	12	15	11	11	9

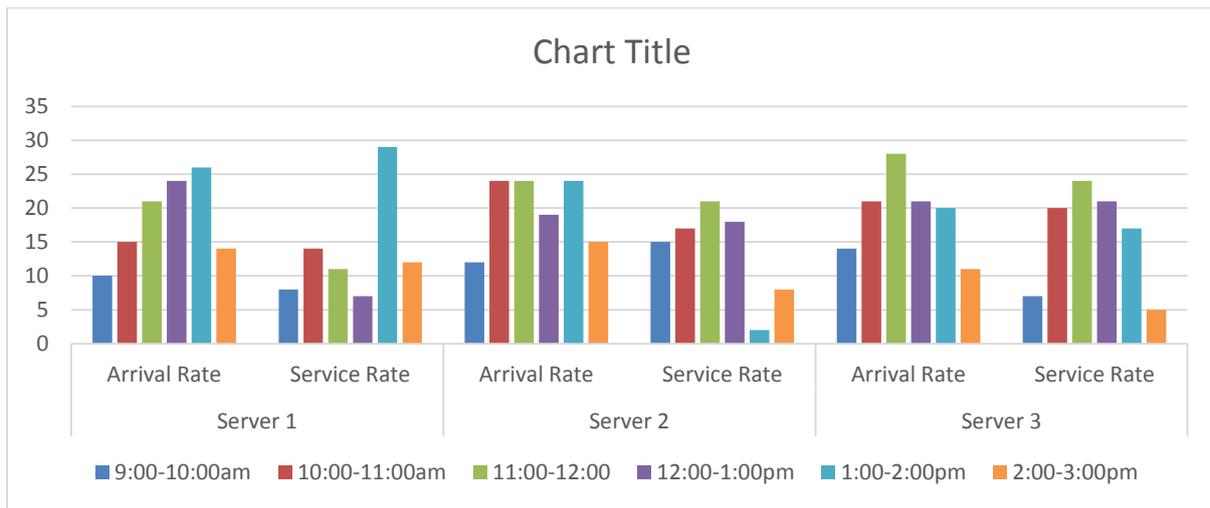


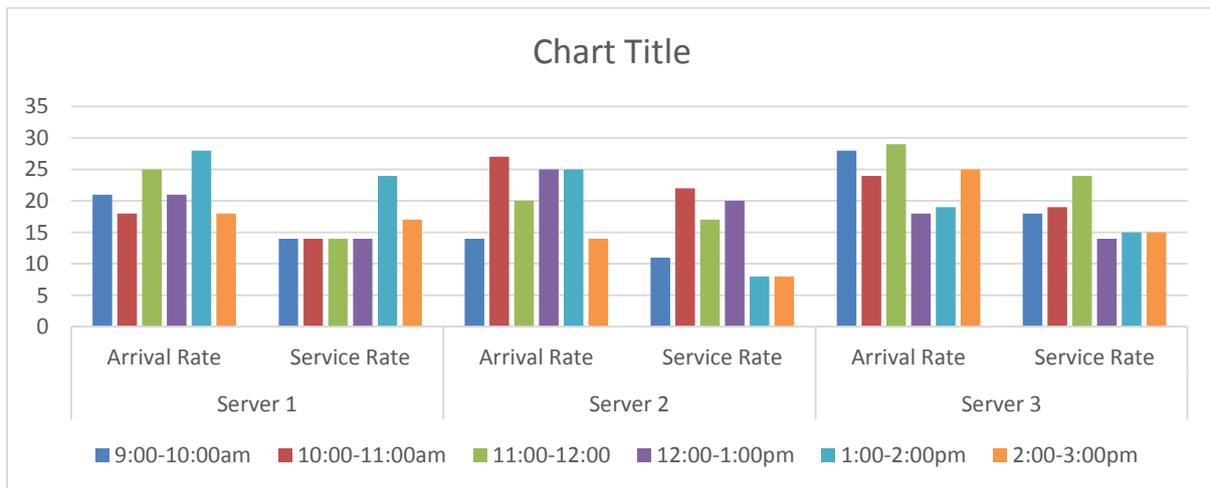
Table 4: Day (Four) 4 Queuing System Analysis of the Servers

Thursday						
	Server 1		Server 2		Server 3	
Time	Arrival Rate	Service Rate	Arrival Rate	Service Rate	Arrival Rate	Service Rate
10:00-11:00am	5	3	10	8	17	15
11:00-12:00	11	8	24	19	15	12
12:00-1:00pm	17	14	15	12	29	25
1:00-2:00pm	21	17	19	18	25	21
2:00-3:00pm	24	21	21	18	24	21
3:00-4:00am	12	10	14	11	14	11



Table 5: Day (Five) 5 Queuing System Analysis of the Servers

Friday						
	Server 1		Server 2		Server 3	
Time	Arrival Rate	Service Rate	Arrival Rate	Service Rate	Arrival Rate	Service Rate
10:00-11:00am	21	19	14	11	28	25
11:00-12:00	18	14	27	22	24	19
12:00-1:00pm	25	22	20	17	29	24
1:00-2:00pm	21	17	25	20	18	14
2:00-3:00pm	28	24	25	21	19	15
3:00-4:00am	18	17	14	12	25	21



Daily Queuing System Analysis of the Server

		Server 1		Server 2		Server 3	
		Arrival Rate	Service Rate	Arrival Rate	Service Rate	Arrival Rate	Service Rate
Day 1(Monday)	Total Arrival or Service Rate	82	65	102	84	118	101
	Average Arrival or Service Rate	16.4	13	20.4	16.8	23.6	20.2
Day 2(Tuesday)	Total Arrival or Service Rate	136	116	136	121	129	109

	Average Arrival or Service Rate	27.2	23.2	27.2	24.2	25.8	21.8
Day 3(Wednesday)	Total Arrival or Service Rate	110	100	118	197	115	295
	Average Arrival or Service Rate	22	20	23.6	39.4	23	59
Day 4(Thursday)	Total Arrival or Service Rate	90	73	103	86	124	105
	Average Arrival or Service Rate	18	14.6	20.6	17.2	24.8	21
Day 5(Friday)	Total Arrival or Service Rate	131	113	125	103	143	118
	Average Arrival or Service Rate	26.2	22.6	25	20.6	28.6	23.6
Total of week	Grand Total of Arrival or Service Rate	549	467	584	591	629	728
	Average of Grand Total of Arrival or Service Rate	109.8	93.4	116.8	118.2	125.8	145.6
	Average System Utilization Rate	1.175588865		0.988155668		0.864010989	

Table 7: Daily System Utilization for each Server

Daily Record	Server 1	Server 2	Server 3
Day 1	1.261538462	1.214285714	1.168316832
Day 2	1.172413793	1.123966942	1.183486239
Day 3	1.1	0.598984772	0.389830508
Day 4	1.232876712	1.197674419	1.180952381
Day 5	1.159292035	1.213592233	1.211864407

III. CONCLUSION

Receptionists with high volume outbound customer workloads and/or those who provide multiple points of service may benefit from queuing theory to reduce long lines and other problematic queuing systems, which can have a negative impact on customer satisfaction. When there is a shortage of a good or service, a line forms.

This study described the banking hall queuing system at Commercial Bank, Delhi Main Branch as Multiple-Channel queuing model with Poisson arrivals and Exponential services times. The economic analysis determined that adding up to 5 teller points would reduce the amount of time clients must wait in line and throughout the system by 98.78% and 87.85%, respectively.

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