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Optimality Principle Approach For Obtaining Minimal Boolean Expression Equivalent To A Given Function

Dr. Y. P. Sabharwal
Consulting Actuary
27 Jaina Building
Roshan Ara Road
Delhi-110007 (India)

Dr. Debasree Goswami
Associate Professor
Department Of Statistics
Hindu College
University Of Delhi (India)

Abstract:
Boolean algebra has played a significant role in the important and complicated task of designing electronic gadgetries. While it is desirable to get the minimal Boolean expressions in designing circuits, it is not necessarily uniquely available. In this paper, we present an algorithm for determining a Minimal Expression from the complete list of Prime Implicants.

Key words:*Minimal Boolean Expression, Prime Implicants, Circuit design, Algorithm, Digital logic, Circuit optimization*

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I. Introduction

In general, a very large number of (apparently) distinct Boolean Expressions are available for a given function $f(\underset{\sim}{x})$ defined over $(x_1, x_2, \dots, x_n)$. The Electronics Engineer responsible for the implementation of $f(\underset{\sim}{x})$ looks for an expression Φ which will require the minimum number of electronic gadgetries. The requisite choice lies in Φ with the number of literals ([1],[2]). Concepts e.g., Canonical Product Term (CPT), Prime Implicant (PI), Cover Table and Irredundant Expression prove useful in this context. Algorithms, e.g. Karnaugh's Map [2], Quinnie's Rule and Birkhoff and Bartee [1], are available for obtaining the complete list of Prime Implicants. Subsequently, the adoption of Essential Prime Implicants (EPI) and Dominance Principle help in determining Irredundant Expressions and finally, the Minimal Expression(s) Equivalent to the given function. These steps, however, remain heuristic in nature. This paper presents an Algorithm for determining a Minimal Expression from the complete list of Prime Implicants.

II. Enunciation of Birkhoff& Bartee's Algorithm for Determining the Complete List of Prime Implicants

Let $x_1, x_2, \dots, x_n$ denote the n variables and $a = (a_1, a_2, \dots, a_n), a_i \in \{0,1\}$ an assignment.

Step 1: Construct $S = \{a | f(a) = 1\}$.

Step 2: For each $a \in S$, define the Canonical Product Term $Y = y_1 \cdot y_2 \cdot \dots \cdot y_n$
where $y_i = x_i$if $a_i = 1$ and $x_i^{\sim}$if $a_1 = 0$.
Then the Sum of Product Terms $\Phi = \bigcup_{a \in S} Y$ will be equivalent to$f$.

Remark: This can invariably be taken as the starting point, even though, one may start with any Sum of Products Expression Equivalent to f.

Step 3: As long as possible, effect the following

(a) If there are two product terms α and β for which consensus γ is defined and γ is not covered by any of the product terms in Φ, then add it to Φ.

(b) If there are two product terms α and β in Φ such that α covers β, then drop β from Φ.

At the end of this step each of the product terms in Φ will be a PI and no PI of $f$will be absent from Φ. Also, this Algorithm is finite.

III. Cover Table, EPI and the Reduced Cover Table

Let $P = \{P_1, P_2, \ldots, P_m\}$ be the set of Prime Implicants as per Section 2 and

L_I = number of Literals in P_I.

Now develop the Cover Table and identify the EPI. Finally, determine the Reduced Cover Table by eliminating the EPI and the CPT covered by these.

Also introduce $E_L = 1$ if P_L is an EPI of 0 otherwise,

$C_1^{'}, C_2^{'}, \ldots C_s^{'}$ the s (uncovered) CPT in this Table,

n_I = number of non-essential PI that cover$C_I^{'}$,

$S_{IJ} = L$ if P_L is the Jth PI in P that covers $C_I^{'}$,

$F_I = F_{I-1} + n_I$, the cumulative sums of n_Is,

$M = (M_{ij})_{F_S \times m}$ with $M_{ij} = 1$,

the Control Matrix to be used for giving dynamic effect to the use of L_Is.

$D_{IJK} = 0$ for $K = 1,2,..,n_{I+1}, J = 1,2,\ldots,n_I$, and $I = 1,2,\ldots,s$

to be updated later for identifying Prime Implicants for inclusion in the Minimal Expression.

IV. The Stage Coach / Optimal Path Formulation

Let the $C_I^{'}$ correspond to the I^{th} stage and $P_{S_{IJ}}$ the J^{th} station at this stage. With $L_{S_{IJ}}$ as the cost of including $P_{S_{IJ}}$ in the Path, we have the usual Stage Coach Formulation, solvable by the Optimally Principle [2], viz. every Sub-Path of an Optimal path is itself Optimal. In the present context, however, inclusion of the same PI at more than one stage should contribute only once to the total number of literals. Following Algorithm will take care of this dynamic nature of the 'cost' structure.

V. Algorithm For Optimal Path from Source To Sink Under The Dynamic Cost Structure &Determining Therefrom A Minimal Sum of Products Expression Equivalent To The Given Function

With

T_{IJ} = cost of the Optimal Path from the J^{th} station of the I^{th} stage to the sink,

the Algorithm comprises of the following Steps:

Step 1: Determining the T_{IJ} and S_{IJ} and updating M

(a) $T_{sJ} = L_{S_{sJ}}$ and $M_{F_{s-1}+J\ S_{sJ}} = 0$

(b) $T_{IJ} = \min_K (T_{I+1K} + M_{F_I+K\ S_{IJ}} * L_{S_{IJ}})$

Let K_0 be the first K for which the minimum value was attained

$D_{IJK_0} = 1$

$M_{F_{I-1}+J\ L} = M_{F_I+K_0\ L}$ for $L = 1,2,\ldots,m$

$M_{F_{I-1}+J\ S_{IJ}} = 0$

Step 2: Identifying the station with least T_{IJ}

$T_{iJ_0} = \min_J (T_{iJ})$

Step 3: The non-essential Prime Implicants to be included in the Optimal Expression

$E_{S_{iJ_0}} = 2$ and $E_{S_{I+i\ K_0}} = 2$ for $I = 1,2,\ldots,s-1$

Step 4: The Sum of Products Expression

$$\Phi = \bigcup_{E_L>0} P_L$$

is a Minimal Expression (with the least number of literals) Equivalent to the given function.

VI. Illustration

The Output depicting various aspects in respect of the function

$$f(x) = \sum(0,2,4,5,10,11,13,15)$$

obtained are given below in the tables.

Table - 1

LISTING THE ENTRIES/DELETIONS WITH MODES												
					MODE OF							
SR.NO.	PRODUCT TERM				ENTRY					EXIT		
1	X1~	X2~	X3~	X4~	INITIAL					S(	9	)
2	X1~	X2~	X3	X4~	INITIAL					S(	9	)
3	X1~	X2	X3~	X4~	INITIAL					S(	10	)
4	X1~	X2	X3~	X4	INITIAL					S(	10	)
5	X1	X2~	X3	X4~	INITIAL					S(	11	)
6	X1	X2~	X3	X4	INITIAL					S(	11	)
7	X1	X2	X3~	X4	INITIAL					S(	12	)
8	X1	X2	X3	X4	INITIAL					S(	12	)
9	X1~	X2~		X4~	C(	1	,	2	)			
10	X1~	X2	X3~		C(	3	,	4	)			
11	X1	X2~	X3		C(	5	,	6	)			
12	X1	X2		X4	C(	7	,	8	)			
13	X1~		X3~	X4~	C(	9	,	10	)			
14		X2~	X3	X4~	C(	9	,	11	)			
15		X2	X3~	X4	C(	10	,	12	)			
16	X1		X3	X4	C(	11	,	12	)			

Table - 2

COVER TABLE									
	CANONICAL PRODUCT TERMS								
PRIME IMPLICANTS	C1	C2	C3	C4	C5	C6	C7	C8	
P1	*	*							
P2			*	*					
P3					*	*			
P4							*	*	
P5	*		*						
P6		*			*				
P7				*			*		
P8						*		*	

Table - 3

PRIME IMPLICANTS TO BE INCLUDED IN THE MINIMAL EXPRESSION						
SR. NO.	PRIME IMPLICANT				NO OF LITERALS	STATUS*
1	X1~	X2~		X4~	3	2
2	X1~	X2	X3~		3	2
3	X1	X2~	X3		3	2
4	X1	X2		X4	3	2
TOTAL					12	
* 1: ESSENTIAL 2: NON-ESSENTIAL						

References

[1]. Birkhoff And . C. Bartee, Modern Applied Algebra, Delhi: Mcgraw-Hill, 1970.

[2]. N. Biswas, Logic Design Theory, India: Prentice Hall, 1993.