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Decomposition Method Solve By Fuzzy Liner Fractional Programming Problem

Shakti Bhatt
Madhyanchal Professional University Bhopal - (M.P.), India-462044

Vijay Dadhore
vijaydadhore@gmail.com
Govt . College Samnapur Dindori- (M.P.), India-481778.

Abstract
The intent of this paper is to establish sovle linear equations by decomposition method is a numerical technique .
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I. INTRODUCTION

This paper proposes a new approach to solve Fuzzy LINER Fractional Programming Problem [FLFPP] by using decomposition method . The cost of the objective function , technological coefficients and the resources are expressed as trainguler fuzzy numbers. The FLFPP is converted into crisp Liner Fractional Programming problem [LFPPP] by using Yager's ranking method. The converted LFPP is then transformed into LPP by using Charnes & Cooper [1962] transformation method. proposes Decomposition Method to solve fuzzy LFPP.

this paper, the coefficients of the problem are taken as triangular fuzzy numbers. The fuzzy LFPP is converted into crisp LFPP using Yager‟s ranking method. By making use of Charnes and Cooper transformation, the LFPP is transformed into LPP and solved by decomposition method. For this decomposition, the objective functions are transformed into constraints and all constraints are of ≤ type inequalities..

The objective function of the LP problem is changed into inequality constraint The system of linear inequalities is then solved by the decomposition. Thus , the optimal solution of the LFP problem is obtained

II. PRELIMINARIES

2.1 Fuzzy Liner Fractional Programming Problem

A FLFPP can mathematically be represented as follows. Maximize Z = $\dfrac{c^T x + \alpha}{d^T x + \beta}$

Subject to the constraints

$$\mathrm{A}x \le \breve{b}$$
$$x \ge 0,$$

where m:number of constraints

x: n- dimensional vectors of decision variables

$c\breve{d}$ **:**n x 1 fuzzy vectors

$\breve{a}$, $\breve{\beta}$:fuzzy scalars

A : m x n constraints fuzzy matrix

$\breve{b}$, : m- dimensional fuzzy vector

2 .2 Charnes and Cooper Transformation

A linear fractional programming problem can mathematically be represented as follows

$$\text{Maximize } Z = \frac{c^T x + \alpha}{d^T x + \phi} \qquad (3.1)$$

Subject to the constraints

$$A\,x \le b$$
$$x \ge 0$$

Where c, d, x $\in \Re^n$, b$\in \Re^m$,A $\in \Re^{m\times n}$, α and β are scalars

Here , d^T x+ $\beta \succ 0$.

Let $\frac{1}{d^T x + \beta}$ and Y = tx Multiply the constraints of the problem by 't' The problem can be written as

$$\text{Maximize } Z = c^T y + \alpha t \qquad (3.2)$$

Subject to the constrains

$$Ay - bt \le 0$$
$$d^T y + \beta t = 1$$
$$y, t \ge 0$$

The objective function in (3.2) can be changed into a constraint as follows .
Since Z $\le$ Max Z , Z – Max Z $\le$ 0.

$$Z - \left(c^T\ y + \alpha t\right) \le 0$$

Thus ,the problem (3.2) becomes Maximize Z
Subject to the constraints

$$Z - \left(c^T\ y + \alpha t\right) \le 0$$
$$d^T y + \beta t = 1$$
$$Ay - bt \le 0$$
$$y, t \ge 0$$

III. MAIN RESULT

The following results are required in the sequel which can be found in [3,5]

3.1 SOLUTION PROCEDURE

3.1 a – cut of Triangular Fuzzy Number

The a – cut of a fuzzy set A is defined by A α = $\{x \in X \setminus \mu A(x) \ge \alpha\ \}$

3.2 Yager's Ranking Method

Let $\breve{a}$ = (a,b,c) be a convex triangular fuzzy number .

The α -cut of the fuzzy number $\breve{a}$ is given by

$$\left(a_a{}^L\ , a_a{}^U\right) = \left((b-a)\alpha + a, c - (c-b)\alpha\right)$$

The Yager"s ranking index is defined by

$$R\left(\breve{a}\right) = d\int_0^1 0.5\left(a_a{}^L + a_a{}^U\ \right) da$$

Where $\left(a_a{}^L, a_a{}^U\right)$ is the α -cut of the fuzzy number $\breve{\alpha}$

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3.2 DECOMPOSITION METHOD

Consider a system of 'n' linear equation in 'n' unknowns . This system can be written as,

$$AY = B \qquad (3.3)$$

1. Write A = LU , where L is the unit lower triangular matrix and U is the upper triangular matrix . Form this equation , L and U can be found.
2. The given system of equation (3.3) can be written as

$$LUY = B \qquad (3.4)$$

3. Let UY = W (3.5)
4. On substituting (3.5) in , (3.4), it gives LW = B , This equation can be solved for W
5. On substituting W in UY = W , it can be solved for Y
6. Y is the solution of the given system of equation .

3.3 Application of Decomposition Method to LFPP

Consider the following LFPP.

$$\text{Maximize } Z = \frac{c_1x_1 + c_2x_2 + \ldots\ldots + c_nx_n + \alpha}{d_1x_1 + d_2x_2 + \ldots\ldots + d_nx_n + \beta}$$

Subject to the constraints

$$a_{11}x_1 + a_{12}x_2 + \ldots.. + a_{1n}x_n \le b_1$$
$$a_{21}x_1 + a_{22}x_2 + \ldots.. + a_{2n}x_n \le b_2$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\ldots\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$a_{n1}x_1 + a_{n2}x_2 + \ldots.. + a_{nn}x_n \le b_n \qquad (3.6)$$

Subject to the constraints

$$d_1y_1 + d_2y_2 + \ldots + d_ny_n + \beta t = 1$$
$$a_{11}y_1 + a_{12}y_2 + \ldots + a_{1n}y_n \le b_1$$
$$ta_{21}y_1 + a_{22}y_2 + \ldots + a_{2n}y_n \le b_2t$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$a_{n1}y_1 + a_{n2}y_2 + \_ + a_{nn}y_n \le b_nt$$
$$y_{1,}y_2, y_nt \ge 0 \qquad (3.7)$$

Since , Z $\le$ Max Z,Z $\le$ $c_1y_1 + c_2y_2 + \ldots + c_ny_n + \alpha t$

Thus the LP problem (3.7) can be written as the following system of inequality Constraints

$$\_c_1y_1 \_c_2y_2 \_\ldots\_c_ny_n \_\alpha t + Z \le 0$$
$$d_1y_1 + d_2y_2 + \ldots + d_ny_n + \beta t = 1$$
$$a_{11}y_1 + a_{12}y_2 + \ldots + a_{1n}y_n - b_1t \le 0$$
$$a_{21}y_1 + a_{22}y_2 + \ldots + a_{2n}y_n - b_2t \le 0$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$a_{n1}y_1 + a_{n2}y_2 + \ldots + a_{nn}y_n - b_nt \le 0$$
$$-y_{1,} - y_2, \ldots, -y_n, -t \le 0$$

Now ,the system of equation can be considered as AY = B where

A= $\begin{bmatrix} -c_1 & -c_2 & \dots & -c_n & -\alpha & 1 \\ d_1 & d_2 & \dots & d_n & \beta & 0 \\ a_{11} & a_{12} & \dots & a_{1n} & -b_1 & 0 \\ a_{21} & a_{22} & \dots & a_{2n} & -b_2 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} & -b_n & 0 \end{bmatrix}$ y = $\begin{bmatrix} y_1 \\ y_2 \\ \dots \\ y_n \\ t \\ Z \end{bmatrix}$ B = $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ \dots \\ 0 \end{bmatrix}$

3.5 NUMERICAL EXAMPLE

Consider the following FLPP

Maximize Z = $\dfrac{(1,2,3,)x_1 + (0,1,2,)x_2}{(1,3,5)x_1 + (1,1,1,)x_2 + (3,6,9)}$

Subject to the constraints

$(4,7,10)x_1 + (0,1,2)x_2 \le (4,6,8)$ (3.8) $x_1, x_2 \ge 0$

Now the FLPP (3.8) is converted into the following crisp LFPP by using Yage's ranking method.

The α _cut of fuzzy number $(1,2,3) is \left(a_{a,}{}^{L} a_{a}{}^{u}\right) = (\alpha + 1, 3\_\alpha)$.

R $(1,2,3) = \int_0^1 0.5\,(\alpha + 1 + 3\_\alpha)d\alpha = 2$

R $(0,1,2) = \int_0^1 0.5\,(\alpha + 0 + 2\_\alpha)d\alpha = 1$

R $(1,3,5) = \int_0^1 0.5\,(2\alpha + 1 + 5\_2\alpha)d\alpha = 3$

R $(1,1,1) = \int_0^1 0.5\,(1 + 1 +)d\alpha = 1$

R $(3,6,9) = \int_0^1 0.5\,(3\alpha + 3 + 9\_3\alpha)d\alpha = 6$

R $(4,7,10) = \int_0^1 0.5\,(3\alpha + 4 + 10\_3\alpha)d\alpha = 7$

R $(0,1,2) = \int_0^1 0.5\,(\alpha + 0 + 2\_\alpha)d\alpha = 1$

R $(4,6,8) = \int_0^1 0.5\,(2\alpha + 4 + 8\_2\alpha)d\alpha = 6$

R $(1,5,9) = \int_0^1 0.5\,(4\alpha + 1 + 9\_4\alpha)d\alpha = 5$

R $(2,3,4) = \int_0^1 0.5\,(\alpha + 2 + 4\_\alpha)d\alpha = 3$

R $(5,6,7) = \int_0^1 0.5\,(\alpha + 5 + 7\_\alpha)d\alpha = 6$

The .problem (3.8)becomes the following crisp LFPP.

Maximize Z = $\dfrac{2x_1 + x_2}{3x_1 + x_2 + 6}$

Subject to the constrains

$7x_1 + x_2 \le 6$

$5x_1 + 3x_2 \le 6$

$x_1, x_2 \ge 0$ (3.9)

Let $\dfrac{1}{3x_1 + x_2 + 6}$ = t and $tx_1 = y_1 \,\&\, tx_2 = y_2$ (3.10)

By multiplying the constraints of the problem $(3.9)$by 't' and using (3.10),the following LPP is obtained.

Maximize Z = 2 $y_1 + y_2$

Subject to the constraints

$3y_1 + y_2 + 6t = 1$

$7y_1 + y_2 \_ 6t \leq 0$

$5y_1 + 3y_2 - 6t \leq 0$

$y_1, y_2, t \geq 0$

Since Z $\leq$ Max Z, (3.11)

$Z \leq 2y_1 + y_2$

The LP problem (3.11) becomes

$-2y_1 - y_2 + Z \leq 0$

$3y_1 + y_2 + 6t \leq 1$

$7y_1 + y_2 - 6t \leq 0$

$5y_1 + 3y_2 - 6t \leq 0$

$-y_1, -y_2, -t \leq 0$ (3.12)

The system (3.11) can be written as AY = B Where

$$A = \begin{bmatrix} -2 & -1 & 0 & 1 \\ 3 & 1 & 6 & 0 \\ 7 & 1 & -6 & 0 \\ 8 & 3 & -6 & 0 \end{bmatrix} \quad y = \begin{bmatrix} y_1 \\ y_2 \\ t \\ Z \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

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