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Integrated Vendor-Buyer Co-Operative Model For Deteriorating Items With Multivariate Demand And Salvage Value

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Abstract: *In the proposed model, we have developed a co-operative relationship between the vendor and the buyer when the expired stock has salvage value. Demand rate is taken to be selling price and stock dependent with constant deterioration and shortages are not allowed. The permissible credit period is used to motivate the buyer to co-operate in the joint inventory system.*

Keywords: ***Deterioration, Multivariate Demand, Salvage Value***.

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I. Introduction:

In most of the inventory models, the demand rates considered were either constant or dependent upon a single factor like stock level, time etc. However, changing market situations have rendered such a consideration quite fruitful. In realistic situation, demand cannot depend exclusively on a single parameter. A combination of two or more factors grants more authenticity to the formulation of the model. There exist many practical cases that force inventory managers to store more items in stock. In the competitive market situation customers are influenced by the marketing policies such as the attractive display of units in the market or the business place. In addition, for the retail organizations, price sensitive demand may lead the customers to buy more. Datta and Pal [5] presented an inventory model in which demand was influenced by stock level display, as well as the selling price of the item. Balkhi and Benkherouf [1] analyzed a stock dependent deteriorating model for finite planning horizon. Mahapatra and Maiti [8] set forth a study on multi-objective inventory model with stock and quality dependent demand. Hence, inventory models should be extended to the situation with multivariate demand.

Generally, it is assumed that the depletion of inventory is due to a constant demand rate. But, in real life situations, when the items of the commodity are kept in stock for fulfilling the future demand, there may be deterioration of items in the inventory system. The best way to dispose these items and avoid deterioration is to reduce the selling price when deterioration rate goes beyond a certain limit. Clark and Scarf [3] studied the vendor- buyer integration for the first time. Ghare and Schrader [6] analyzed the problem of decaying inventory and developed a simple EOQ model. Covert and Philip [4] extended Ghare and Schrader's [6] model by considering a two parameter Weibull distribution for variable rate of deterioration. In this study, we have developed a two-echelon supply chain, comprising a vendor and a buyer.

The whole method of business dealing has been assumed to be in accordance with progressive credit period, which conform to the practical market situation. A joint optimal inventory strategy for both the buyer and the vendor is considered when the expired stock has salvage value and are subject to constant rate of deterioration. There are many models in inventory which have no sale value after deterioration. They are considered lost. Salvage value is the estimated value each asset will have after it is no longer going to be used in the operation of a business. Then the vendor can offer his buyer reduced unit cost for the deteriorated stocks.

Banerjee [2] extended Clark and Scarf's [3] model by introducing finite replenishment rate. Raafat [9] gave survey of literature on continuously deteriorating inventory model. Goyal and Giri [7] gave up-to-date review of the research articles on deteriorating inventory.

Present chapter deals with a joint vendor- buyer inventory model for deteriorating items with salvage value where demand is a function of selling price and stock on display. Numerical examples are presented to demonstrate the developed model and to illustrate the procedure.

II. Assumptions And Notations:

1. The demand rate R(t) is taken to be selling price and stock dependent and is given by
$R(t) = a - bs + cq(t);$ a, b, c are positive constants and 's' is selling price per unit.

2. The deteriorating rate is constant $(0 < \theta \le 1)$ and is proportional to on hand inventory.
3. The system consists of single vendor and single buyer.
4. There is no repair or replenishment of deteriorated units.
5. The replenishment occurs instantaneously at an infinite rate.
6. Shortages are not allowed.
7. The model has been developed for a finite planning horizon.
8. Lead time is zero.
9. 'H' is holding cost per unit time for the buyer and 'F' is holding cost per unit time for the vendor.
10. 'C_b' is unit purchase cost for the buyer and 'C_v' is unit purchase cost for the vendor.
11. 'βC_b' is salvage value associated with deteriorated units for the buyer and 'βC_v' is salvage value associated with deteriorated units for the vendor.
12. 'η' is buyer's order time during [0, T].
13. '$q_b(t)$' is the inventory level for the buyer and '$q_v(t)$' is the inventory level for the vendor.
14. 'K_b' is ordering cost for buyer and 'K_v' is ordering cost for vendor.
15. 'T_b' is buyer's replenishment cycle time and 'Tv' is vendor's replenishment cycle time.
16. 'TC' is annual total cost for both the vendor and the buyer.

III. Mathematical Model:

The differential equations describing the instantaneous states of q(t) for both the vendor and the buyer at any time 't' are given by

$$\frac{dq_b(t)}{dt} + \theta q_b(t) = -\left[a - bs + cq_b(t)\right], \quad 0 \le t \le \frac{T}{\eta} \qquad \text{... (1)}$$

and $$\frac{dq_v(t)}{dt} + \theta q_v(t) = -\left[a - bs + cq_v(t)\right], \quad 0 \le t \le T \qquad \text{...(2)}$$

The solutions of the above differential equations after applying the boundary conditions $q_b\left(\frac{T}{\eta}\right) = 0 : q_v(T) = 0$

$q_b(t) = q_{mb}$ at $t = 0$ [Maximum inventory carried by the buyer]

$q_v(t) = q_{mv}$ at $t = 0$ [Maximum inventory carried by the vendor]

are

$$q_b(t) = \frac{(a - bs)}{(\theta + c)}\left[e^{(\theta+c)\left(\frac{T}{\eta} - t\right)} - 1\right]; \quad 0 \le t \le \frac{T}{\eta} \qquad \text{... (3)}$$

and

$$q_v(t) = \frac{(a - bs)}{(\theta + c)}\left[e^{(\theta+c)(T-t)} - 1\right]; \quad 0 \le t \le T \qquad \text{... (4)}$$

Hence, the maximum lot-sizes for the buyer and vendor are

$$q_{mb}(t) = \frac{(a - bs)}{(\theta + c)}\left[e^{(\theta+c)\frac{T}{\eta}} - 1\right] \qquad \text{... (5)}$$

$$q_{mv}(t) = \frac{(a - bs)}{(\theta + c)}\left[e^{(\theta+c)T} - 1\right] \qquad \text{... (6)}$$

respectively.

During [0, T], the total inventory holding cost for the buyer is

$$HC_b = \eta H \int_0^{T/\eta} q_b(t)\,dt$$

$$= \eta H \frac{(a-bs)}{(\theta+c)^2}\left[e^{(\theta+c)\frac{T}{\eta}} - (\theta+c)\frac{T}{\eta} - 1\right] \qquad \text{... (7)}$$

The actual vendor's holding cost in the interval [0, T] is

$$HC_v = F\left[\int_0^{T} q_v(t)\,dt - \eta \int_0^{T/\eta} q_b(t)\,dt\right]$$

$$HC_v = F\frac{(a-bs)}{(\theta+c)^2}\left[e^{(\theta+c)T} - 1 - \eta\left\{e^{(\theta+c)\frac{T}{\eta}} - 1\right\}\right] \qquad \text{... (8)}$$

In the interval [0, T], the deterioration cost for the buyer is

$$CD_b = \eta C_b\left[q_{mb}(t) - \left\{a - bs + c\,q_b(t)\right\}\frac{T}{\eta}\right]$$

$$= \eta C_b \frac{(a-bs)}{(\theta+c)}\left[e^{(\theta+c)\frac{T}{\eta}} - 1 - c\,\frac{T}{\eta}\left\{e^{(\theta+c)\left(\frac{T}{\eta}-t\right)} - 1\right\} - \frac{T}{\eta}(\theta+c)\right] \qquad \text{... (9)}$$

And for the vendor is

$$CD_v = C_v\left[q_{mv}(t) - \left\{a - bs + c\,q_v(t)\right\}T - \eta\left\{q_{mb}(t) - \left(a - bs + c\,q_b(t)\right)\frac{T}{\eta}\right\}\right]$$

$$= C_v\frac{(a-bs)}{(\theta+c)}\left[e^{(\theta+c)T} - 1 - cT\left\{e^{(\theta+c)(T-t)} - 1\right\} - \eta\left\{e^{(\theta+c)\frac{T}{\eta}} - 1 - \frac{cT}{\eta}\left(e^{(\theta+c)\left(\frac{T}{\eta}-t\right)} - 1\right)\right\}\right] \qquad \text{... (10)}$$

The salvage value of the deteriorated units for the buyer in the interval [0, T] is

$$SV_b = \eta\beta C_b \frac{(a-bs)}{(\theta+c)}\left[e^{(\theta+c)\frac{T}{\eta}} - 1 - c\frac{T}{\eta}\left\{e^{(\theta+c)(\frac{T}{\eta}-t)} - 1\right\} - \frac{T}{\eta}(\theta+c)\right] \qquad \text{... (11)}$$

And for the vendor is

$$SV_v = \beta C_v\frac{(a-bs)}{(\theta+c)}\left[e^{(\theta+c)T} - 1 - cT\left\{e^{(\theta+c)(T-t)} - 1\right\} - \eta\left\{e^{(\theta+c)\frac{T}{\eta}} - 1 - \frac{cT}{\eta}\left(e^{(\theta+c)\left(\frac{T}{\eta}-t\right)} - 1\right)\right\}\right] \qquad \text{... (12)}$$

The ordering cost for the buyer is

$$OC_b = \eta K_b \qquad \text{... (13)}$$

And for the vendor is

$$OC_v = K_v \qquad \text{... (14)}$$

The buyer's total cost per unit time is given by

$$TC_b = \frac{1}{T}\left[HC_b + CD_b + OC_b - SV_b\right] \qquad \text{... (15)}$$

And the vendor's total cost per unit time is given by

$$TC_v = \frac{1}{T}\left[HC_v + CD_v + OC_v - SV_v\right] \qquad \text{... (16)}$$

The annual joint total cost; TC is the sum of TC_b and TC_v. Since, $T_b = \frac{T}{\eta}$, TC is a function of continuous variable: T_b and discrete variable: η.

IV. Numerical Example:

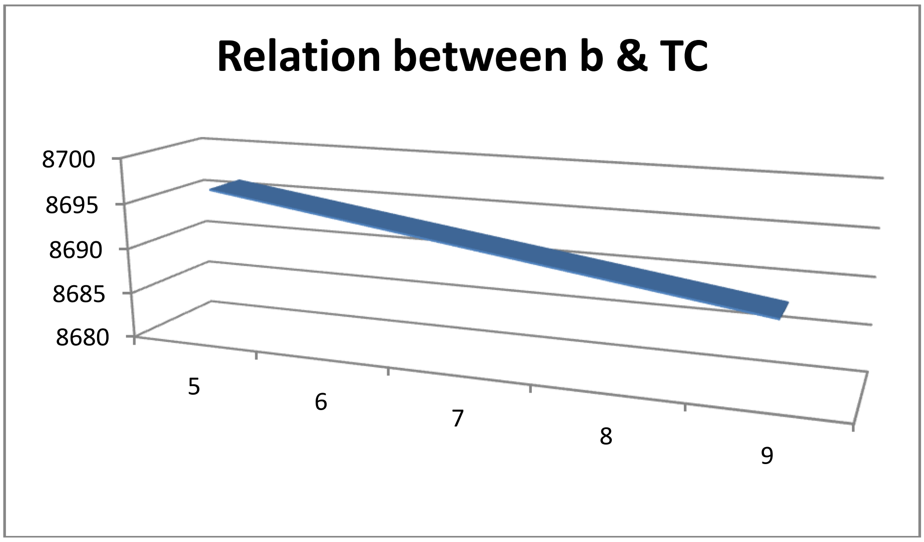
To illustrate the model numerically, the following parametric values are considered:
H=1.34, a=1200, b=7, c=0.50, s=0.50, F=1, θ=0.60, C_b=20, K_b=600, β=0.30, K_v=3000, C_v=10.

Cases	without joint decision	with joint decision
η	1.1	3
T_b	0.6182	0.245385
T	0.68002	0.736155
TC_b	5533.909	3719.065
TC_v	3158.115	3705.043
TC	8692.025	7424.108

Case I: When the buyer and the vendor make decision independently

Table 1: **Relation between 'b' and 'TC'**

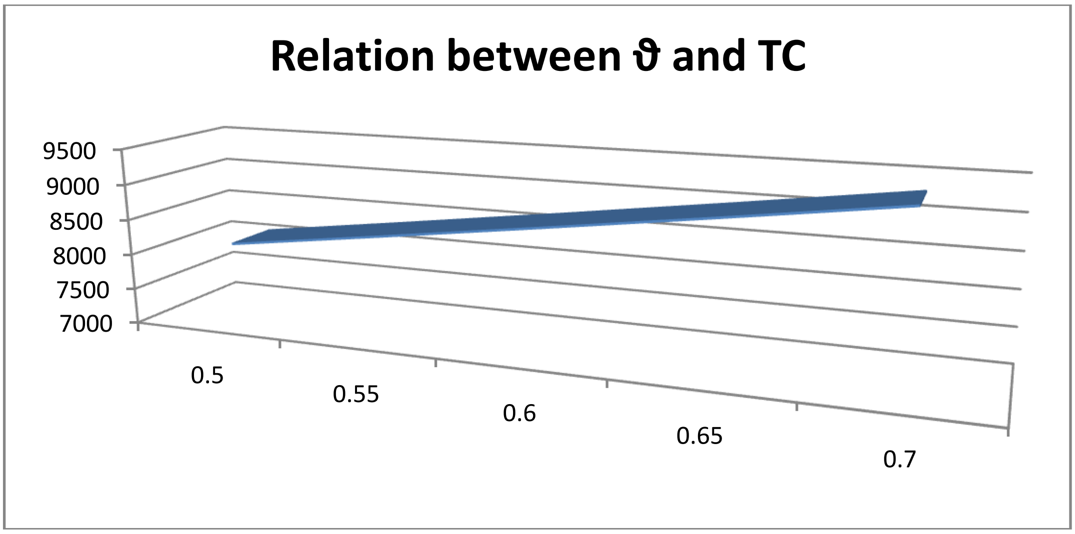
b	TC_b	TC_v	TC
5	5537.983	3158.248	8696.23
6	5535.946	3158.182	8694.128
7	5533.909	3158.115	8692.025
8	5531.873	3158.049	8689.922
9	5529.836	3157.983	8687.819



When 'b' increases, then the total cost for both the buyer and vendor decreases and total annual cost also decreases.

Table 2: **Relation between 'θ' and 'TC'**

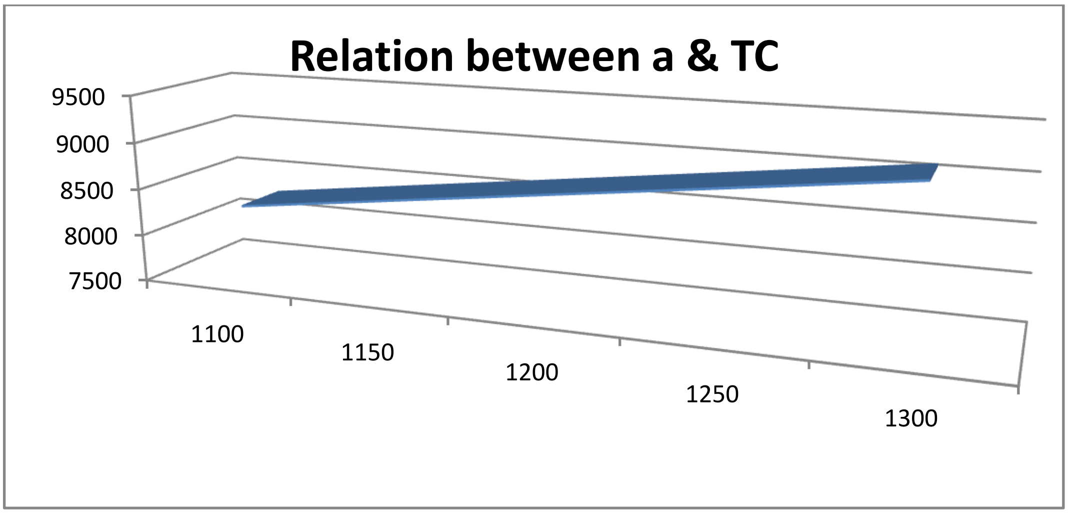
θ	TC_b	TC_v	TC
0.5	4977.145	3116.942	8094.087
0.55	5252.514	3137.115	8389.63
0.6	5533.909	3158.115	8692.025
0.65	5821.475	3179.972	9001.448
0.7	6115.362	3202.716	9318.078



When 'θ' increases, then the total cost for both the buyer and vendor increases and total annual cost also increases.

Table 3: Relation between 'a' and 'TC'

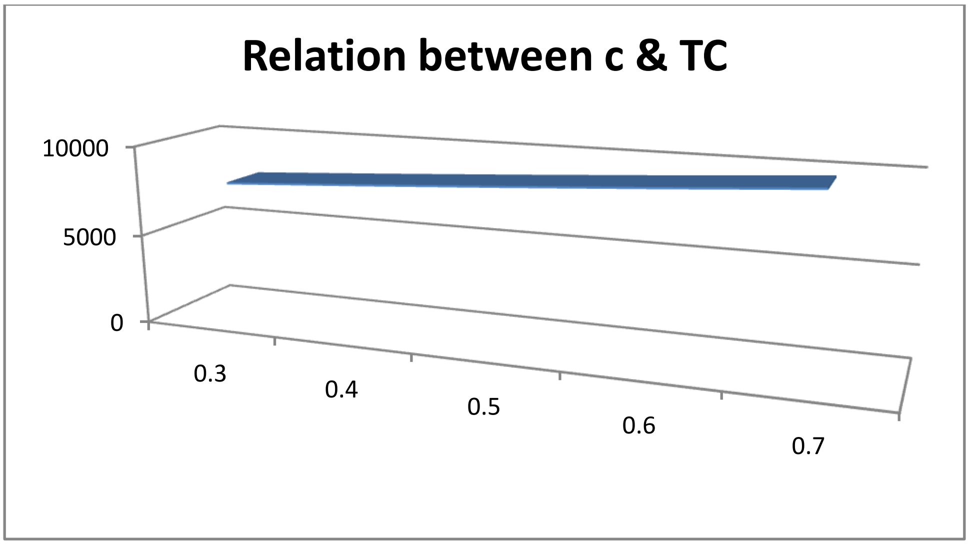
a	TC_b	TC_v	TC
1100	5126.562	3144.901	8271.463
1150	5330.236	3151.508	8481.744
1200	5533.909	3158.115	8692.025
1250	5737.583	3164.723	8902.306
1300	5941.257	3171.33	9112.587



When 'a' increases, then the total cost for both the buyer and vendor increases and total annual cost also increases.

Table 4: Relation between 'c' and 'TC'

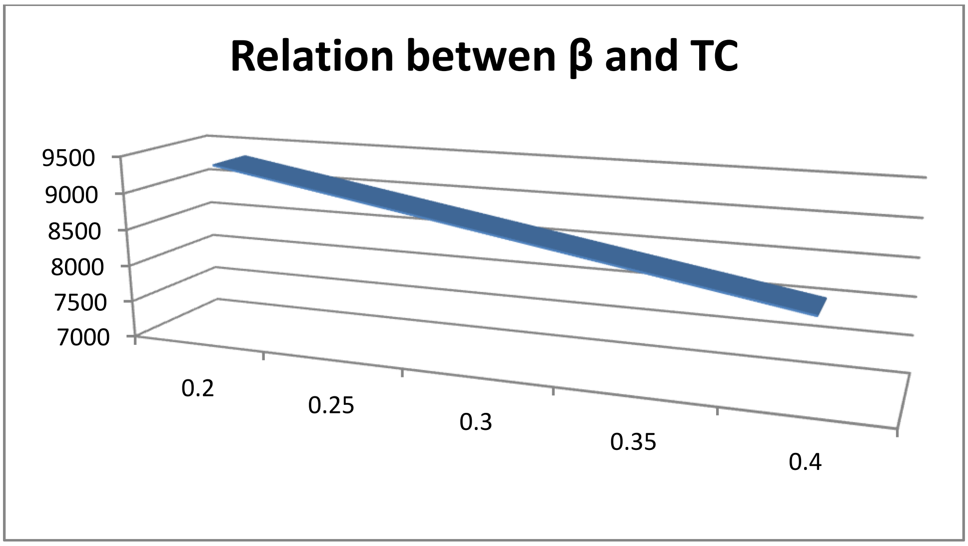
c	TC_b	TC_v	TC
0.3	4633.387	3156.797	7790.184
0.4	5072.264	3156.296	8228.56
0.5	5533.909	3158.115	8692.025
0.6	6019.465	3162.475	9181.941
0.7	6530.135	3169.611	9699.746



When 'c' increases, then the total cost for both the buyer and vendor increases and total annual cost also increases.

Table 5: Relation between 'β' and 'TC'

β	TC_b	TC_v	TC
0.2	6168.986	3174.84	9343.826
0.25	5851.448	3166.478	9017.925
0.3	5533.909	3158.115	8692.025
0.35	5216.371	3149.753	8366.124
0.4	4898.832	3141.391	8040.224

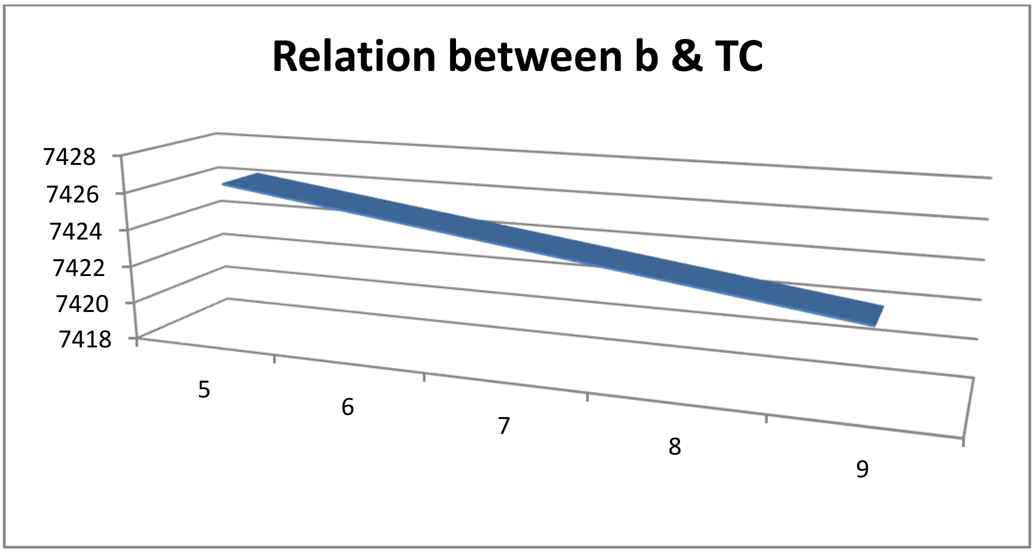


When 'β' increases, then the total cost for both the buyer and vendor decreases and total annual cost also decreases.

Case II: When the buyer and the vendor make decision jointly.

Table 1: **Relation between 'b' and 'TC'**

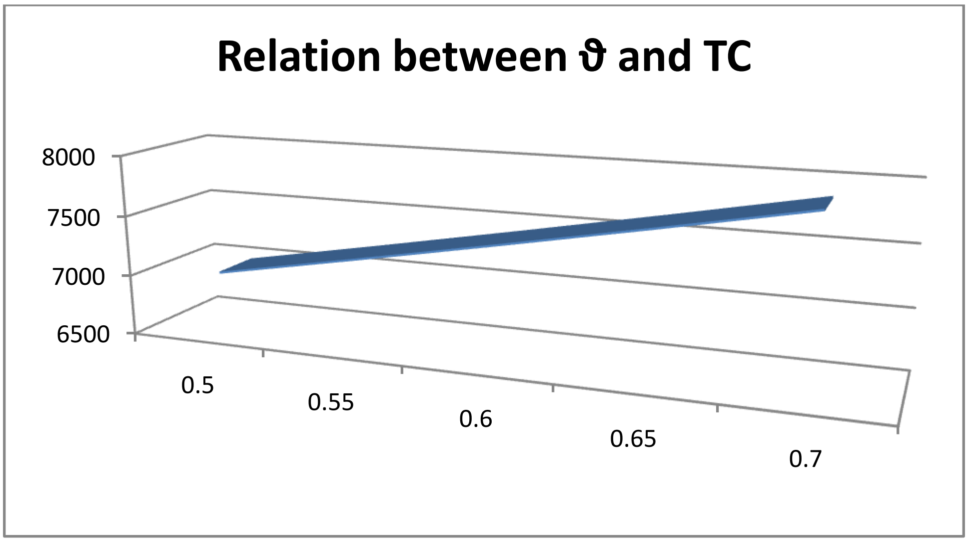
b	TC_b	TC_v	TC
5	3720.669	3705.632	7426.301
6	3719.867	3705.337	7425.204
7	3719.065	3705.043	7424.108
8	3718.263	3704.748	7423.011
9	3717.461	3704.454	7421.915



When ‘b’ increases, then the total cost for both the buyer and vendor decreases and total annual cost also decreases.

Table 2: **Relation between ‘θ’ and ‘TC’**

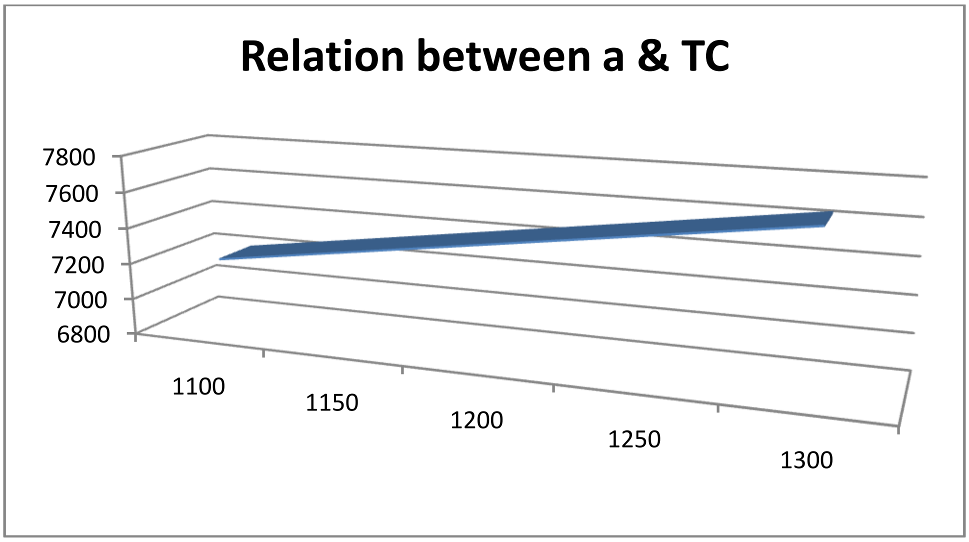
θ	TC_b	TC_v	TC
0.5	3537.766	3453.416	6991.182
0.55	3628.038	3577.088	7205.126
0.6	3719.047	3705.034	7424.081
0.65	3810.836	3837.404	7648.241
0.7	3903.395	3974.333	7877.728



When ‘θ’ increases, then the total cost for both the buyer and vendor increases and total annual cost also increases.

Table 3: Relation between 'a' and 'TC'

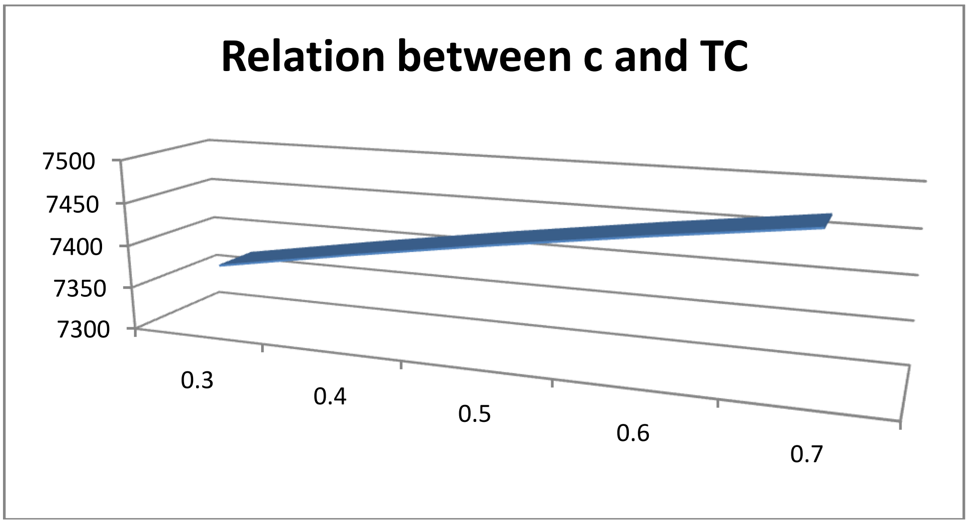
a	TC_b	TC_v	TC
1100	3558.675	3646.117	7204.792
1150	3638.87	3675.58	7314.45
1200	3719.065	3705.043	7424.108
1250	3799.26	3734.506	7533.765
1300	3879.455	3763.968	7643.423



When 'a' increases, then the total cost for both the buyer and vendor increases and total annual cost also increases.

Table 4: **Relation between 'c' and 'TC'**

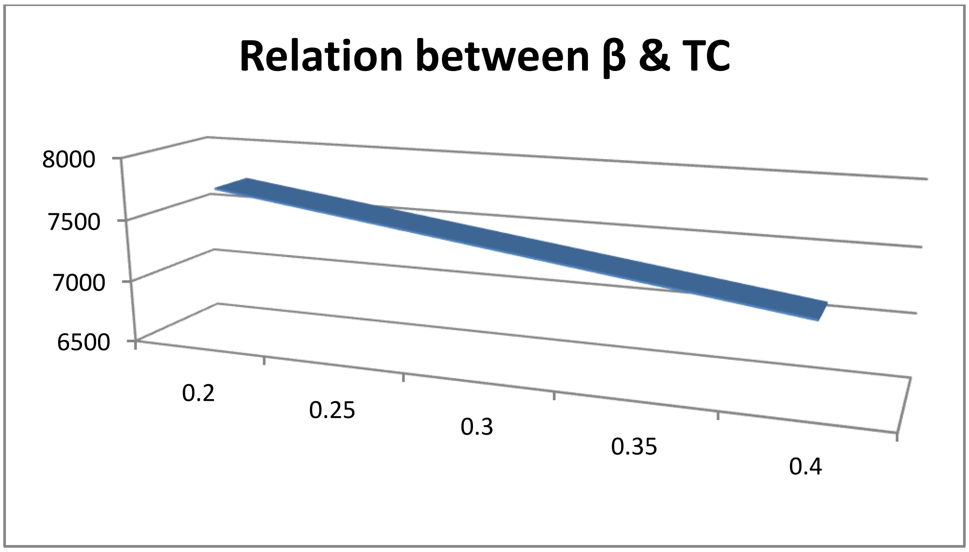
c	TC_b	TC_v	TC
0.3	3385.2	3986.449	7371.649
0.4	3550.641	3848.141	7398.782
0.5	3719.065	3705.043	7424.108
0.6	3890.527	3557.075	7447.602
0.7	4065.085	3404.164	7469.249



When ‘c’ increases, then the total cost for the buyer increases and it decreases for the vendor and total annual cost also increases.

Table 5: **Relation between ‘β’ and ‘TC’**

β	TC_b	TC_v	TC
0.2	3970.535	3760.869	7731.404
0.25	3844.8	3732.956	7577.756
0.3	3719.065	3705.043	7424.108
0.35	3593.33	3677.13	7270.459
0.4	3467.595	3649.217	7116.811



When ‘β’ increases, then the total cost for both the buyer and vendor decreases and total annual cost also decreases.

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