IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 19, Issue 2 Ser. II (Mar. – Apr. 2023), PP 32-39
www.iosrjournals.org

Solving Assignment Problem, a Special Case of Transportation Problem by Various Methods and Their Comparison: A Case Study

[1]Varinder Singh, [2]Pooja Rani,[*3]Leena Jain
[1, 2, 3]Department of Computer Applications, Global Group of Institutes, Amritsar, 143501(Punjab) India
**Corresponding Author: -Prof. (Dr.) Leena Jain*
Head-Department of Computer Applications
Global Group of Institutes, Amritsar (Punjab)

Abstract:
The assignment problems are a well-studied topic in combinatorial optimization. These problems find numerous applications in production planning, telecommunication VLSI design, economic etc. The assignment problem is a special case of the transportation problem where the supply from every source and the demand at every sink are equal to 1. Such a situation arises naturally in the setting of assigning workers to jobs, or of assigning workers to a time schedule. In this paper we consider the assignment problem as LPP with objective is to assign n persons to n jobs, so as to minimize the total assignment cost and then we solve this problem with the help of two phase &big M method by using TORA software. As assignment problem is special case of transportation problem so this problem also solved with the help of North- West Corner Method, Least (Minimum)-Cost Method & Vogel's Approximation Method (VAM) by using TORA software and compared optimal solution with number of required iteration to get optimal solution of different methods.

Keywords: *Assignment problem, Hungarian assignment method (HA) method, Linear Integer Programming, optimization Problem, NWCM,LCM,VAM, Two Phase method , Big M method*

Date of Submission: 24-03-2023 Date of Acceptance: 06-04-2023

I. Introduction:

In linear programming problem, assignment problem is introducing instantly after transportation problem [1]. The main aim of the assignment problem is to minimize total cost or time of several resources to an equal number of activities [1]. Assignment problem holds a condition that one resource can connect with only one activity [2]. Applications of assignment problem are many and widespread because its uses just not only confined to assign jobs to worker or machines but also uses in personnel to offices, transportation, airbus to a destination, teachers to classrooms and so on[3,4]. Actually, assignment problem is one of the initial problems which is important in decision making [2]. Assignment problem was first discussed as the degenerate case of transportation problem in 1951 in the SCOOP symposium on linear inequalities and programming[5].D.F. Votaw & A. Orden, in 1952 was first formulated assignment problem as a type of transportation problem[5].Harold Kuhn in 1955 was developed and published assignment problem[6]. He named the method as "Hungarian method" because his method was based on the methods of two Hungarian mathematicians: Dénes Kőnig and Jenő Egerváry[6]. In 1953 Dénes Konig provided an algorithm for linear assignment problem which was based on the translation of a paper of Jeno Egerváry[7].Recently Jain L. & Singh C. Solved Assignment problem using Lingo software and get optimal solution which is same as the optimal solutions of Hungarian assignment method [8]. In this paper assignment problem represented as linear programming problem (LPP) and then different methods like two-phase method &Big-M methods are used to solve LPP. As assignment problem is a special case of transportation problem so North-West Corner Method (NWCM), Least (Minimum)-Cost Method (LCM) & Vogel's Approximation Method (VAM) are used to solve the problem using TORA software. TORA Package is a computer application software package used for statistical computation and analysis. It is an already written program or suite of programs written for statistical application. It is basically applied for Operations Research (OR) analysis. TORA Optimization Window is a graphical user interface (GUI). This is what makes it unique from other statistical packages that have spreadsheets windows. Although, the input grid is edited much like spreadsheets, the number of rows and columns depend on setting. TORA is automated for screen display setting of 800 X 600 and 1024 X 768 pixels. The second setting is recommended because it produces a more proportionate layout of the screen.

Experimental Setup:
A Company is faced with the problem of assigning five jobs to five machine, each job must be done only one machine, the cost of processing each job on each machine is given below(in Rs)

Table 3: Cost Matrix

↓Jobs \ Machine →	M1	M2	M3	M4	M5
J1	**7**	**5**	**9**	**8**	**11**
J2	**9**	**12**	**7**	**11**	**10**
J3	**8**	**5**	**4**	**6**	**9**
J4	**7**	**3**	**6**	**9**	**5**
J5	**4**	**6**	**7**	**5**	**11**

The problem is to determine the assignment of jobs to machines so that it will result in minimum cost.
Source: Operations Research by K. K. Chawla, Vijay Gupta & Bhushan K. Sharama16th edition Page 9.4, question 1

Mathematical Formulation

Min

Z= $7*X_{11}+5*X_{12}+9*X_{13}+8*X_{14}+11*X_{15}+9*X_{21}+12*X_{22}+7*X_{23}+11*X_{24}+10*X_{25}+8*X_{31}+5*X_{32}+4*X_{33}+6*X_{34}+9*X_{35}+7*X_{41}+3*X_{42}+6*X_{43}+9*X_{44}+5*X_{45}+4*X_{51}+6*X_{52}+7*X_{53}+5*X_{54}+11*X_{55}$

Subject to Constraints:

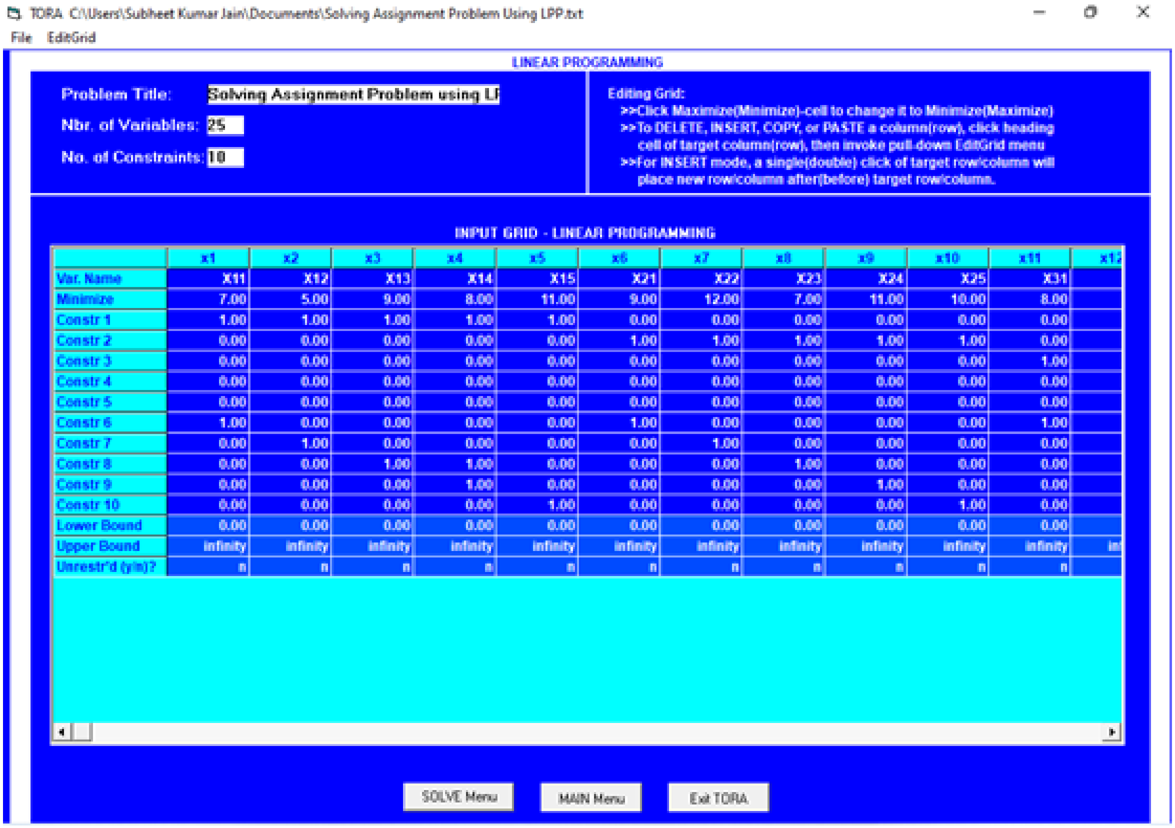
$X_{11}+X_{12}+X_{13}+X_{14}+X_{15}=1$
$X_{21}+X_{22}+X_{23}+X_{24}+X_{25}=1$
$X_{31}+X_{32}+X_{33}+X_{34}+X_{35}=1$
$X_{41}+X_{42}+X_{43}+X_{44}+X_{45}=1$
$X_{11}+X_{21}+X_{31}+X_{41}+X_{51}=1$
$X_{12}+X_{22}+X_{32}+X_{42}+X_{52}=1$
$X_{13}+X_{23}+X_{33}+X_{43}+X_{53}=1$
$X_{14}+X_{24}+X_{34}+X_{44}+X_{45}=1$
$X_{15}+X_{25}+X_{35}+X_{45}+X_{55}=1$
where $X_{ij}=0$ or 1, i=1,2,3,4,5& j=1,2,3,4,5

Solving Using Two-Phase& Big M Method
he simplex method was applied to linear programming problems with less than or equal to (≤) type constraints. Thus, there we could introduce slack variables which provide an initial basic feasible solution of the problem. Generally, the linear programming problem can also be characterized by the presence of both less than or equal to " (≤) ' type or 'greater than or equal to " (≥)' type constraints.
In such case it is not always possible to obtain an initial basic feasible solution using slack variables.
The greater than or equal to type of linear programming problem can be solved by using the following methods:
1.Two Phase Method
2.Big M- Method
he simplex method was applied to linear programming problems with less than or equal to (≤) type constraints. Thus, there we could introduce slack variables which provide an initial basic feasible solution of the problem. Generally, the linear programming problem can also be characterized by the presence of both less than or equal to " (≤) ' type or 'greater than or equal to " (≥)' type constraints.
In such case it is not always possible to obtain an initial basic feasible solution using slack variables.
The greater than or equal to type of linear programming problem can be solved by using the following methods:
1.Two Phase Method
2.Big M- Method
he simplex method was applied to linear programming problems with less than or equal to (≤) type constraints. Thus, there we could introduce slack variables which provide an initial basic feasible solution of the problem. Generally, the linear programming problem can also be characterized by the presence of both less than or equal to " (≤) ' type or 'greater than or equal to " (≥)' type constraints.
In such case it is not always possible to obtain an initial basic feasible solution using slack variables.
The greater than or equal to type of linear programming problem can be solved by using the following methods:
1.Two Phase Method

2.Big M- Method

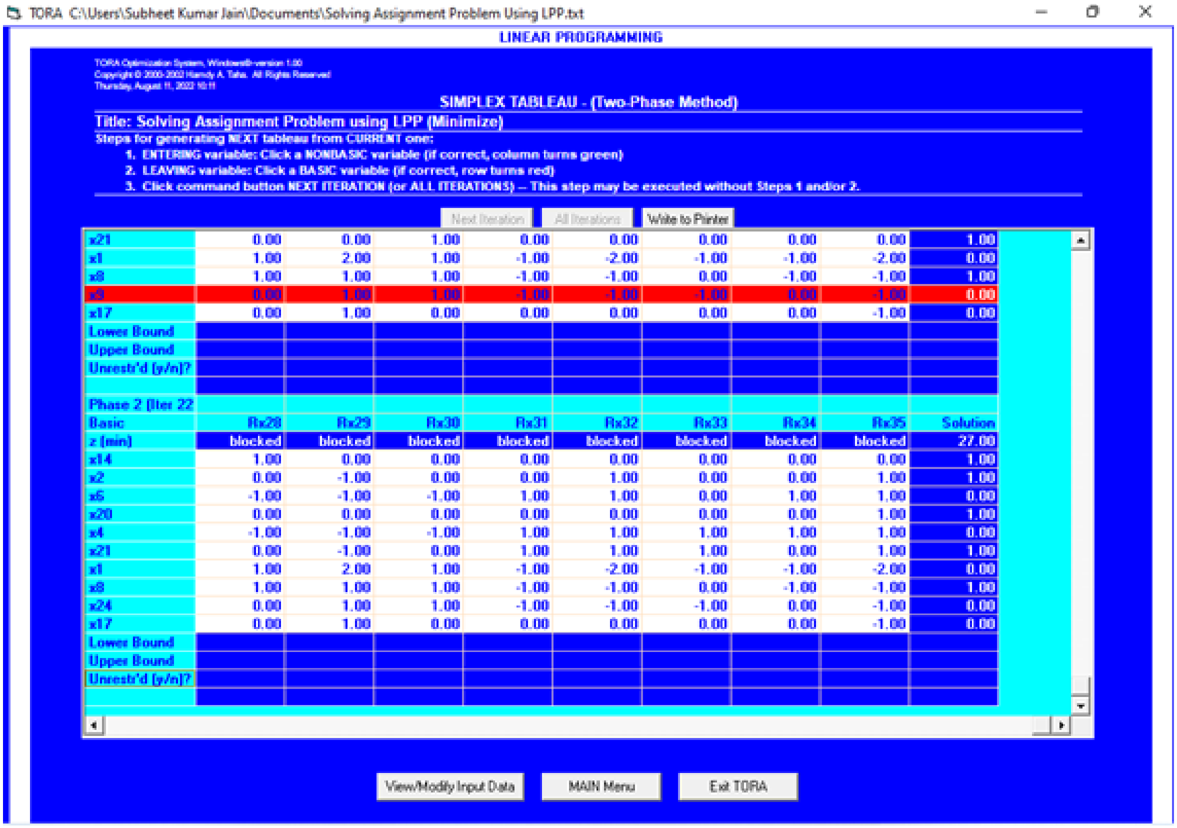
The simplex method was applied to linear programming problems with less than or equal to (≤) type constraints. Thus, there we could introduce slack variables which provide an initial basic feasible solution of the problem.
Generally, the linear programming problem can also be characterized by the presence of both less than or equal to " (≤) ' type or 'greater than or equal to " (≥)' type constraints.
In such case it is not always possible to obtain an initial basic feasible solution using slack variables.
The greater than or equal to type of linear programming problem can be solved by using the following methods:
1.Two Phase Method
2.Big M- Method
The Simplex method was applied to solve linear programming problems with ≤ type constraints. Generally, the linear programming problem can also be characterized by the presence of both ≤ & ≥ type constraints. In that case problem can be solved by two-phase & big M Method.
One way to guarantee that the new optimal solution is optimal for the original LP is to modify the objective function, so that the artificial variable will take value zero in the new optimal solution. In other words, a very large penalization is added to the objective function if the artificial variable takes positive value in big M method.
The two-phase method, on the other hand, does not involve the big number M and hence all the problems are avoided. The two-phase method, as it is called, divides the process into two phases.
Phase 1: The goal is to find a Basic feasible solution for the original LP. Indeed, we will ignore the original objective for a while, and instead try to minimize the sum of all artificial variables. At the end of phase 1, a Basic feasible solution is obtained if the minimal value of this LP is zero.
Phase 2: Drop all the artificial variables, change the objective function back to the original one. Use just the regular simplex algorithm, with the starting Basic feasible solution obtained in Phase 1.
The two-phase method and big-M method are equivalent. In practice, however, most computer codes utilize the two-phased method. The reasons are that the inclusion of the big number M may cause round-off error and other computational difficulties. Here, LPP solved using two phase method and big M method by TORA Software. Fig1 represent the input grid of linear programming and fig 2 and fig 3 represents output of two phase and big M method respectively. Both methods produce same optimal solution 27 for the given problem with 15 and 22 iterations by big M and two phase methods respectively.

Fig 1 Input Grid- Linear Programming



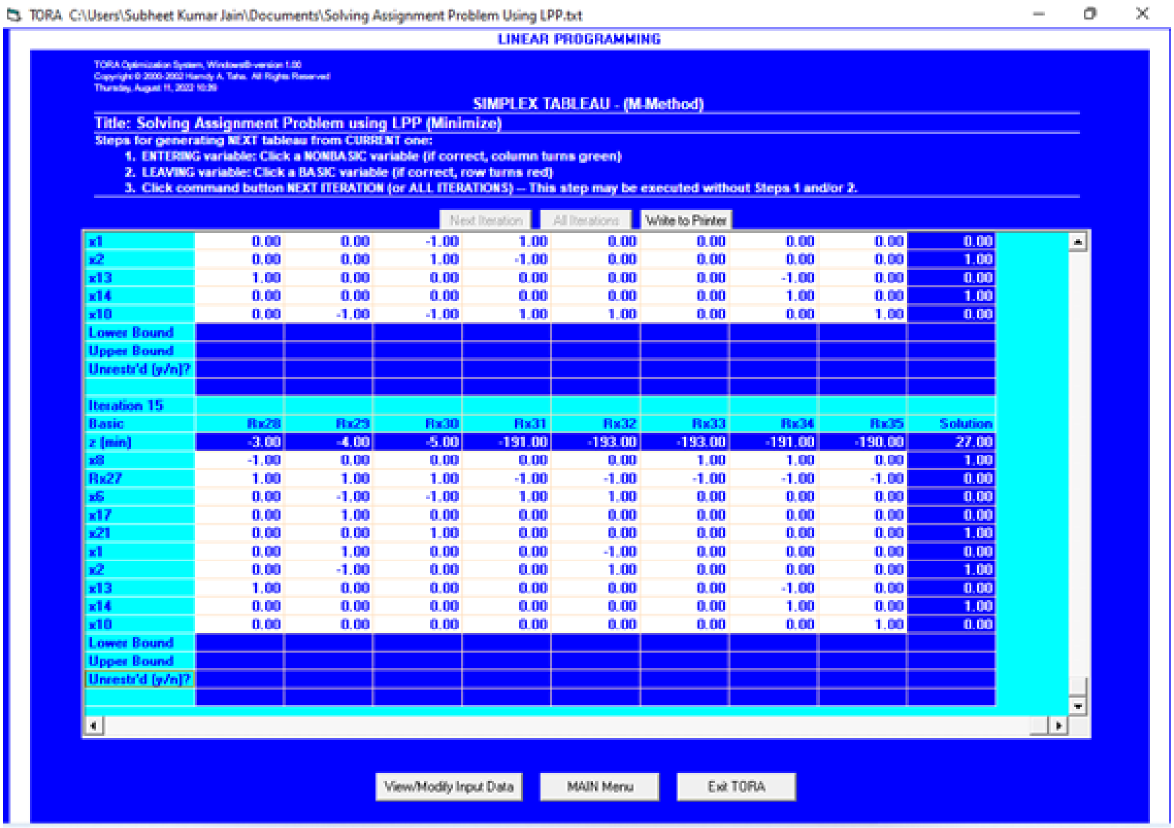
	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11
Var. Name	X11	X12	X13	X14	X15	X21	X22	X23	X24	X25	X31
Minimize	7.00	5.00	9.00	8.00	11.00	9.00	12.00	7.00	11.00	10.00	8.00
Constr 1	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
Constr 2	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00	0.00
Constr 3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00
Constr 4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Constr 5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Constr 6	1.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00
Constr 7	0.00	1.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
Constr 8	0.00	0.00	1.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00
Constr 9	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
Constr 10	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	0.00
Lower Bound	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Upper Bound	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity
Unrestr'd (y/n)?	n	n	n	n	n	n	n	n	n	n	n

Fig 2. Output Screen Using Two-phase method



x21	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00
x1	1.00	2.00	1.00	-1.00	-2.00	-1.00	-1.00	-2.00	0.00
x8	1.00	1.00	1.00	-1.00	-1.00	0.00	-1.00	-1.00	1.00
x9	0.00	1.00	1.00	-1.00	-1.00	-1.00	0.00	-1.00	0.00
x17	0.00	1.00	0.00	0.00	0.00	0.00	0.00	-1.00	0.00
Lower Bound									
Upper Bound									
Unrestr'd (y/n)?									
Phase 2 (Iter 22									
Basic	Rx28	Rx29	Rx30	Rx31	Rx32	Rx33	Rx34	Rx35	Solution
z (min)	blocked	blocked	blocked	blocked	blocked	blocked	blocked	blocked	27.00
x14	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00
x2	0.00	-1.00	0.00	0.00	1.00	0.00	0.00	1.00	1.00
x6	-1.00	-1.00	-1.00	1.00	1.00	0.00	1.00	1.00	0.00
x20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00
x4	-1.00	-1.00	-1.00	1.00	1.00	1.00	1.00	1.00	0.00
x21	0.00	-1.00	0.00	1.00	1.00	1.00	0.00	1.00	1.00
x1	1.00	2.00	1.00	-1.00	-2.00	-1.00	-1.00	-2.00	0.00
x8	1.00	1.00	1.00	-1.00	-1.00	0.00	-1.00	-1.00	1.00
x24	0.00	1.00	1.00	-1.00	-1.00	-1.00	0.00	-1.00	0.00
x17	0.00	1.00	0.00	0.00	0.00	0.00	0.00	-1.00	0.00
Lower Bound									
Upper Bound									
Unrestr'd (y/n)?									

Fig 3. Output Screen Using Big-M method



x1	0.00	0.00	-1.00	1.00	0.00	0.00	0.00	0.00	0.00
x2	0.00	0.00	1.00	-1.00	0.00	0.00	0.00	0.00	1.00
x13	1.00	0.00	0.00	0.00	0.00	0.00	-1.00	0.00	0.00
x14	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	1.00
x10	0.00	-1.00	-1.00	1.00	1.00	0.00	0.00	1.00	0.00
Lower Bound									
Upper Bound									
Unrestr'd (y/n)?									
Iteration 15									
Basic	Rx28	Rx29	Rx30	Rx31	Rx32	Rx33	Rx34	Rx35	Solution
z (min)	-3.00	-4.00	-5.00	-191.00	-193.00	-193.00	-191.00	-190.00	27.00
x8	-1.00	0.00	0.00	0.00	0.00	1.00	1.00	0.00	1.00
Rx27	1.00	1.00	1.00	-1.00	-1.00	-1.00	-1.00	-1.00	0.00
x6	0.00	-1.00	-1.00	1.00	1.00	0.00	0.00	0.00	0.00
x17	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
x21	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00
x1	0.00	1.00	0.00	0.00	-1.00	0.00	0.00	0.00	0.00
x2	0.00	-1.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00
x13	1.00	0.00	0.00	0.00	0.00	0.00	-1.00	0.00	0.00
x14	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	1.00
x10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
Lower Bound									
Upper Bound									
Unrestr'd (y/n)?									

Assignment Problem a Special Case of Transportation Problem

Each assignment problem has a table or matrix associated with it. Generally the row contain the objects or people we wish to assign, and the column comprise the jobs or task we want them assigned to. Consider a problem of assignment of n resources to m activities so as to minimize the overall cost or time in such a way that each resource can associate with one and only one job. The cost matrix (Cij) is given as under:

		Activity				
		A_1	A_2	.....	A_n	Available
Resource	R_1	c_{11}	c_{12}	......	c_{1n}	1
	R_2	c_{21}	c_{22}	......	c_{2n}	1
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	R_n	c_{n1}	c_{n2}	......	c_{nm}	1
Required		1	1	.....	1	

The cost matrix is same as that of a Transportation Problem except that availability at each of the resource and the requirement at each of the destinations is unity.

Let x_{ij} denote the assignment of i^{th} resource to j^{th} activity, such that

$$x_{ij} = \begin{cases} 1 \text{ if job j is performed by worker i} \\ 0 \text{ otherwise} \end{cases}$$

Then the mathematical formulation of the assignment problems as a special case of transportation problem is

$$\text{Minimize } z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}\, x_{ij}$$

Subject to the constraints

$$\sum_{i=1}^{n} x_{ij} = 1 \text{ and } \sum_{j=1}^{n} x_{ij} = 1 \;:\; x_{ij} = 0 \text{ or } 1$$

For all i = 1,2,…n & j = 1,2…n

There are three methods to determine the solution for balanced transportation problem:

1. Northwest Corner method
2. Minimum cost method
3. Vogel's approximation method

The three methods differ in the "quality" of the starting basic solution they produce and better starting solution yields a smaller objective value. All three methods are applied on given problem and same optimal solution found with 27, 3 & 4 iteration by NWCM, LCM & VAM methods respectively. Fig 4 represents the input grid of transportation problem and Fig 5, 6 & 7 represents output produced by NWCM, LCM & VAM methods respectively. Table 1 & Fig 8 represent result of different methods for solving assignment problem Using TORA Software.

Fig 4 Input Grid- Transportation Problems

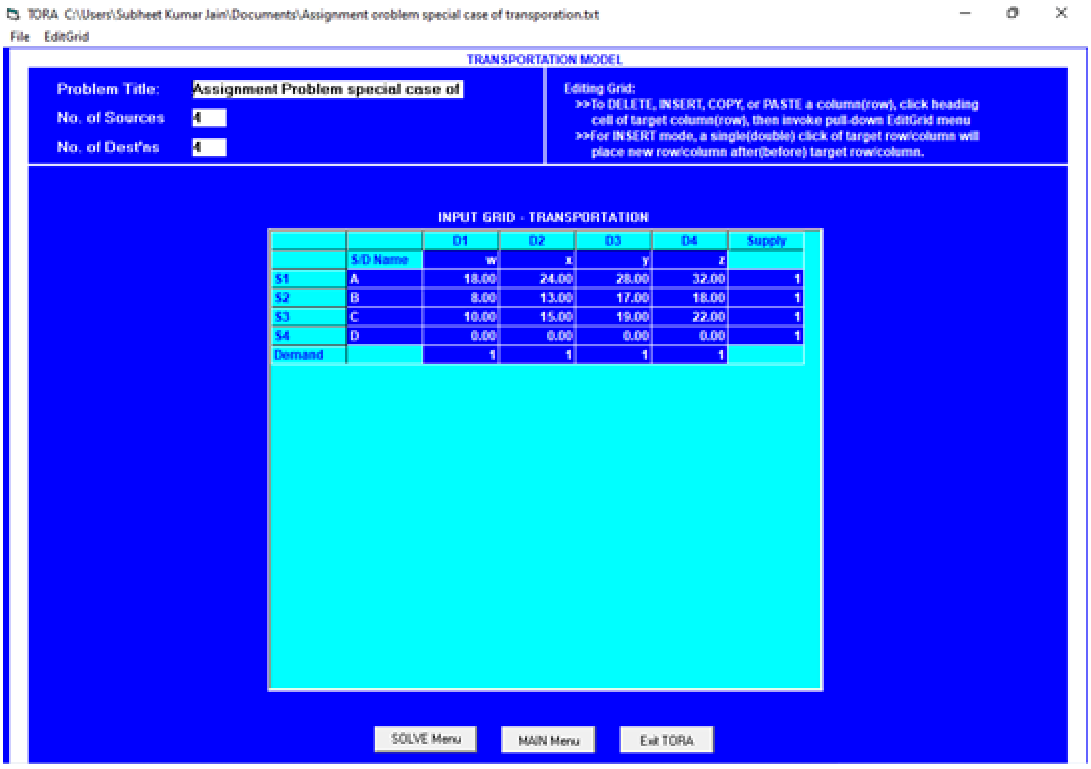


Fig 5. Output Screen Using North-West Corner method

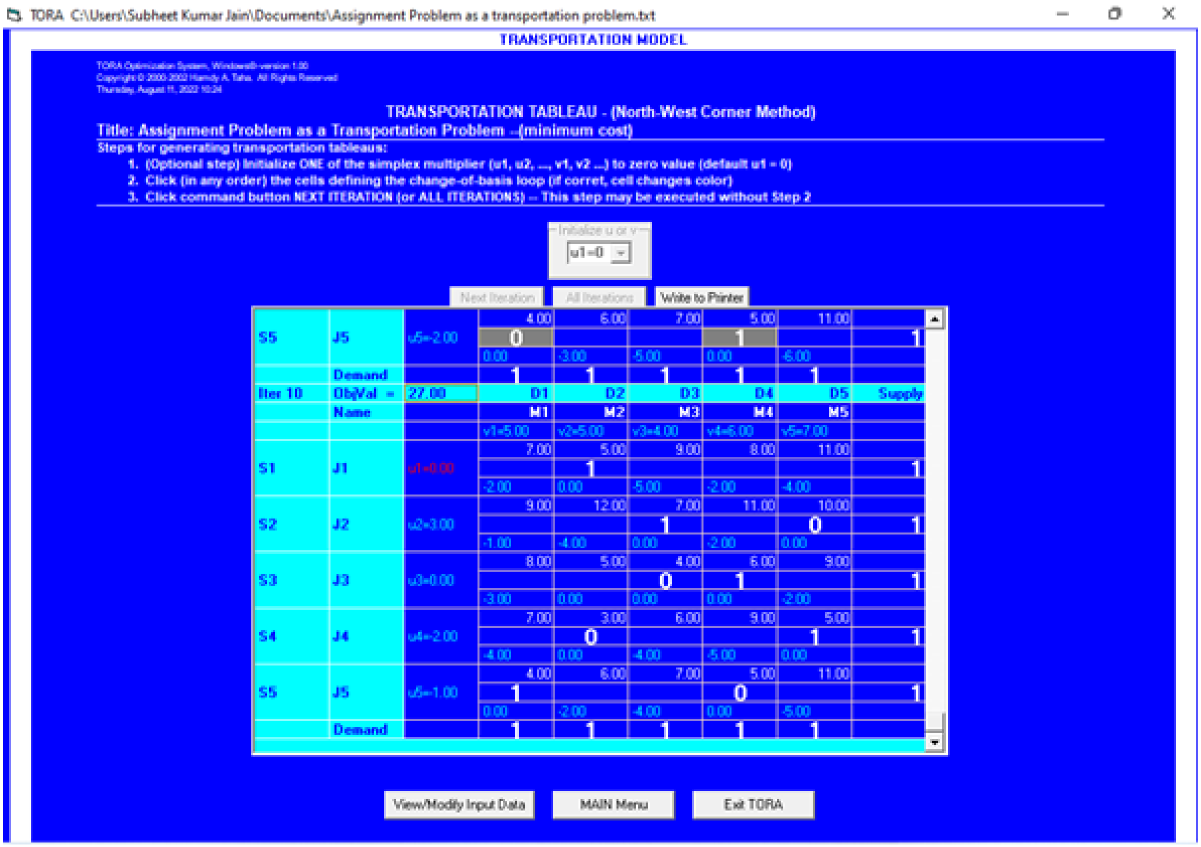


Fig 6. Output Screen Using Least Cost Method

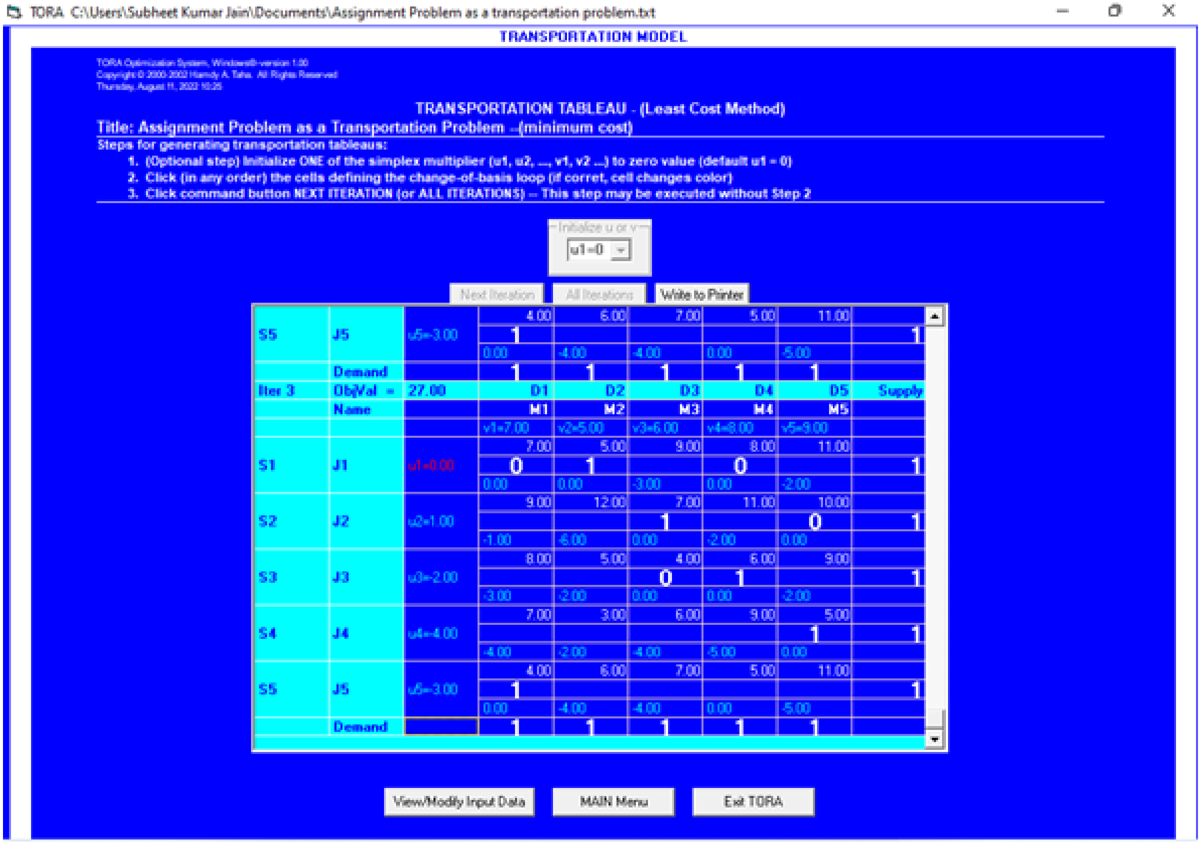


Fig 7. Output Screen Using VAM Method

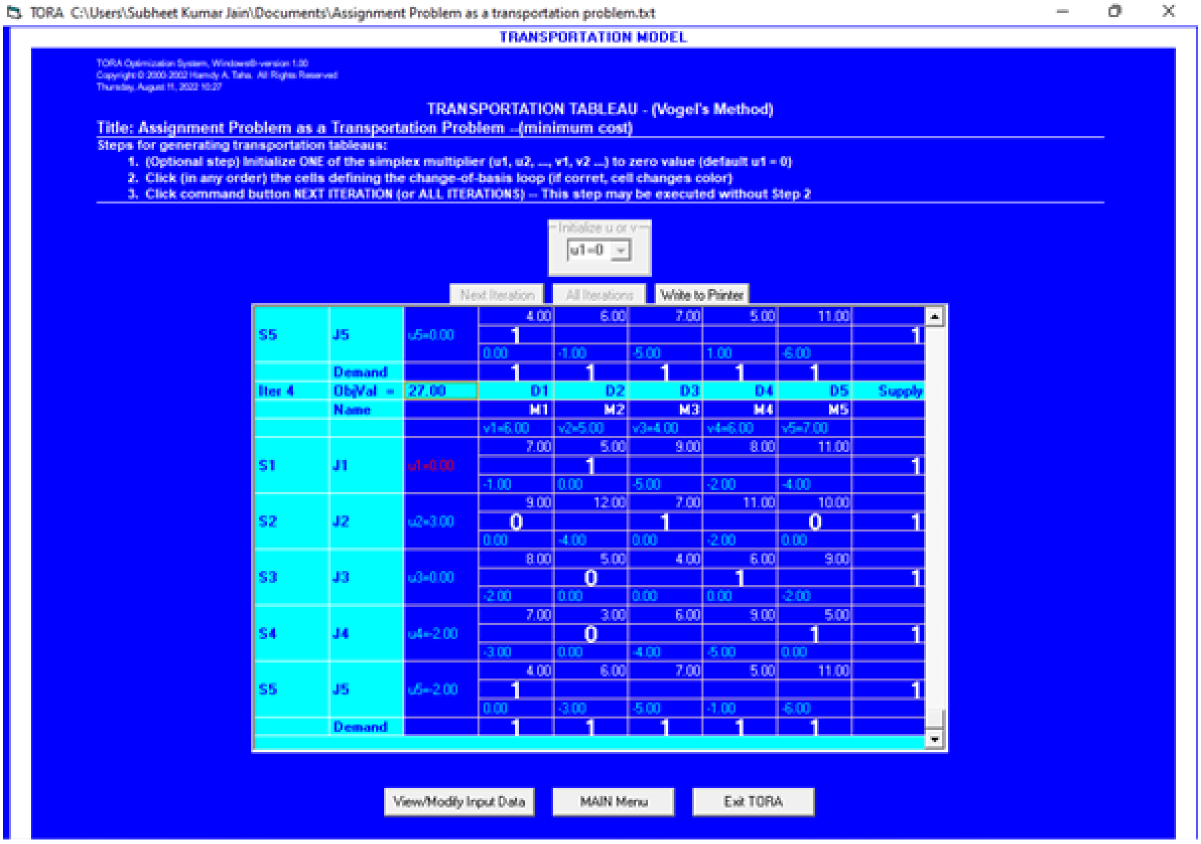
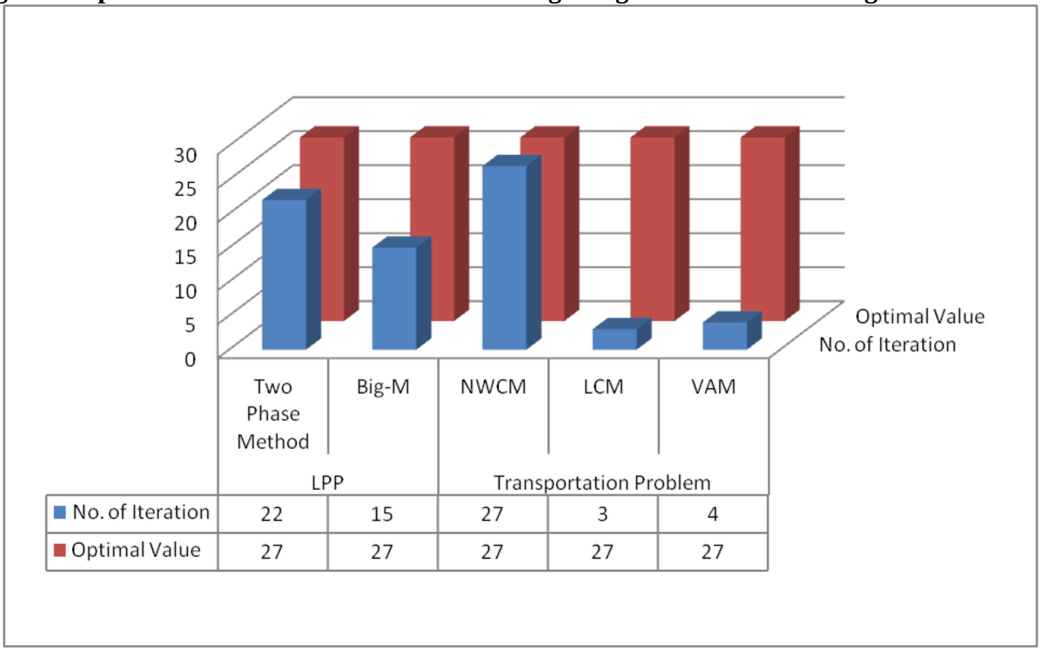


Table 1: Result of different methods for solving assignment problem Using TORA Software

Problem considers as	Method	No. of Iteration	Optimal Value
LPP	Two Phase Method	22	27
	Big-M	15	27
Transportation Problem	NWCM	27	27
	LCM	3	27
	VAM	4	27

Fig 8: Comparison of Different Methods for Solving Assignment Problem Using TORA Software



II. Conclusion:

In this paper we consider the assignment problem as LPP with objective is to assign n persons to n jobs, so as to minimize the total assignment cost and then we solve this problem by two phase & big M method by using TORA software. As assignment problem is special case of transportation problem so this problem also solved with the help of North- West Corner Method, Least (Minimum)-Cost Method & Vogel's Approximation Method (VAM) by using TORA software and compared optimal solution with number of required iterations to get optimal solution of different methods. When we consider assignment problem as linear programming problem and solve LPP by using two-phase & Big-M method both methods produce same optimal value but two phase methods required large no of comparison than Big-M Method. Similarly, when we consider assignment problem as a transportation problem and solved by NWCM, LCM & VAM, we get optimal solution in less number of iteration by LCM.

References

[1]. H. Basirzadeh, "Ones Assignment Method for Solving Mathematical formulation of assignment prob-," vol. 6, no. 47, pp. 2345–2355, 2012.

[2]. H. D. Afroz and D. M. A. Hossen, "New Proposed Method for Solving Assignment Problem and Comparative Study with the Existing Methods," IOSR J. Math., vol. 13, no. 02, pp. 84–88, 2017, doi: 10.9790/5728-1302048488.

[3]. A. Rashid, "An Alternative Approach for Solving Unbalanced Assignment Problems," vol. 40, no. 2, pp. 45–56, 2017.

[4]. M. Khalid, M. Sultana, and F. Zaidi, "New improved ones assignment method," Appl. Math. Sci., no. 81–84, pp. 4171–4177, 2014, doi: 10.12988/ams.2014.45327.

[5]. P. Jaskowski, "Assignment problem and its extensions for construction project scheduling" Czas. Tech., vol. 2014, no. January 2014, pp. 241–248, 2014, doi: 10.4467/2353737XCT.14.133.2583.

[6]. M. Fischetti, "Lezioni di Ricerca Operativa", Edizioni Libreria Progetto Padova, Italia, 1995.

[7]. Y. J. L. Kamm, D. J. T. Wagener, I. M. C. M. Rietjens, and C. J. A. Punt, "5-Fluorouracil in colorectalcancer: Rationale and clinical results of frequently used schedules," Anticancer. Drugs, vol. 9, no. 5,pp. 371–380, 1998, doi: 10.1097/00001813-199806000-00001.

[8]. L. Jain & C. Singh, "Finding an Optimal Solution of Assignment Problem", "YMER", Page no. 259-272, VOLUME 21: ISSUE 8 (Aug) – 2022 DOI: 10.37896/YMER21.08/23 ISSN : 0044-0477