IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 18, Issue 6 Ser. II (Nov. – Dec. 2022), PP 52-60
www.iosrjournals.org

Intuitionistic Anti Fuzzy Ideal Graph of Semigroup

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Abstract:
Zadeh introduced the concept of fuzzy sets in 1965, while Atanassov developed the concept of intuitionistic fuzzy sets in 1986. Many researchers apply these two concepts to graph structures, so as to produce a fuzzy graph structure, an anti-fuzzy graph, a fuzzy graph of semigroup, ananti-fuzzy graph of semigroup, an intuitionistic fuzzy graph (IFG), an intuitionistic anti-fuzzy graph (IAFG) and anintuitionistic fuzzy graph of semigroup (IFGS). In this article, we introduce the concept of an intuitionistic anti-fuzzy graph of a semigroup which is a development of two concepts namely IAFG and IFGS. The general properties of IAFGS are built and proved and also introduces the concept of an intuitionistic anti-fuzzy (k_1, k_2)*-regular graph of a semigroup (*(k_1, k_2)*-IAFRGS), an intuitionistic anti-fuzzy* (r_1, r_2)*-total regular graph of semigroup (*(r_1, r_2)*-IAFTRGS), concept of complement of IAFGS, isomorphism of IAFGS, equivalence of* (k_1, k_2)*-IAFRGS and* (r_1, r_2)*-IAFTRGS) and the relationship between the complement and isomorphism of IAFGS.*
Key Word:*Fuzzyideal graph ofsemigroup,Anti fuzzyideal graph ofsemigroup,Intuitionistic fuzzy ideal graph ofsemigroup,Intuitionistic anti fuzzyideal graph ofsemigroup.*

Date of Submission: 05-12-2022 Date of Acceptance: 17-12-2022

I. Introduction

A semigroup is a non-empty set equipped with one binary operation and fulfilling closed and associative axioms[1]. Meanwhile, an intuitionistic fuzzy set (HFI) was introduced by Atanssov in 1986[2] and is the result of the development of the fuzzy set (HF) developed by Zadeh in 1965 [3]. A fuzzy set is a set that is equipped with functions and degrees of membership, while HFI, besides being equipped with functions and degrees of membership, is also equipped with degrees of non-membership. These two sets have recently been widely studied by researchers, both in terms of theory and application.

One of the applications of fuzzy sets is the development of graph structures so as to produce fuzzy graph structures. Here are some of the research results. Rosenfeld in 1975 developed a fuzzy graph structure, where each point and edge on a fuzzy graph has a degree of membership which is in closed intervals [0 1][4]. The concept of order and size of fuzzy graphs was developed by Nagoorgani in 2003 [5], then the concept of the degree of a point on a fuzzy graph was developed by Nagoorgani in 2009 [6] and the concept of complement of fuzzy graphs was developed by Sunitha in 2013 [7]. The development of the anti-fuzzy graph concept was carried out by Mathuraj in 2017 [8] by modifying the structure of the fuzzy graph. In the following year, it was developed by Sangoor on the concept of order, size and degree of anti-fuzzy graphs [9], and Kalaivani developed the concept of complement on anti-fuzzy graphs in 2019 [10].

Besides developing the graph on fuzzy sets, the researchers also developed it on HFI, so that an intuitionistic fuzzy graph structure is formed. Nagoorgani in 2010 developed the concepts of degree, order and size of intuitionistic fuzzy graphs [11]. Mathuraj in 2019 developed the concept of operations on intuitionistic anti-fuzzy graphs [12] and in the same year, Mathuraj developed the concept of complement and anti-complement on intuitionistic anti-fuzzy graphs [13].

In this article, a new structure will be developed which is the result of a combination of three concepts, namely fuzzy theory, graph theory, and semigroups. This research was inspired by Krisna about developing the concept of a fuzzy graph of semigroup[14]. In his research, Krisna introduced the definition of a fuzzy graph of semigroup, a fuzzy regular graph of semigroup, and isomorphism of a fuzzy graph of semigroup, then also introduced the definition of an anti-fuzzy graph of semigroupand some properties of an anti-fuzzy graph of semigroup. In the same year, Krisna introduced the concept of an ideal fuzzy graph of semigroup by adding a completeness requirement to the graph under study [15]. Krisna's research was then continued by Sunitha, et al in 2019, who developed an intuitionistic fuzzy ideal graph of semigroup[16]. In her research, Sunitha, et al introduced definitions of an intuitionistic fuzzy graph of semigroup, an intuitionistic fuzzy ideal graph of semigroup, and isomorphism of an intuitionistic fuzzy graph semigroup. In 2021, Jabrullah develops the concept of an intuitionistic anti-fuzzy graphs, but in Jabarullah's research this has been not yet developed on semigroup algebraic structures [17].

Therefore, in this study the authors combined the research concepts of Krisna (2018), Sunitha, et al (2019) and Jabarullah (2021), to build a new structure, namely the concept of an intuitionistic anti-fuzzy graph of semigroup(IAFGS). The research was carried out by constructing the definition of an intuitionistic anti-fuzzy graph of semigroup (IAFGS), the definition of an intuitionistic anti-fuzzy ideal graph of semigroup(IAFIGS) and the general properties that apply to IAFGS such as order, size, degree and total degree. In addition, it also introduces the concept of an intuitionistic anti-fuzzy(k_1, k_2)-regular graph of semigroup ((k_1, k_2)-IAFRGS), an intuitionistic anti-fuzzy(r_1, r_2)-total regular graph ofsemigroup ((r_1, r_2)-IAFTRGS), the concept of complement ofIAFGS, and isomorphism ofIAFGS. This research also shows properties that explain the equivalence of ((k_1, k_2)-IAFRGS) and ((r_1, r_2)-IAFTRGS) and properties that explain the relationship between the concept of complement and isomorphism of IAFGS.

II. Preliminaries

In this section we will recall some of the fundamental concepts and definitions, which are necessary for this paper.

Definition 2.1[18]A graph G is a pair (V, E), where V is a non-empty set of vertices and E is a set of unordered pairs of vertices in $V \times V$, which is called edges. Denoted by $G(V, E)$.

Definition 2.2[18]Two vertices u and v in a graph $G(V, E)$ is called adjacent if (u, v) is an edge of $G(V, E)$.

Definition 2.3[18]Edge e is said to be incident to point u and point v if the side $e = (u, v)$ is formed.

Definition 2.4[18]A complete graph is a graph where every vertex is adjacent to all other vertices.

Definition 2.5[18]The number of vertices in a graph $G(V, E)$is called on order of $G(V, E)$and it is denoted by $|V|$.

Definition 2.6[18]The number of edges in a graph $G(V, E)$ is called a size of $G(V, E)$and it is denoted by $|E|$.

Definition 2.7[18]The *degree* of vertex $v$of graph $G(V, E)$ is defifined as the number of edges incident on $v$and it is denoted by (v) .

Definition 2.8[18]A graph $G(V, E)$ is said to be k-regular graph if $d(v) = k$, for all $v \in V$.

Definition 2.9[18]The complement of a graph G and it is denoted by $\bar{G}$ is a graph that has the same set of vertices as V and two points u and v in $\bar{G}$are adjacent if the points u and v are not adjacent in G for each $(u, v) \in E$.

Definition 2.10[18]Let $G(V_1, E_1)$and $G(V_2, E_2)$ be graphs. Then there exists a map $h : V_1 \to V_2$ such that

(i) h is a bijection

(ii) if two vertices v_1 and v_2 of$V_1$are adjacent in $G(V_1, E_1)$, then $h(v_1)$ and $h(v_2)$are adjacent in $G(V_2, E_2)$if and only if h is said to be isomorphism of graphs.

Definition 2.11[3]Let Sbe a non-empty set.A mapping $\mu: S \to [0, 1]$ is called a fuzzy subset of S.

Definition 2.12[2]Let X be a non-empty set. The intuitionistic fuzzy set A over X is defined as $A = \{(x, \mu(x), \nu(x)) \mid x \in X\}$, where the functions $\mu : X \to [0,1]$ and $\nu : X \to [0\ 1]$ respectively indicate the degree of membership and degree of non- membership and apply $0 \leq \mu(x) + \nu(x) \leq 1$, for every $x \in X$.

Definition 2.13[1]Let S be a non-empty set and $*$ is a binary operation on S. $(S,*)$ is called a semigroup if it satisfies the following axioms:

(i) closed, for every $x, y \in S$ holds $x * y \in S$

(ii) associative, for every $x, y, z \in S$ applies$(x * y) * z = x * (y * z)$.

Definition 2.14[14]Let $G(V, E)$ be a graph, $(V,*)$ be a finite commutative semigroup and μ be a fuzzy subset of Vsuch that $\mu(u * v) \geq max\{\mu(u), \mu(v)\}$,for all $(u, v) \in E$. Then $G(V, E)$ is called a fuzzy graph of semigroup. It is denoted by (V, E, μ) .

Definition 2.15[15]Let $G(V, E)$ be a graph, $(V,*)$ be a finite commutative semigroup and μ be a fuzzy subset of Vsuch that $\mu(u * v) \leq min\{\mu(u), \mu(v)\}$,for all $(u, v) \in E$. Then $G(V, E)$ is called ananti-fuzzy graph of semigroup. It is denoted by $G_A(V, E, \mu)$.

Definition 2.16[16]Let $G(V\ ,\ E)$ be a graph, $(V,*\ )$ be a finite commutativesemigroup and $(\mu\ , \nu)$ be a intuitionistic fuzzy subset of V such that $\mu(u * v) \geq max\{\mu(u), \mu(v)\}, \nu(uv) \leq min\{\nu(u), \nu(v)\}$, where$\mu$ denotes the membership and ν denotes the non-membership value for all $\{u, v\} \in E$. Then the graph $G(V, E)$ is called a intuitionistic fuzzy graph of a semigroup and denoted by $G(V, E, \mu, \nu)$.

III. Main Result

Definition 3.1 Let $G(V,E)$ be a graph with $(V,*)$ is a finite commutativesemigroup and edge set $E \subseteq V \times V$.Intuitionistic anti fuzzy graph of semigroup (IAFGS) is a graph $G(V,E)$whit(μ,ν) is an intuitionistic fuzzy subset of V, such that

$$\mu(u*v) \leq min\{\mu(u),\mu(v)\}, \nu(u*v) \geq max\{\nu(u),\nu(v)\},$$

whereμ is the membership function and ν is the non-membership function for all $(u,v) \in E$ and denoted by $G_A(V,E,\mu\,\nu)$.

Example 3.1 Suppose $G_A(V_1,E_1)$ is a graph with $V_1 = \{v_1,v_2,v_3,v_4\}$ with the binary operation $'*'$ on V_1 defined in Table 3.1 below,

Table 3.1 Binary operations on V_1

$*$	v_1	v_2	v_3	v_4
v_1	v_1	v_2	v_3	v_4
v_2	v_2	v_2	v_2	v_4
v_3	v_3	v_2	v_3	v_4
v_4	v_4	v_4	v_4	v_4

and the set $E_1 = \{(v_1,v_2),(v_1,v_4),(v_2,v_3),(v_3,v_4)\}$ as shown in Figure 3.1 below,

Figure 3.1IAFGS $G_A(V_1,E_1,\mu,\nu)$

Defined intuitionistic fuzzy subset (μ,ν) over V_1 with the degree of membership for each point in V_1 is $\mu(v_1) = 0.4, \mu(v_2) = 0.2, \mu(v_3) = 0.3, \mu(v_4) = 0.2$and the degree of non-membership for each point in V_1 is $\nu(v_1) = 0.5, \nu(v_2) = 0.6, \nu(v_3) = 0.6, \nu(v_4) = 0.7$.

Definition 3.2 Let $G_A(V,E,\mu,\nu)$be anIAFGS. The order of $G_A(V,E,\mu,\nu)$, the denoted by $o(G_A(V,E,\mu,\nu))$is defined as

$$o(G_A(V,E,\mu,\nu)) = \left(\sum_{v\in V}\mu(v), \sum_{v\in V}\nu(v)\right).$$

Definition 3.3 Let $G_A(V,E,\mu,\nu)$beIAFGS.The sizeof $G_A(V,E,\mu,\nu)$, the denoted by $s(G_A(V,E,\mu,\nu))$is defined as

$$s(G_A(V,E,\mu,\nu)) = \left(\sum_{(u,v)\in E}\mu(u*v), \sum_{(u,v)\in E}\nu(u*v)\right).$$

Definition 3.4 Let $G_A(V,E,\mu,\nu)$be anIAFGS.The degree of vertex $v$of $G_A(V,E,\mu,\nu)$, the denoted by $d(v)$is defined as $d(v) = (d_\mu(v), d_\nu(v))$, where

$$d_\mu(v) = \sum_{u\neq v,(u,v)\in E}\mu(u*v), d_\nu(v) = \sum_{u\neq v,(u,v)\in E}\nu(u*v).$$

Definition 3.5 Let $G_A(V,E,\mu,\nu)$be anIAFGS.The total degree of vertex $v$of $G_A(V,E,\mu,\nu)$, the denoted by $Td(v)$is defined as $Td(v) = (Td_\mu(v), Td_\nu(v))$, where

$$Td_\mu(v) = d_\mu(v) + \mu(v), \qquad Td_\nu(v) = d_\nu(v) + \nu(v).$$

Theorem 3.1 Let $G_A(V,E,\mu,\nu)$be an IAFGS. The number of degrees of membership of each vertex in graph $G_A(V,E,\mu,\nu)$ as

$$\sum_{v\in V}d(v) = 2\left(\sum_{(u,v)\in E}\mu(u*v), \sum_{(u,v)\in E}\nu(u*v)\right).$$

Proof:

Let $G_A(V,E,\mu,\nu)$be an IAFGSwith $V = \{v_1,v_2,v_3,\ldots,v_n\}$, then

$$\sum_{v_i \in V} d(v) = \left(\sum_{i=1}^{n} d_\mu(v_i), \sum_{i=1}^{n} d_\nu(v_i)\right) = \left(d_\mu(v_1) + \cdots + d_\mu(v_n), \qquad d_\nu(v_1) + \cdots + d_\nu(v_n)\right).$$

Based on Definition 3.4, that $d_\mu(v_i) = \sum_{(v_i,v_j)\in E} \mu(v_i * v_j)$, $d_\nu(v_i) = \sum_{(v_i,v_j)\in E} \nu(v_i * v_j)$.Then we obtained,

$d(v_1) = \left(\mu(v_1 * v_2), \nu(v_1 * v_2) + \cdots + \mu(v_1 * v_n), \nu(v_1 * v_n)\right)$

$d(v_2) = \left(\mu(v_2 * v_1), \nu(v_2 * v_1) + \cdots + \mu(v_2 * v_n), \nu(v_2 * v_n)\right)$

$\vdots$

$d(v_n) = \left(\mu(v_n * v_1), \nu(v_n * v_1) + \cdots + \mu(v_{n-1} * v_n), \nu(v_{n-1} * v_n)\right)$.

Therefore,

$$\begin{aligned}
\sum_{v_i \in V} d(v) &= \left(d_\mu(v_1) + \cdots + d_\mu(v_n), \qquad d_\nu(v_1) + \cdots + d_\nu(v_n)\right) \\
&= \Big(\left(\mu(v_1 * v_2), \nu(v_1 * v_2) + \cdots + \mu(v_1 * v_n), \nu(v_1 * v_2)\right) \\
&\qquad + \left(\mu(v_2 * v_1), \nu(v_2 * v_1) + \cdots + \mu(v_2 * v_n), \nu(v_2 * v_n)\right) \\
&\qquad \vdots \\
&\qquad + \left(\mu(v_n * v_1), \nu(v_n * v_1) + \cdots + \mu(v_{n-1} * v_n), \nu(v_{n-1} * v_n)\right)\Big) \\
&= 2\left(\mu(v_1 * v_2), \nu(v_1 * v_2) + \mu(v_1 * v_3), \nu(v_1 * v_3) + \ldots + \mu(v_{n-1} * v_n), \nu(v_{n-1} * v_2)\right) \\
&= \left(2 \sum_{(v_i,v_j)\in E} \mu(v_i * v_j), 2 \sum_{(v_i,v_j)\in E} \nu(v_i * v_j)\right) \\
&= 2\left(\sum_{(v_i,v_j)\in E} \mu(v_i * v_j), \sum_{(v_i,v_j)\in E} \nu(v_i * v_j)\right).
\end{aligned}$$

■

Definition 3.6 Let $G_A(V, E, \mu, \nu)$be anIAFGS. $G_A(V, E, \mu, \nu)$ is called an intuitionistic anti fuzzy (k_1, k_2)-regular graph of semigroup, the denoted by (k_1, k_2)-IAFRGS, if for every $v \in V$ applies$d(v) = (k_1, k_2)$ and $k_1, k_2 \in R$.

Definition 3.7 Let $G_A(V, E, \mu, \nu)$be aIAFGS. $G_A(V, E, \mu, \nu)$ is called an intuitionistic anti fuzzy a (r_1, r_2)-total regular graph ofsemigroup, the denoted by (r_1, r_2)-IAFTRGS, if for every $v \in V$ applies $Td(v) = (r_1, r_2)$and $r_1, r_2 \in R$.

Example 3.2 Suppose $G_A(V_2, E_2)$ is a graph with $V_2 = \{v_1, v_2, v_3, v_4\}$ with the binary operation $' * '$on V_2 as Table 3.1 and the set $E_2 = \{(v_1, v_2), (v_1, v_4), (v_2, v_3), (v_3, v_4)\}$ as shown in Figure 3.2 below,

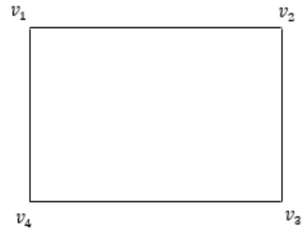

Figure 3.2(k_1, k_2)-IAFRGS$G_A(V_2, E_2, \mu, \nu)$

Defined intuitionistic fuzzy subset (μ, ν) over V_2 with the degree of membership for each point in V_2 is $\mu(v_1) = 0.5, \mu(v_2) = 0.5, \mu(v_3) = 0.5, \mu(v_4) = 0.5$and the degree of non-membership for each point in V_2 is $\nu(v_1) = 0.5, \nu(v_2) = 0.5, \nu(v_3) = 0.5, \nu(v_4) = 0.5$.

Theorem 3.2 If$G_A(V, E, \mu, \nu)$be IAFGS, and an intuitionistic subset anti-fuzzy (μ, ν) is a constant function, then the following statements are equivalent,

(i) $G_A(V, E, \mu, \nu)$ is a (k_1, k_2)-IAFRGS,

(ii) $G_A(V, E, \mu, \nu)$is a (r_1, r_2)-IAFTRGS.

Proof:

(i) Let $G_A(V, E, \mu, \nu)$ is a (k_1, k_2)-IAFRGS, then for all $v \in V$ hold,

$$Td(v) = \left(Td_\mu(v), Td_\nu(v)\right) = \left(d_\mu(v) + \mu(v), d_\nu(v) + \nu(v)\right) = (k_1 + c, k_2 + c) = (r_1, r_2).$$

So, it is proven that $G_A(V, E, \mu, \nu)$ is a IAFRGS with total degrees (r_1, r_2).

(ii) Let $G_A(V, E, \mu, \nu)$ is a (r_1, r_2)-IAFRGS, then for all $v \in V$ hold,

$Td(v) = (r_1, r_2)$

$\left(d_\mu(v) + \mu(v), d_\nu(v) + \nu(v)\right) = (r_1, r_2)$

$$\left(d_\mu(v), d_\nu(v)\right) = (r_1 - \mu(v), r_2 - \nu(v)) = (r_1 - c, r_2 - c) = (k_1, k_2).$$

So, it is proven that $G_A(V, E, \mu, \nu)$ is a IAFRGS with total degrees (r_1, r_2).

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Theorem 3.3 If$G_A(V, E, \mu, \nu)$be an (k_1, k_2)-IAFRGS then the size of a (k_1, k_2)-IAFRGS is $|V|\ (k_1\ , k_2)/2$.

Proof:

Based on Theorem 3.1, we can write

$$\left(\sum_{v\in V} d_\mu(v), \sum_{v\in V} d_\nu(v)\right) = 2\left(\sum_{(u,v)\in E} \mu(u*v), \sum_{(u,v)\in E} \nu(u*v)\right),$$

$$\sum_{v\in V}(k_1, k_2) = 2\left(s\big(G_A(V, E, \mu, \nu)\big)\right)$$

$$|V|(k_1, k_2) = 2\left(s\big(G_A(V, E, \mu, \nu)\big)\right)$$

$$s\big(G_A(V, E, \mu, \nu)\big) = \frac{|V|}{2}(k_1, k_2).$$

■

Definition 3.8 Let $G_A(V, E, \mu, \nu)$ be an IAFGS. If $G_A(V, E, \mu, \nu)$ is a complete graph, that is $E = V \times V$ then $G(V, E, \mu, \nu)$ is called an intuitionistic anti fuzzy ideal graph of a semigroup (IAFIGS).

Example 3.3 Suppose $G_A(V_3, E_3\ )$ is a graph with $V_3 = \{v_1, v_2, v_3, v_4\}$ with the binary operation $'*'$ on V_3 defined in Table 3.2 below,

Table 3.2 Binary operations on V_3

$*$	v_1	v_2	v_3	v_4
v_1	v_1	v_2	v_3	v_4
v_2	v_2	v_2	v_3	v_4
v_3	v_3	v_3	v_3	v_4
v_4	v_4	v_4	v_4	v_4

and the set $E_3 = \{(v_1, v_2), (v_1, v_3), (v_1, v_4), (v_2, v_3), (v_2, v_4), (v_3, v_4)\}$as shown in Figure 3.3 below,

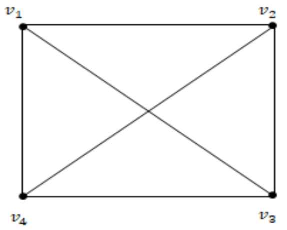

Figure 3.3IAFIGS$G_A(V_3, E_3, \mu, \nu)$

Defined intuitionistic fuzzy subset (μ, ν) over V_3 with the degree of membership for each point in V_3 is,

$$\mu(v_i) = \begin{cases} 0.4, & v_i = v_1 \\ 0.2, & v_i \neq v_1 \end{cases},$$

and the degree of non-membership for each point in V_3 is,

$$\nu(v_i) = \begin{cases} 0.3, & v_i = v_1 \\ 0.5, & v_i \neq v_1 \end{cases}.$$

Theorem 3.4 Let $G_A(V, E, \mu, \nu)$ be an IAFIGS whit element 0and 1, then

(i) $\mu(0) \leq \mu(v); \nu(0) \geq \nu(v)$,for all $v \in V$ and

(ii) $\mu(0) \leq \mu(1); \nu(0) \geq \nu(1)$.

Proof:

Let $G_A(V, E, \mu, \nu)$ be an IAFIGS whit element 0and 1, then

(i) $\mu(0) \leq \mu(0 * v) \leq \min\{\mu(0), \mu(v)\} \leq \min\{1, \mu(v)\} \leq \mu(v)$,
$\nu(0) \leq \nu(0 * v) \leq \max\{\nu(0), \nu(v)\} \leq \max\{0, \nu(v)\} \leq \nu(v)$,
So, we get $\mu(0) \leq \mu(v); \nu(0) \geq \nu(v)$,for all $v \in V$.

(ii) $\mu(0) \leq \mu(0 * 1) \leq \min\{\mu(0), \mu(1)\} \leq \min\{1, \mu(1)\} \leq \mu(1)$,
$\nu(0) \leq \nu(0 * 1) \leq \max\{\nu(0), \nu(1)\} \leq \max\{0, \nu(1)\} \leq \nu(1)$,
So, we get $\mu(0) \leq \mu(1); \nu(0) \geq \nu(1)$,for all $v \in V$.

■

Definition 3.9 Let $G_A(V,E,\mu,\nu)$be anIAFGS. The complement of $G_A(V,E,\mu,\nu)$ is denoted as $\overline{G_A}(V,E,\bar{\mu},\bar{\nu})$ with the terms of degree membership and non-membership from its side are as follows,

$$\big(\bar{\mu}(u*v),\bar{\nu}(u*v)\big)=\big(min\{\mu(u),\mu(v)\}-\mu(u*v),max\{\nu(u),\nu(v)\}-\nu(u*v)\big),\text{for all } (u,v)\in E.$$

Theorem 3.5 If$G_A(V,E,\mu,\nu)$be an IAFGS, and $\overline{G_A}(V,E,\bar{\mu},\bar{\nu})$ is the complement of $G_A(V,E,\mu,\nu)$then the complement of$\overline{G_A}(V,E,\bar{\mu},\bar{\nu})$ is a $G_A(V,E,\mu,\nu)$.

Proof:

Let the complement of $G_A(V,E,\mu,\nu)$ is $\overline{G_A}(V,E,\bar{\mu},\bar{\nu})$ then,

$$\big(\bar{\mu}(u*v),\bar{\nu}(u*v)\big)=\big(min\ \{\mu(u),\mu(v)\}-\mu(u*v),max\ \{\nu(u),\nu(v)\}-\nu(u*v)\big),\text{for all } (u,v)\in E.$$

And we can write,

$$\begin{aligned}\big(\overline{(\bar{\mu})}(u*v),\overline{(\bar{\nu})}(u*v)\big)&=\big(\min\{\mu(u),\mu(v)\}-\bar{\mu}(u*v),\max\ \{\nu(u),\nu(v)\}-\bar{\nu}(u*v)\big),\\&=\big(\min\{\mu(u),\mu(v)\}-\big(\min\ \{\mu(u),\mu(v)\}-\mu(u*v)\big),\\&\qquad\max\ \{\nu(u),\nu(v)\}-(\max\ \{\nu(u),\nu(v)\}-\nu(u*v))\big)\\&=\big(\mu(u*v),\nu(u*v)\big),\text{for all } (u,v)\in E.\end{aligned}$$

So, it is proven thatthe complement of$\overline{G_A}(V,E,\bar{\mu},\bar{\nu})$ is a $G_A(V,E,\mu,\nu)$.

■

Theorem 3.5 Let $G_A(V,E,\mu,\nu)$ be an IAFIGS.

$$(\mu(u*v),\nu(u*v))=\left(\frac{1}{2}\min\{\mu(u),\mu(v)\},\frac{1}{2}\max\{\nu(u),\nu(v)\}\right),\text{for all } (u,v)\in E.$$

If only if $\overline{G_A}(V,E,\bar{\mu},\bar{\nu})=G_A(V,E,\mu,\nu)$.

Proof:

(⇒)It will be shown that if, $\big(\mu(u*v),\nu(u*v)\big)=\left(\frac{1}{2}\min\{\mu(u),\mu(v)\},\frac{1}{2}\max\{\nu(u),\nu(v)\}\right)$, for all $(u,v)\in E$, then$\overline{G_A}(V,E,\bar{\mu},\bar{\nu})=G_A(V,E,\mu,\nu)$ which means that $(\bar{\mu}(u*v),\bar{\nu}(u*v))=\big(\mu(u*v),\nu(u*v)\big)$.

Let the complement of $G_A(V,E,\mu,\nu)$ is $\overline{G_A}(V,E,\bar{\mu},\bar{\nu})$ then based on Definition 3.9, we get

$$\big(\bar{\mu}(u*v),\bar{\nu}(u*v)\big)=\big(\min\ \{\mu(u),\mu(v)\}-\mu(u*v),\max\ \{\nu(u),\nu(v)\}-\nu(u*v)\big). \quad (1)$$

Therefore, $\big(\mu(u*v),\nu(u*v)\big)=\left(\frac{1}{2}\min\{\mu(u),\mu(v)\},\frac{1}{2}\max\{\nu(u),\nu(v)\}\right)$,then equation (1) can be written as follows,

$$\begin{aligned}(\bar{\mu}(u*v),\bar{\nu}(u*v))&=\big(\min\ \{\mu(u),\mu(v)\}-\mu(u*v),\max\ \{\nu(u),\nu(v)\}-\nu(u*v)\big)\\&=\Big(\min\ \{\mu(u),\mu(v)\}-\frac{1}{2}\min\{\mu(u),\mu(v)\},\max\ \{\nu(u),\nu(v)\}\\&\qquad-\frac{1}{2}\max\{\nu(u),\nu(v)\}\Big)\\&=\left(\frac{1}{2}\min\{\mu(u),\mu(v)\},\frac{1}{2}\max\{\nu(u),\nu(v)\}\right)\\&=\big(\mu(u*v),\nu(u*v)\big).\end{aligned}$$

We get,$(\bar{\mu}(u*v),\bar{\nu}(u*v))=\big(\mu(u*v),\nu(u*v)\big)$, which shows that $\overline{G_A}(V,E,\bar{\mu},\bar{\nu})=G_A(V,E,\mu,\nu)$.

(⇐) Next it will be shown that if $\overline{G_A}(V,E,\bar{\mu},\bar{\nu})=G_A(V,E,\mu,\nu)$ then

$$\big(\mu(u*v),\nu(u*v)\big)=\left(\frac{1}{2}\min\{\mu(u),\mu(v)\},\frac{1}{2}\max\{\nu(u),\nu(v)\}\right),\text{for all } (u,v)\in E.$$

Noted that $\overline{G_A}(V,E,\bar{\mu},\bar{\nu})=G_A(V,E,\mu,\nu)$which means that $(\bar{\mu}(u*v),\bar{\nu}(u*v))=\big(\mu(u*v),\nu(u*v)\big)$.

Therefore, equation (1) can be written as follow,

$$\begin{aligned}(\bar{\mu}(u*v),\bar{\nu}(u*v))=(\mu(u*v),\nu(u*v))&=\big(\min\ \{\mu(u),\mu(v)\}-\mu(u*v),\max\ \{\nu(u),\nu(v)\}-\nu(u*v)\big)\\\big(2\mu(u*v),2\nu(u*v)\big)&=(\min\ \{\mu(u),\mu(v)\},\max\ \{\nu(u),\nu(v)\})\\2\big(\mu(u*v),\nu(u*v)\big)&=(\min\ \{\mu(u),\mu(v)\},\max\ \{\nu(u),\nu(v)\})\\\big(\mu(u*v),\nu(u*v)\big)&=\frac{1}{2}(\min\ \{\mu(u),\mu(v)\},\max\ \{\nu(u),\nu(v)\})\\\big(\mu(u*v),\nu(u*v)\big)&=\left(\frac{1}{2}\min\ \{\mu(u),\mu(v)\},\frac{1}{2}\max\ \{\nu(u),\nu(v)\}\right).\end{aligned}$$

So it is proved that if a$\overline{G_A}(V,E,\bar{\mu},\bar{\nu})=G_A(V,E,\mu,\nu)$ then

$$\big(\mu(u*v),\nu(u*v)\big)=\left(\frac{1}{2}\min\{\mu(u),\mu(v)\},\frac{1}{2}\max\{\nu(u),\nu(v)\}\right),\text{for all } (u,v)\in E.$$

■

Proposition 3.1 If$\overline{G_A}(V,E,\bar{\mu},\bar{\nu})=G_A(V,E,\mu,\nu)$ then

$$s\big(G_A(V,E,\mu,\nu)\big)=\left(\frac{1}{2}\min\{\mu(u),\mu(v)\},\frac{1}{2}\max\{\nu(u),\nu(v)\}\right),\text{for all } (u,v)\in E.$$

Proof:
According to Theorem 3.5 if the complement of IAFIGS$\overline{G_A}(V,E,\bar{\mu},\bar{\nu}) = G_A(V,E,\mu,\nu)$then

$$\big(\mu(u*v),\nu(u*v)\big) = \left(\frac{1}{2}\min\{\mu(u),\mu(v)\},\frac{1}{2}\max\{\nu(u),\nu(v)\}\right), \text{for all } (u,v)\in E.$$

Based on Definition 3.3 we get,

$$s\big(G_A(V,E,\mu,\nu)\big) = \left(\sum_{(u,v)\in E}\mu(u*v), \sum_{(u.v)\in E}\nu(u*v)\right)$$
$$= \left(\sum_{(u,v)\in E}\frac{1}{2}\min\{\mu(u),\mu(v)\}, \sum_{(u,v)\in E}\frac{1}{2}\max\{\nu(u),\nu(v)\}\right)$$
$$= \left(\frac{1}{2}\sum_{(u,v)\in E}\min\{\mu(u),\mu(v)\}, \frac{1}{2}\sum_{(u,v)\in E}\max\{\nu(u),\nu(v)\}\right)$$
$$= \frac{1}{2}\left(\sum_{(u,v)\in E}\min\{\mu(u),\mu(v)\}, \sum_{(u,v)\in E}\max\{\nu(u),\nu(v)\}\right).$$

■

Definition 3.9 Let $G_{A1}(V_1,E_1,\mu_1,\nu_1)$ and $G_{A2}(V_2,E_2,\mu_2,\nu_2)$are two IAFGS. Isomorphismof IAFGSis a mapping$h: V_1 \to V_2$ such that,
(i) $h$is an isomorphism of semigroups.
(ii) $\big(\mu_1(v),\nu_1(v)\big) = \Big(\mu_2\big(h(v)\big),\nu_2\big(h(v)\big)\Big)$, for all $v\in V_1$
(iii) $\big(\mu_1(u*v),\nu_1(u*v)\big) = \Big(\mu_2\big(h(u)*h(v)\big),\nu_2\big(h(u)*h(v)\big)\Big)$, for all $(u,v)\in E_1$, $(h(u),h(v))\in E_2$.
Graph $G_{A1}(V_1,E_1,\mu_1,\nu_1)$ and $G_{A2}(V_2,E_2,\mu_2,\nu_2)$ is called to be isomorphic and denoted by $G_{A1}(V_1,E_1,\mu_1,\nu_1)\cong G_{A2}(V_2,E_2,\mu_2,\nu_2)$.
Example 3.4 Suppose $G_A(V_4,E_4)$ is a graph with $V_4 = \{v_1,v_2,v_3,v_4\}$ with the binary operation $'*'$on V_4 as Table 3.1and the set $E_4 = \{(v_1,v_2),(v_1,v_4),(v_2,v_3),(v_3,v_4)\}$ as shown in Figure 3.4 below,

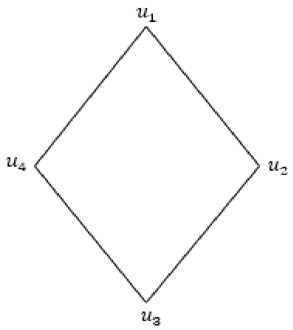

Figure 3.4 Graph Isomorphic of IAFGS$G_A(V_4,E_4,\mu',\nu')$

Defined intuitionistic fuzzy subset (μ',ν') over V_2 with the degree of membership for each point in V_2 is $\mu'(v_1) = 0.4, \mu'(v_2) = 0.2, \mu'(v_3) = 0.3, \mu'(v_4) = 0.2$and the degree of non-membership for each point in V_2 is $\nu'(v_1) = 0.5, \nu'(v_2) = 0.6, \nu'(v_3) = 0.6, \nu'(v_4) = 0.7$.
Theorem 3.6 If$G_{A1}(V_1,E_1,\mu_1,\nu_1)$ and $G_{A2}(V_2,E_2,\mu_2,\nu_2)$are isomorphic then order and size of $G_{A1}(V_1,E_1,\mu_1,\nu_1)$ and $G_{A2}(V_2,E_2,\mu_2,\nu_2)$ are same.
Proof:
Let $G_{A1}(V_1,E_1,\mu_1,\nu_1)\cong G_{A2}(V_2,E_2,\mu_2,\nu_2)$ then,
(i) $\big(\mu_1(v),\nu_1(v)\big) = \Big(\mu_2\big(h(v)\big),\nu_2\big(h(v)\big)\Big)$, for all $v\in V_1$
(ii) $\big(\mu_1(u*v),\nu_1(u*v)\big) = \Big(\mu_2\big(h(u)*h(v)\big),\nu_2\big(h(u)*h(v)\big)\Big)$, for all $(u,v)\in E_1$, $(h(u),h(v))\in E_2$.
Thus,

$$o\big(G_{A1}(V_1,E_1,\mu_1,\nu_1)\big) = \left(\sum_{v\in V}\mu_1(v), \sum_{v\in V}\nu_1(v)\right)$$

$$= \left(\sum_{v\in V}\mu_2(h(v)), \sum_{v\in V}\nu_2(h(v))\right)$$
$$= o(G_{A2}(V_2,E_2,\mu_2,\nu_2)),$$

and,

$$s(G_{A1}(V_1,E_1,\mu_1,\nu_1)) = \left(\sum_{(u,v)\in E_1}\mu_1(u*v), \sum_{(u,v)\in E_1}\nu_1(u*v)\right)$$
$$= \left(\sum_{(u,v)\in E_1}\mu_2(h(u*v)), \sum_{(u,v)\in E_1}\nu_2(h(u*v))\right)$$
$$= \left(\sum_{(h(u),h(v))\in E_2}\mu_2(h(u)*h(v)), \sum_{(h(u),h(v))\in E_2}\nu_2(h(u)*h(v))\right)$$
$$= s(G_{A2}(V_2,E_2,\mu_2,\nu_2)).$$

■

Theorem 3.7 If $G_{A1}(V_1,E_1,\mu_1,\nu_1)$ and $G_{A2}(V_2,E_2,\mu_2,\nu_2)$ are isomorphic then the degrees of each point of $G_{A1}(V_1,E_1,\mu_1,\nu_1)$ is maintained which means that$(v) = d(h(v))$.

Proof:

Let $G_{A1}(V_1,E_1,\mu_1,\nu_1)$ and $G_{A2}(V_2,E_2,\mu_2,\nu_2)$ are isomorphic then there is a semigroup isomorphism $h: V_1 \to V_2$ such that,

$$(\mu_1(u*v), \nu_1(u*v)) = (\mu_2(h(u)*h(v)), \nu_2(h(u)*h(v))), \text{for all } (u,v) \in E_1, (h(u),h(v)) \in E_2.$$

Based on Definition 3.4 can be written,

$$d(v) = (d_\mu(v), d_\nu(v))$$
$$= \left(\sum_{u\neq v,(u,v)\in E}\mu_1(u*v), \sum_{u\neq v,(u,v)\in E}\nu_1(u*v)\right)$$
$$= \left(\sum_{u\neq v,(u,v)\in E}\mu_2(h(u)*h(v)), \sum_{u\neq v,(u,v)\in E}\nu_2(h(u)*h(v))\right)$$
$$= d(h(v)).$$

■

Theorem 3.8 Let $G_{A1}(V_1,E_1,\mu_1,\nu_1)$and $G_{A2}(V_2,E_2,\mu_2,\nu_2)$are two IAFGS. $G_{A1}(V_1,E_1,\mu_1,\nu_1) \cong G_{A2}(V_2,E_2,\mu_2,\nu_2)$ifonly if $\overline{G_{A1}}(V_1,E_2,\overline{\mu_1},\overline{\nu_1}) \cong \overline{G_{A2}}(V_2,E_2,\overline{\mu_2},\overline{\nu_2})$.

Proof:

(⇒) Let $G_{A1}(V_1,E_1,\mu_1,\nu_1)$ and$G_{A2}(V_2,E_2,\mu_2,\nu_2)$ are isomorphic then there is a semigroup isomorphism $h: V_1 \to V_2$ such that,

(i) $(\mu_1(v), \nu_1(v)) = (\mu_2(h(v)), \nu_2(h(v)))$, $\forall\, v \in V_1$

(ii) $(\mu_1(u*v), \nu_1(u*v)) = (\mu_2(h(u)*h(v)), \nu_2(h(u)*h(v)))$, $\forall\, (u,v) \in E_1, (h(u),h(v)) \in E_2$.

Thus,

$$(\overline{\mu_1}(u*v), \overline{\nu_1}(u*v)) = (\min\{\mu_1(u),\mu_1(v)\} - \mu_1(u*v), \max\{\nu_1(u),\nu_1(v)\} - \nu_1(u*v))$$
$$= (\min\{\mu_2(h(u)),\mu_2(h(v))\} - \mu_2(h(u)*h(v)),$$
$$\max\{\nu_2(h(u)),\nu_2(h(v))\} - \nu_2(h(u)*h(v)))$$
$$= (\overline{\mu_2}(h(u)*h(v)), \overline{\nu_2}(h(u)*h(v))).$$

So it's proven that $\overline{G_{A1}}(V_1,E_2,\overline{\mu_1},\overline{\nu_1}) \cong \overline{G_{A2}}(V_2,E_2,\overline{\mu_2},\overline{\nu_2})$.

(⇐) Let $\overline{G_{A1}}(V_1,E_2,\overline{\mu_1},\overline{\nu_1})$ and$\overline{G_{A2}}(V_2,E_2,\overline{\mu_2},\overline{\nu_2})$are isomorphic then there is a semigroup isomorphism $h: V_1 \to V_2$ such that,

$$(\overline{\mu_1}(u*v), \overline{\nu_1}(u*v)) = (\overline{\mu_2}(h(u)*h(v)), \overline{\nu_2}(h(u)*h(v))), \text{for all } (u,v) \in E_1, (h(u),h(v)) \in E_2. \quad (2)$$

Based on Definition 3.4 can be written,

$$\left(\overline{\mu_1}(u*v), \overline{\nu_1}(u*v)\right) = \left(\min\{\mu_1(u), \mu_1(v)\} - \mu_1(u*v), \max\{\nu_1(u), \nu_1(v)\} - \nu_1(u*v)\right),$$

then equation (4.17) can be rewritten as follows,

$(\min\{\mu_1(u), \mu_1(v)\} - \mu_1(u*v), \max\{\nu_1(u), \nu_1(v)\} - \nu_1(u*v)) = \Big(\min\{\mu_2(h(u)), \mu_2(h(v))\} -$ $\mu 2hu*hv,$ max $\nu 2hu,\nu 2hv-\nu 2hu*hv.$

And we get,

(i) $\left(\mu_1(v), \nu_1(v)\right) = \left(\mu_2(h(v)), \nu_2(h(v))\right)$,

(ii) $\left(\mu_1(u*v), \nu_1(u*v)\right) = \left(\mu_2(h(u)*h(v)), \nu_2(h(u)*h(v))\right)$.

So it's proven that $G_{A1}(V_1, E_1, \mu_1, \nu_1) \cong G_{A2}(V_2, E_2, \mu_2, \nu_2)$.

■

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