IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 18, Issue 6 Ser. II (Nov. – Dec. 2022), PP 39-45
www.iosrjournals.org

On The Mathematical Model of the Biomechanics of Green Plants (Part 2)

Uka N. O[1]*, Abbey,T. M[2], Alex,M[3]
[1,3]Department of Mathematics and Statistics, University of Port-Harcourt, Nigeria
[2]*Department of Physics, University of Port-Harcourt, Nigeria*

Abstract
In this study, the effects of aspect ratio, porosity, buoyancy forces (thermal and concentration Grashof numbers) and Schmidt number on the flow of soil mineral salt water in the xylem of a non-bifurcating green plant are examinedon the velocity and temperature fields. The coupled non-linear differential equations governing the motion of the flow are non-dimensionlized and then solved using the homotopy perturbation method. It was discovered from the analysis that;the velocity and temperature of flow fields increases as the porosity of the fluid carrying vessel and aspect ratio increases;increase in the Schmidt number and buoyancy forcesresulted to a corresponding increase in the flow velocity and thus enhancing the effective growth and yield of the plant.

Keywords: *Biomechanics, xylem flow, homotopy perturbation method (HPM)*

Date of Submission: 05-12-2022 Date of Acceptance: 17-12-2022

I. Introduction

Green plants are among the most successful organism on earth in terms of biomass and individual size range Jensen, *et al.*, [10]. They are the earth's primary solar energy collectors and are the ultimate source of food for both man and animals. Rand, [24].

The mechanics of green plants when compared to that of animals is open. In the sense that it consists of the roots, stems and leaves. The roost absorbs water and minerals from the soil and transports it through the stem and then to the leaves. The stem consists basically two vascular tissues, namely xylem and phloem Dolger *et al.*, [5]; Yu, [29]. The xylem is made up of the tracheids and vessel elements that die after reaching maturity while the phloem contains sieve elements that still lives after maturity Rand, [24]. The water and minerals that enters the stem are transported upwards through the xylem and then to the leaves through its petiole. Most of the water leaving the xylem (tracheary elements) moves into the leaf mesophyll (sieve elements) and then evaporates into the atmosphere through the stomata (this process is called transpiration). Carbon dioxide enters the leaves from the atmosphere through the stomata by diffusion and then combines with part of the water that entered the mesophyll cells in the presence of sunlight to form carbohydrate by the process of photosynthesis. The carbohydrate produced is pushed downward with the aid of water into the phloem vessel and then translocated downwards to the fruits, shoots and roots where they are needed (this process is known as translocation). Bestman, [1]; Jensen, *et al.*, [2]; Rand, [24]; Kizilova, [7].

The force that drives the upward flow in the xylem is enhanced by suction pressure generated in the leaves by evaporation of water vapor into the atmosphere Jensen, *et al.*, [9] and the environmental thermal differences resulting from free convective motion of the fluid Okuyade and Abbey, [15]. The downward phloem on the other hand is driven by concentration differences resulting from active transport Rand, [24].

From literature, it is observed that fluid carrying vessels of green plants are porous and the flow naturally convective. Studies have also shown that flow through porous channels are affected by certain parameters such as, thermal gradient, concentration gradient, pressure gradient(for example suction and root pressure), the porosity, permeability, the physical properties of the fluid(for example, viscosity, density), body forces(for example, gravity, magnetic field, buoyancy force)and bifurcation (Muskat, [14]; Zami-Pierre, et al., [30]; El-dabe, *et al.*, [6]; Okuyade and Abbey, [16]) etc. Several methods such as Laplace transform, perturbation, direct numerical simulations have also been used to examine the effects of these parameters on the flow structure.

The effect of increasing values of the porosity at a small value of the aspect ratio were examined on the concentration field of a fully developed flow for the phloem and xylem of a green plant by Bestman, [1] using Laplace transform method. Bestman, [2] went further to consider the case where the flow is not fully developed for larger value of the aspect ratio using perturbation and finite Fourier sine and cosine technique.Hoad, [8] studied translocation of hormones in the phloem of higher plans. Problems associated with collection of sieve tube exudates and the analysis of samples were discussed. More so, possible functions of hormones were investigated. From the study, it was established that mobile hormones played a part in controlling the structure of the plant as their concentration in sieve tube have been shown to be influenced by the environment and developmental stage of the plant. Peuke *et al.,* [20] used the nuclear magnetic resonance (NMR) spectrometry to study the measurement of rates of flow in xylem and phloem. The effect of light regime on water flow on xylem phloem was monitored using this same method. Rengel, [26] studied the transport of micronutrients (manganese and zinc) from leaves to roots, leaves and stems to developing grains and then from one root to another in the xylem and phloem of a developing plant species. Result showed that zinc solute was more mobile in phloem while manganese solute had poor mobility in the phloem and therefore occurred mainly on the xylem. Pitterman, [21] studied the evolution of plant vascular system, highlighted the recent developments that contributed to a better understanding of the xylem evolution, discussed the functions of vascular structure in terms of support, drought and freeze-thaw stress resistance and also discussed in details the impacts of plant transport on hydrology and climate. Cabrita, [3] investigated the magnitude of radial fluxes in the stem (that is, water and solute exchanges along the long pathway) and what controlled them using experiment and theory. A steady state model of phloem transport was constructed using the Navier-Stokes and convection-diffusion equations. It was observed from the model that radial water exchange affects the pressure gradient and solute exchange which depends on the permeability of the phloem also affects the pressure gradient.Rand and Cooke, [23] studied flow through sieve tubes with sieve plates in the phloem of plants using an idealized single-pore axisymmetrical model. Rand *et al.*, [25] also studied the flow using an approximate formula. Jensen *et al.,* [10] presented an experimental and theoretical study of transient osmotically driven flows through pipes with semi-permeable walls. Cabrita *et al.,* [4] studied the transport phloem which allows leakage of solute of a steady state model. The sieve tube membrane permeability strongly influenced the results of the model. Payvandi, *et al.,* [19] studied the transport of water and nutrient in xylem vessels of a wheat plant. Solution to the transport of the nutrient was obtained considering convection and diffusion.Uka and Olisa, [28] studied the transport of sap in the stem of a of a non-bifurcating green plant using the homotopy perturbation method. The effects of varying values of the aspect ratio, porosity parameter, buoyancy forces and Schmidt number were examined on the concentration flow field. Prakash *et al.,* [22]. Okuyade, [18] studied MHD blood flow through bifurcated porous fine capillaries of humans using perturbation method. Effects of magnetic field and environmental thermal parameters on the flow structure were examined. Tadjfar and Smith [27] examined the effects of bifurcation angle on a 3-dimensional laminar steady flow of an incompressible viscous fluid through a straight mother tube bifurcating into two straight but divergent daughter tubes by direct numerical simulations. Liou, *et al.,* [13] studied the effect of bifurcation angles on the steady flow structure in a straight terminal aneurysm model with asymmetric outflow through the branches using the Laser-Doppler velocity and fluctuating intensity distribution. Okuyade and Abbey, [16] studied a steady MHD fluid flow in a bifurcating rectangular porous medium using perturbation method. The effects of bifurcation angle, magnetic field, thermal and concentration Grashof numbers on the flow were examined. This study will however consider the effects of increasing values of the porosity, aspect ratio, Schmidt number, buoyancy forces (thermal and concentration Grashof number) on the velocity and temperature fields on the steady flow of sap in a non-bifurcating green plant using the homotopy perturbation method.

II. Materials And Method

The sap in green plants is well describe as viscous, incompressible, Newtonian liquid. The liquid carrying vessels are cylindrical and porous in nature and the flow itself naturally convective since they are not driven by any physical means. It is assumed that the flow is creepy with a very low Reynolds number. The velocity vectors with respect to the orthogonal coordinate directions (r',θ',z') are (u',v',w'). Assuming also that the flow is fully developed and the velocity is symmetrical about the θ' axis such that the variations about θ' is zero. The coordinate and velocity vectors becomes (r',z') and (u',w') respectively. By the usual Boussinesq approximation, the mathematical models describing the motion of the flow in cylindrical coordinate for the steady case can be written as

$$\frac{1}{r'}\frac{\partial}{\partial r'}(r'u') + \frac{\partial w'}{\partial z'} = 0, \qquad (1)$$

$$0 = -\frac{\partial p'}{\partial r'} + v\left(\frac{\partial^2 u'}{\partial r'^2} + \frac{1}{r'}\frac{\partial u'}{\partial r'} - \frac{u'}{r'^2} + \frac{\partial^2 u'}{\partial z'^2}\right) - \frac{v}{K}u', \qquad (2)$$

$$0 = -\frac{\partial p'}{\partial z'} + v\left(\frac{\partial^2 w'}{\partial r'^2} + \frac{1}{r'}\frac{\partial w'}{\partial r'} + \frac{\partial^2 w'}{\partial z'^2}\right) - \frac{v}{K}w' + \rho g\beta_t(T - T_\infty) + \rho g\beta_c(C' - C_\infty), \qquad (3)$$

$$\rho C_p\left(u'\frac{\partial T}{\partial r'} + w'\frac{\partial T}{\partial z'}\right) = \alpha\left(\frac{\partial^2 T}{\partial r'^2} + \frac{1}{r'}\frac{\partial T}{\partial r'} + \frac{\partial^2 T}{\partial z'^2}\right), \qquad (4)$$

$$\left(u'\frac{\partial C'}{\partial r'} + w'\frac{\partial C'}{\partial z'}\right) = D\left(\frac{\partial^2 C'}{\partial r'^2} + \frac{1}{r}\frac{\partial C'}{\partial r'} + \frac{\partial^2 C'}{\partial z'^2}\right), \qquad (5)$$

where p is the pressure, v is the viscosity, $T\ and\ C'$ are the fluid temperature and concentration respectively, $T_\infty\ and\ C_\infty$ are the temperature and concentration at equilibrium, K is the permeability, g is the gravitation which acts in opposite direction to the flow, ρ is the fluid density, $\beta_t\ and\ \beta_c$ are the coefficient of volume expansion for temperature and concentration respectively, C_p is the heat capacity, α is the thermal conductivity,k_0 is the thermal diffusivity and D is the mass diffusion coefficient.

$(u', w')(0,z) = 1\,;\quad T(0,z) = T_\infty C'(0,z) = C_\infty \qquad \text{at } r' = 0,$

$(u', w')(1,z) = 0;\quad T(1,z) = T_w C'(1,z) = C_w \qquad \text{at } r' = 1.$ (6)

The following non-dimensional quantities are used to normalize the governing equations.

$$r = \frac{r'}{r_0};\ z = \frac{z'}{l};\ (u,w) = (u',w')\frac{r_0}{v};\ \theta = \frac{T - T_\infty}{T_w - T_\infty};\ \emptyset = \frac{C' - C_\infty}{C_w - C_\infty};p = \frac{r_0^3(p' - p_\infty)}{\rho_\infty l v^2};\ R = \frac{r_0}{l},$$

$$x^2 = \frac{r_0}{\sqrt{K}};\ Gr = \frac{g\beta_t(T_w - T_\infty)r_0^3}{v^2}\ ;\qquad Gc = \frac{g\beta_c(C_w - C_\infty)r_0^3}{v^2}\ ;\ Sc = \frac{v}{D};Pr = \frac{v}{k_0}\ , k_0 = \frac{\alpha}{\rho C_\rho}. \tag{7}$$

Thus, the normalized governing equations are;

$$\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial z} = 0, \tag{8}$$

$$0 = -\frac{\partial p}{\partial r} + \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} - x^2 u - R^2\frac{\partial^2 u}{\partial z^2}, \tag{9}$$

$$0 = -R\frac{\partial p}{\partial z} + \frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} - x^2 w + R^2\frac{\partial^2 w}{\partial z^2} + Gr\theta + Gc\emptyset, \tag{10}$$

$$Pr\left(u\frac{\partial\theta}{\partial r} + Rw\frac{\partial\theta}{\partial z}\right) = \frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r} + R^2\frac{\partial^2\theta}{\partial z^2}, \tag{11}$$

$$Sc\left(u\frac{\partial\emptyset}{\partial r} + Rw\frac{\partial\emptyset}{\partial z}\right) = \frac{\partial^2\emptyset}{\partial r^2} + \frac{1}{r}\frac{\partial\emptyset}{\partial r} + R^2\frac{\partial^2\emptyset}{\partial z^2}. \tag{12}$$

For convenience, we assume a solution of the form

$$\theta^{(0)} = \theta(r) - \gamma z,\ \emptyset = \emptyset^{(0)}(r) - \gamma z,\ p = Kz - \frac{\gamma}{R}z^2\,, \tag{13}$$

as given by Bestman, [1]. Substituting the assumed solution (13) into equations (8) - (12), we have

$$K = w'' + \frac{1}{r}w' - x^2 w + Gr\theta^{(0)} + Gc\emptyset^{(0)}, \tag{14}$$

$$-P_r R\gamma w = \theta^{(0)''} + \frac{1}{r}\theta^{(0)'}, \tag{15}$$

$$-S_c R\gamma w = \emptyset^{(0)''} + \frac{1}{r}\emptyset^{(0)'}, \tag{16}$$

where γ is a constant, R is the aspect ratio, x is the porosity parameter, Gr is the thermal Grashof number, Gc is the concentration Grashof number, Sc is the Schmidt number, Pr is the Prandtl number. The transformed boundary conditions are;

$$\emptyset^{(0)}(0) = 1;\qquad \emptyset^{(0)}(1) = 0,$$
$$w(0) = 1;\qquad w(1) = 1,$$
$$\theta^{(0)}(0) = 1;\qquad \theta^{(0)}(1) = 1\,. \tag{17}$$

According to the HPM of He, [7] , the homotopy form of (14), (15) and (16) are constructed as follows

$$(1-p)\{w''\} + p\left[w'' + \frac{1}{r}w' - x^2 w + Gr\theta^{(0)} + Gc\emptyset^{(0)} - K\right] = 0, \tag{18}$$

$$(1-p)\left[\theta^{(0)''}\right] + p\left[\theta^{(0)''} + \frac{1}{r}\theta^{(0)'} + P_r R\gamma w\right] = 0, \tag{19}$$

$$(1-p)\left[\emptyset^{(0)''}\right] + p\left[\emptyset^{(0)''} + \frac{1}{r}\emptyset^{(0)'} + S_c R\gamma w\right] = 0. \tag{20}$$

We assume w, $\theta^{(0)}$ and $\emptyset^{(0)}$ as

$$w = w_0 + pw_1 + p^2 w_2 + \cdots, \tag{21}$$

$$\theta^{(0)} = \theta^{(0)}{}_0 + p\theta^{(0)}{}_1 + p^2\theta^{(0)}{}_2 + \cdots, \tag{22}$$

$$\emptyset^{(0)} = \emptyset^{(0)}{}_0 + p\emptyset^{(0)}{}_1 + p^2\emptyset^{(0)}{}_2 + \cdots, \tag{23}$$

Substituting equations (21) – (23) into equation (18) and simplifying, we have

$$w_0'' + pw_1'' + p^2 w_2'' + p\left[\frac{1}{r}\left(w_0' + pw_1' + p^2 w_2'\right) - x^2(w_0 + pw_1 + p^2 w_2) + Gr\left(\theta^{(0)}{}_0 + p\theta^{(0)}{}_1 + p^2\theta^{(0)}{}_2\right) + \right.$$
GcØ00+pØ01+p2Ø02−K=0. (24)

Substituting equation (21) and (23) into equation (19) and simplifying, we have

$$\theta^{(0)''}{}_0 + p\theta^{(0)''}{}_1 + p^2\theta^{(0)''}{}_2 + p\left[\frac{1}{r}\left(\theta^{(0)'}{}_0 + p\theta^{(0)'}{}_1 + p^2\theta^{(0)'}{}_2\right) + PrR\gamma(w_0 + pw_1 + p^2 w_2)\right] = 0. \tag{25}$$

Substituting equations (21) and (23) into equation (20) and simplifying, we have

$$\emptyset^{(0)''}{}_0 + p\emptyset^{(0)''}{}_1 + p^2\emptyset^{(0)''}{}_2 + p\left[\frac{1}{r}\left(\emptyset^{(0)'}{}_0 + p\emptyset^{(0)'}{}_1 + p^2\emptyset^{(0)'}{}_2\right) + ScR\gamma(w_0 + pw_1 + p^2 w_2)\right] = 0. \tag{26}$$

Rearranging equations (24) - (26) based on the powers of p-terms together with its boundary conditions, we have

$$p^0;\quad w_0'' = 0 \qquad ;\qquad w_0(0) = 1;\ w_0(1) = 0,$$
$$\theta^{(0)''}{}_0 = 0\ \theta^{(0)}{}_0(0) = 1; \theta^{(0)}{}_0(1) = 1,$$

$\emptyset^{(0)}{}''_0 = 0\emptyset^{(0)}{}_0(0) = 1; \emptyset^{(0)}{}_0(1) = 1,$ (27)

$P^1;\quad w_1'' = -\frac{1}{r}w_0' + x^2 w_0 - Gr\theta^{(0)}{}_0 - Gc\emptyset^{(0)}{}_0 + K \quad ; \quad w_1(0) = 0;\; w_1(0) = 0,$

$\theta^{(0)}{}''_1 = -\frac{1}{r}\theta^{(0)}{}'_0 - PrR\gamma w_0 \theta^{(0)}{}_1(0) = 0; \theta^{(0)}{}_1(1) = 0,$

$\emptyset^{(0)}{}''_1 = -\frac{1}{r}\emptyset^{(0)}{}'_0 - ScR\gamma w_0 \emptyset^{(0)}{}_0(0) = 0; \emptyset^{(0)}{}_1(1) = 0,$ (28)

$P^2;\; w_2''' = -\frac{1}{r}w_1' + x^2 f_1 - Gr\theta^{(0)}{}_1 - Gc\emptyset^{(0)}{}_1 \quad ; \quad w_2(0) = 0;\; w_0(1) = 0,$

$\theta^{(0)}{}''_2 = -\frac{1}{r}\theta^{(0)}{}'_1 - PrR\gamma w_1 \theta^{(0)}{}_2(0) = 0; \theta^{(0)}{}_2(1) = 0,$

$\emptyset^{(0)}{}''_2 = -\frac{1}{r}\emptyset^{(0)}{}'_1 - ScR\gamma w_1 \emptyset^{(0)}{}_2(0) = 0; \emptyset^{(0)}{}_2(1) = 0,$ (29)

Solving (27) - (29) we have

$w_0 = A1r + A2,$ (30)

$w_1 = -A1(r\,In\,r - r) + \frac{1}{6}(x^2A1 - GrB1 - GcD1)r^3 + \frac{1}{2}(x^2A2 - GrB2 - GcD2 + K)r^2 + E1r + E2,$ (31)

$w_2 = \frac{A1}{2}(r(In\,r)^2 - In\,r) - E1(r\,In\,r - r) - \frac{x^2A1}{2}\left(\frac{r^3In\,r}{3} - \frac{r^3}{9}\right) + \frac{Gr\,B1}{2}\left(\frac{r^3In\,r}{3} - \frac{r^3}{9}\right) + \frac{1}{120}(x^4A1 - x^2GrB1 -$
x2GcD1+GrPrRγA1+GcScRγA1r5+124x4A2−x2GrB2−x2GcD2+x2K+GrPrRγA2+GcScRγA2r4+16x2 A13−x2A1+x2E1−GrB13−GrB1−GrE3−GcD13−GcD1−GcE5r3−12x2A2−GrB2−GcD2−x2E2+GrE4+GcE6+Kr2+E11r+E12, (32)

$\theta^{(0)}{}_0 = B1r + B2,$ (33)

$\theta^{(0)}{}_1 = -B1(r\,In\,r - r) - \frac{PrR\gamma A1}{6}r^3 - \frac{PrR\gamma A2}{2}r^2 + E3r + E4,$ (34)

$\theta^{(0)}{}_2 = \frac{B1}{2}(r(In\,r)^2 - In\,r) - E3(r\,In\,r - r) + \frac{PrR\gamma A1}{2}\left(\frac{r^3In\,r}{3} - \frac{r^3}{9}\right) - \frac{1}{120}(PrR\gamma x^2A1 - PrR\gamma GrB1 -$
PrRγGcD1r5−124PrRγx2A2−PrRγGrB2−PrRγGcD2r4−16PrRγA1+PrRγE1r3+12PrRγA2+PrRγE2r2 +E7r+E8, (35)

$\emptyset^{(0)}{}_0 = C1r + C2,$ (36)

$\emptyset^{(0)}{}_1 = -C1(r\,In\,r - r) - \frac{ScR\gamma A1}{6}r^3 - \frac{ScR\gamma A2}{2}r^2 + E5r + E6,$ (37)

$\emptyset^{(0)}{}_2 = \frac{C1}{2}(r(In\,r)^2 - In\,r) - E5(r\,In\,r - r) + \frac{ScR\gamma A1}{2}\left(\frac{r^3In\,r}{3} - \frac{r^3}{9}\right) - \frac{1}{120}(ScR\gamma x^2A1 - ScR\gamma GrB1 -$
ScRγGcC1r5−124ScRγx2A2−ScRγGrB2−PrRγGcC2+ScRγKr4−16ScRγA1+ScRγE1r3+12ScRγA2−ScRγE2r2+E9r+E10, (38)

III. Results And Discussion

The effects of varying values of theporosity parameter (x), aspect ratio (R), Schmidt number (Sc), buoyancy forces (Thermal and Concentration Grashof number, (Gr/Gc))embedded in the fully developed flow at a very low Reynolds number are examined on the velocity and temperature fields.The results obtained are examined at a fixed value of $Pr = 7.0,\; K = \eta = 0.5.$ and varying values of $x = 1.0, 5.0, 10.0, 15.0, R = 0.5, 1.0, 3.0, 5.0, Gc = Gr = 1.0, 5.0, 10.0, 15.0, Sc = 1.0, 5.0, 10.0, 15.0, 20.0.$ as shown in figures (1) –(9). Nusselt number(Nu) effect is also shown.

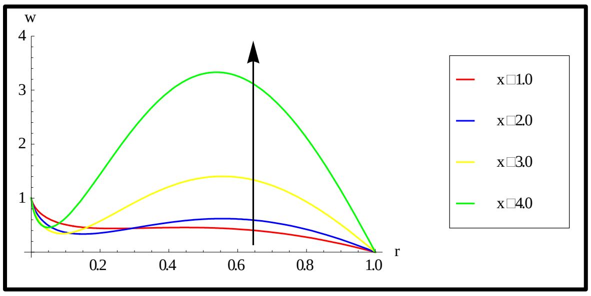

Figure 1: Effects of x on velocity at $Pr = 7.0, \gamma = R = K = 0.5, Sc = Gr = Gc = 1.0.$

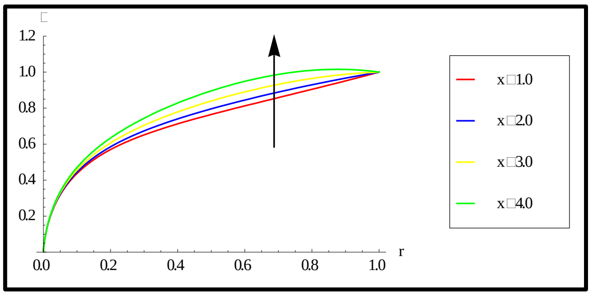

Figure 2: Effects of x on temperature at $Pr = 7.0, \gamma = R = K = 0.5, Sc = Gr = Gc = 1.0.$

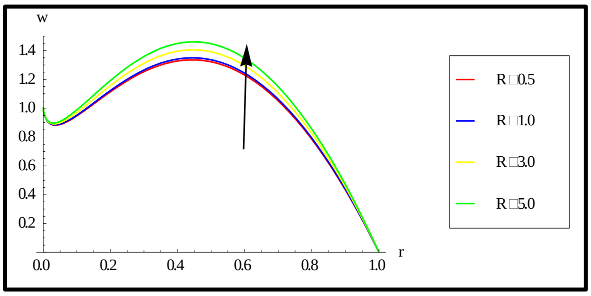

Figure 3: Effects of R on velocity at $Pr = 7.0, \gamma = K = 0.5, Sc = x = Gr = Gc = 1.0$.

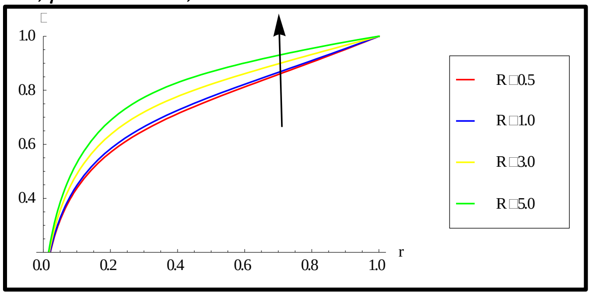

Figure 4: Effects of R on temperature at $Pr = 7.0, \gamma = K = 0.5, Sc = x = Gr = Gc = 1.0$.

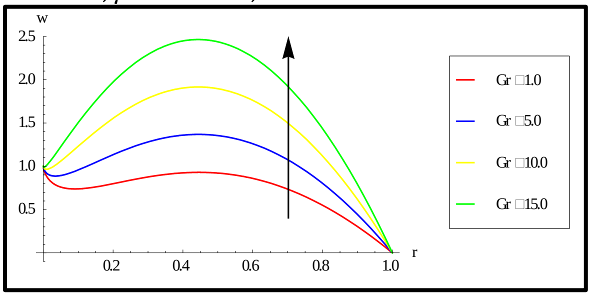

Figure 5: Effects of Gr on velocity at $Pr = 7.0, \gamma = R = K = 0.5, Sc = x = Gc = 1.0$.

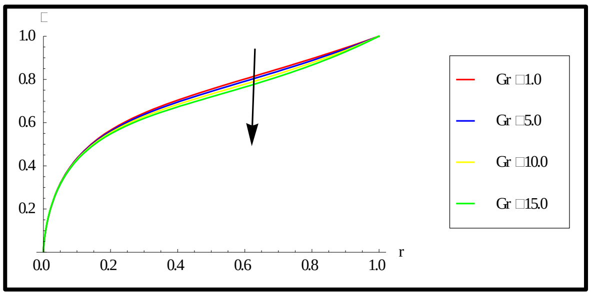

Figure 6: Effects of Gr on temperature at $Pr = 7.0, \gamma = R = K = 0.5, Sc = x = Gc = 1.0$.

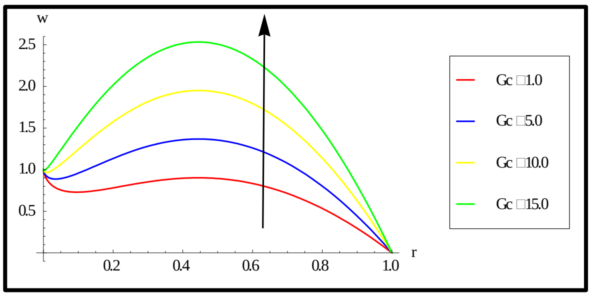

Figure 7: Effects of Gc on velocity at $Pr = 7.0, \gamma = R = K = 0.5, Sc = x = Gr = 1.0$

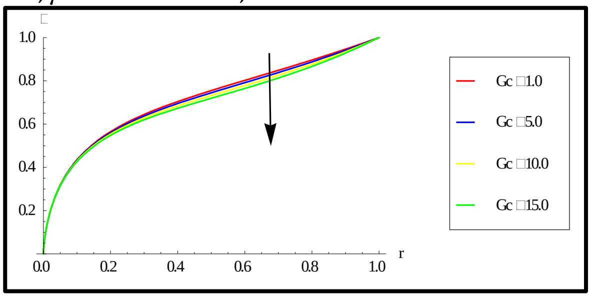

Figure 8: Effects of Gc on temperature at $Pr = 7.0, \gamma = R = K = 0.5, Sc = x = Gr = 1.0$.

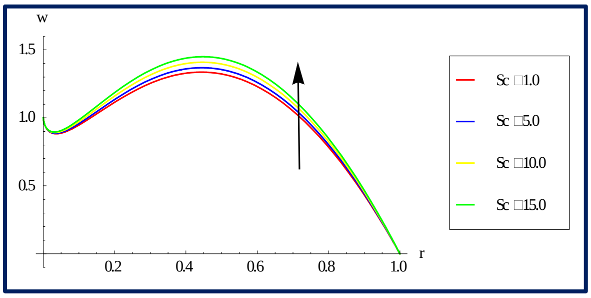

Figure 9: Effects of Sc on velocity at $Pr = 7.0$,
$\gamma = R = K = 0.5, \quad Gc = x = Gr = 1.0$.

Table 1: The effect of porosity parameter (x) variation on Nusselt number (Nu)

x	k	Pr	Nu
1.0	**0.5**	**7.0**	**1.02049**
5.0	**0.5**	**7.0**	**0.08716**
10.0	**0.5**	**7.0**	**-2.82951**
15.0	**0.5**	**7.0**	**-7.69063**

It is observed from figures$(1) - (4)$that at larger values of the porosity parameter andaspect ratio,the flow velocity and temperatureincreases.Figures $(5) - (8)$displays the effects of the buoyancy forces. It shows that increasing values of Gr/Gc resulted to increase in the flow velocity but its temperature decelerated.From figure (9), the flow velocity increased with increase in the Schmidt number. Thus, as the rateat which the fluid is transported into the plant increases, more nutrients are absorbed into the plant, hence enhancing its growth and productivity.Finally, fromTable (11), the rate of heat transfer (Nusselt number) decreases as the porosity parameter increases.

IV. Conclusion

A steady, two-dimensional flow of a viscous incompressible, Newtonian fluid in the stem of a non-bifurcating green plant has just been analyzed. The coupled non-linear governing equations of the flow were non-dimensionalized and then solved by homotopy perturbation method. Analytical results for various parametric conditions of the fully developed flow were presented on the velocity and temperature fields. Results showed that increasing the porosityand aspect ratio resulted to anincrease in the velocity and temperature flow fields. Increase in the buoyancy forces had a positive effect on the flow velocity. In order words, as more nutrients are absorbed into the plant, the rate at which they are transportedwithin the vessel increases. This enhancesits growth and productivity.

COMPETING INTEREST

No competing interest exists.

References

[1]. Bestman, A. R. (1991). "Global Models for the Biomechanics of Green Plants Part 1"International Journal of Energy Research,16:677-684.
[2]. Bestman, A. R. (1992). "Global Models for the Biomechanics of Green Plants Part 2" International Journal of Energy Research,16:685-689.
[3]. Cabrita, P. (2012). "Experimental and Theoretical Studies of Phloem Transport with the Inclusion of Lateral Solute Exchange and Apoplastic Conditions" M.Sc. Thesis,institute of Bio and Geo-Sciences: Plant Science (IBG-2)
[4]. Cabrita, P., Thorpe, M. & Huber, G. (2013). "Hydrodynamics of Steady State Phloem Transport with Radial Leakage of Solute" Frontiers in Plant Science, 4
[5]. Dolger, J., Rademaker, H., Liesche, J., Schulz, A. & Bohr, T. (2014). "Diffusion and Bulk Flow in Phloem Loading: A Theoretical Analysis of the Polymer Trap Mechanism for Sugar Transport in Plants" PHYSICAL REVIEW E 90
[6]. El-dabe, N. T. M., Abou-zeid, M. Y. & Sayed, H. A. (2014). "Pulsatile Motion of Non- Newtonian Fluid with Heat and Mass Transfer through a Porous Medium in a Solid Cylindrical Pipe in the Presence of Magnetic Field" International Journal of Scientific and Innovative Mathematical Research (IJSIMR), 2(10) :844-854.
[7]. He, J. H. (1999). "Homotopy Perturbation Method"Computer methods in applied mechanics and engineering, 178 (3):257-262.
[8]. Hoad, G. V. (1995). "Transport of hormones in the phloem of higher plants" Plant Growth Regulation 16(2): 173-182
[9]. Jensen, K. H., Berg-Sorensen, K. Bruus, H.,Holbrook, N. M., Leische, J., Schulz, A., Zwieniecki, M. A. & Bohr, T. (2016). "Sap Flow and Sugar Transport in Plant" Reviews of Modern Physics, 88.
[10]. Jensen, K. H., Rio, E., Hansen, R., Clanet, C. & Bohr, T. (2009). "Osmotically Driven Pipe Flows and their Relation to Sugar Transport in Plants" J. Fluid Mech., 636 :371-396
[11]. Kililova, N. N. (2008). "Long-distance Liquid Transport in Plants" Proceeding of the Estonian Academy of Science, 57 (3): 179-203.
[12]. Knudson D. (2007). "Fundamentals of Biomechanics" Springer Science+Business Media, LLC, 233 Spring Street,New York, NY 10013, USA
[13]. Liou, T. M., Chang, T. W. & Chang, W. C. (1993). "Effects of Bifurcation angle on the steady flow Structure in model Saccular Aneurysms" Experiments in fluids, 289-295.
[14]. Muskat, (1937). "Flow of Fluids Through Porous Media" Journal of Applied Physics, 8:274.
[15]. Okuyade, W. I. A. & Abbey, T. M. (2016). "Biomechanics of a Bifurcating Green Plant, Part 1" Asian Journal of Physics and Chemical, 1 (2): 1-22
[16]. Okuyade, W. I. A. & Abbey, T. M. (2016). "Steady MHD fluid flow in a Bifurcating Rectangular Porous Channel" Advances in Research, 8: 3
[17]. Okuyade, W. I. A. & Abbey, T. M. (2017). "Biomechanics of a Bifurcating Green Plant, Part 2: Environmental Thermal Effects" Asian Journal of Physics and Chemical, 2 (3): 1-14
[18]. Okuyade, W. I. A., (2015). "MHD Blood Flow in Bifurcating Porous Fine capillaries" African Journal of Science Research, 4(4) :56-59.
[19]. Payvandi, S.,Daly, K. R., Jones, D. L., Talboys, P., Zygalakis, K. C. & Roose, T. (2014). "A Mathematical Model of Water and Nutrients Transport in Xylem Vessels of a Wheat Plant" Bulletin of Mathematical Biology, 76 (3) :566-596.
[20]. Peuke, A. D., Rokitta, M., Zimmermann, U., Schreiber, L., & Haase, A (2001). "Simultaneous Measurement of Water Flow Velocity and Solute Transport in Xylem and Phloem of Adult Plants of Ricinus Communis over a Daily Time Course by Nuclear Magnetic Resonance Spectrometry" Plant Cell and Environment 24:491–503
[21]. Pittermann, J. (2010). "The Evolution of Water Transport in Plants: An Integrated Approach" Geobiology, 8(2): 112–139
[22]. Prakash, O., Singh, S. P., Kumar, D. & Dwivedi, Y. K. (2011). "A Study of Effects of Heat Source on MHD Blood Flow Through Bifurcated Arteries" AIP ADVANCES 1: 2158-3226.
[23]. Rand, R. H. & Cooke, J. R. (1978). "Fluid Dynamics of Phloem Flow: An Axisymmetric Model" American Society of Agricultural Engineers
[24]. Rand, R. H. (1983). "Fluid Mechanics of Green Plants" Ann. Rev. Fluid Mechanics, 15: 29-45
[25]. Rand, R. H., Upadhyaya, S. K. & Cooke, J. R. (1980). "Fluid Dynamics of Phloem Flow: Part II An Approximate Formula"American Society of Agricultural Engineers

[26]. Rengel, Z. (2001). "Xylem and Phloem Transport of Micronutrients" Plant Nutrition – Food Security and Sustainability of Agro-Ecosystems, 628~629

[27]. Tadjfar, M. & Smith, F. T. (2004). "Direct Simulations and Modelling of basic Three-dimensional Bifurcating Tube Flow" J. Fluid Mech., 519: 1-32.

[28]. Uka, N. O. & Olisa, J. D. (2019). "On the Mathematical Model of the Biomechanics ofGreen Plants" Asian Journal of Physical and Chemical Sciences, 7(3): 1-9

[29]. Yu, T. (2008). "Translocation in the Phloem" Plant Physiology

[30]. Zami-Pierre, F., de Lonbens, R., Quintard, M. & Davit, Y. (2016). "Transition in the Flow of Power –law Fluids Through Isotropic Porous Media" Physical Review Letters, 117.

Uka N. O. et. al. "On The Mathematical Model of the Biomechanics of Green Plants (Part 2)." *IOSR Journal of Mathematics (IOSR-JM)*, 18(6), (2022): pp. 39-45.