

Mathematical Model Of Two Layered Non-Newtonian Blood Flow Through Artery In Presence Of Stenosis During Malaria

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Abstract:

In this paper, our aim is to verify non-Newtonian Power law model according to stress and strain rate of hepatic artery in presence of stenosis which is axis-symmetric shape during malaria and employing Navier Stoke's equation in cylindrical co-ordinate system. Here we considered blood flow two phase in which first phase is of blood plasma and second is that of red blood cells. All required formulations are written in tensorial form and to calculate the value of parameter by numerical method for clinical data of blood pressure and hemoglobin. We study the graph of blood pressure drop versus hematocrit along the length of stenosis and also found that graphical presentation for particular parametric value is much closer to clinical observation.

Keywords –Stenosis, Hematocrit, Blood pressure drop, Non-Newtonian power law model, Stress, Strain rate.

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I. INTRODUCTION

Structure and function of liver

The liver consists of two primary lobes, the right lobe and smaller left lobe. These lobes are further divided into smaller lobes called lobules. Within each lobule, these are specialized liver cells called hepatocytes. Hepatocytes are the functional units of the liver and are arranged in rows or cords separated by blood-filled spaces called sinusoids. The liver is supported by two major blood vessels: the hepatic artery which delivers oxygenated blood, and portal vein which carries nutrient-rich blood from the digestive organs. The liver also has a complex network of bile ducts that collect and transport bile; a substance produced by the liver and essential for digestion [5]. Detoxification is one of the liver's primary functions. It removes harmful substances, such as drugs, alcohol and metabolic waste products from the blood. The liver metabolizes these substances into less toxic forms, which are then eliminated from the body. The liver produced bile a greenish-yellow fluid that helps in the digestion and absorption of fats. Bile contains bile salts that emulsify fat, making it easier to break down by enzymes in the small intestine [42].

The function of hepatic artery

The hepatic artery arises from the celiac trunk which is branch of the abdominal aorta. The hepatic artery carries nutrient-rich blood to the liver, providing it with oxygen and other essential substances necessary for its metabolic functions. This blood supply is critical for maintaining liver health and enabling its various functions, such as detoxification, protein synthesis and bile production [6]. It is generally believed that half of the oxygen requirements of the liver are provided by portal venous blood and the other half is delivered from the hepatic artery. Any blockage or damage to the hepatic artery can have serious consequences for liver function and overall health. Conditions such as liver tumor, arterial thrombosis or hepatic artery stenosis can disrupt blood flow to the liver, leading to tissue damage or liver dysfunction [7].

Stenosis

Stenosis is the narrowing or constricting of artery due to the development of arteriosclerotic plaques or other types of abnormal tissue development. The development of stenosis in an artery can have serious consequences and disrupt the normal functioning of the circulatory system. When blood flow reduced, nutrients and oxygen cannot travel to the tissue that needs it. The presence of stenosis can lead to the serious circulatory blood flow in porous vessel having double stenosis in the presence of an external magnetic field has been

instigated by sinha et al. [11] while shit and Roy [9] analyzed a mathematical model for unsteady flow of blood through arteries having stenosis, in which blood was treated as Newtonian viscous incompressible fluid and also investigated that the wall shear stress decreases as stenosis shape parameter increases, but in the case of increasing stenosis size, stenosis length and peripheral layer viscosity wall shear stress is increases. Bali and Awasthi [7] analyzed the effect of external magnetic field on blood flow in stenotic artery.

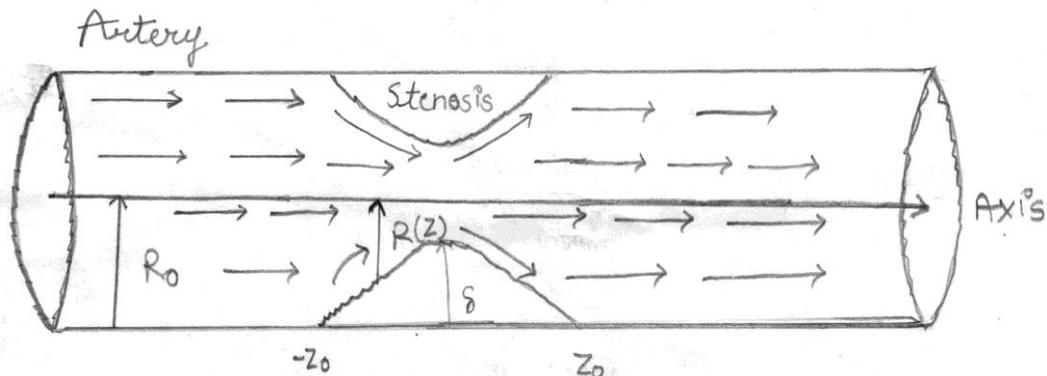


Figure 1: Artery with stenosis

The Blood

Blood can be divided into two components – a yellow fluid, plasma and cells which are suspended in it. Plasma is that part of the extracellular fluid which is limited to the blood vessels. Plasma which constitutes 55% of blood fluid is mostly water (92%) by the volume [8]. Blood account for 7% of the human body weight [9] by volume; red blood cells constitute about 45% of whole blood, plasma about 54.3% and white cells about 0.7% [10]. Whole blood (plasma and cells) displays non-Newtonian fluid dynamics. Red blood cells comprise the blood hemoglobin and distribute oxygen [11]. Since the blood is complex non-Newtonian flow various sign of non-Newtonian rheology such as shear thinning [35], yield stress [36] and viscoelasticity [37]. The blood is also characterized by distinctive thixotropy behavior [38] revealed by the appearance of hysteresis loops during shearing cycles [39]. These non-Newtonian properties do not affect the flow patterns inside the flow paths and the fluid transportation only but they also affect the mechanical stress on the blood vessel walls and the surrounding tissues [40] especially in case of irregular lumen geometry like stenosed arteries [41].

Description of disease

Malaria is a mosquito borne infectious disease of human caused by plasmodium parasites. Malaria parasite invades the erythrocyte and assumes different roles by molecular mimicry of human proteins, thus dodging the human immune system [19]. Some of the prominent changes due to malaria parasite are also associated with the functional and morphological properties of erythrocytes [20]. This parasite progressively consumes/degrades intracellular protein [21], alters the membrane integrity [22], increase the cell stickiness [23], affect the membrane permeability [24] and reduces the regional charge distribution [25] leading to decrease in the RBC deformability [26].

II. REAL MODEL

Selection of Parameter and Frame of Reference

We choose generalized three dimensional orthogonal curvilinear co-ordinate system, shortly prescribed as E^3 , addressed as 3-dim Euclidean space for mathematical modeling of hepatic blood flow. The biophysical laws thus articulated completely hold well in any coordinate system, which is mandatory for the veracity of the law (1990) [13]. Now let the coordinate axes be OX^i where O is the origin and superscript $i = 1, 2, 3$ let X^i be the coordinate of any point P in space, the mathematical description of state of a moving blood is affected by means of functions which give the distribution of blood velocity $v^i = v^k(X^i, t)$, $k = 1, 2, 3$ and of any two thermodynamic quantities pertaining to the blood for the pressure $p = p(X^i, t)$ and the density $\rho = \rho(X^i, T)$. Blood is the mixed fluid it is divided in two phase first is plasma and other is blood cells. The blood cells are enclosed with a semi-permeable membrane whose density is greater than that of plasma; these blood cells are uniformly distributed in plasma. Thus blood can be considered as homogeneous mixture of two phases.

Mathematical Formulation

According to V.Upadhyay and P.N.Pandey the blood flow in hepatic artery remote from heart to be non-Newtonian and effective viscosity of blood flowing in the arteries remote from the heart depends upon the strain rate [16]. In this situation, the blood flow becomes non Newtonian for strain rate between 5 to 200 per second, and the power law $T' = \eta_m e^n$ where $0.68 \leq n \leq 0.80$ hold good for blood flow. The stenosis is taken to be axially-symmetric whose surface is given by the following equation

$$\frac{R(z)}{R_0} = 1 - \frac{\delta}{2R_0} \left(1 + \frac{\cos \pi z}{z_0} \right), -z_0 \leq z \leq z_0 \quad (2.1)$$

We shall assume that $\frac{\delta}{R_0} \ll \ll 1$ and $R_e \frac{\delta}{R_0} \ll \ll 1$, where R_e the Reynolds number of fluid flows

The equation of continuity for power law flow will be as follows:

$$\frac{1}{\sqrt{g}} (\sqrt{g} v^i)_{,i} = 0 \quad (2.2)$$

again the

equation of motion in tensorial form is as follows:

$$\rho_m \frac{\partial v^i}{\partial t} + (\rho_m v^j) v^i_{,j} = \tau^{ij}_{,j} \quad (2.3)$$

The

constitutive equations for the power law non-Newtonian blood flow is as follows:

$$\tau^{ij} = -p g^{ij} + \eta_m (e^{ij}) + \tau'^{ij}$$

According to Singh P. and Upadhyay K.S. the flow of blood is affected by the presence of blood cells and this effect is directly proportional to the volume occupied by blood cells [15]. Let the volume portion covered by blood cells in unit volume be X, this X is replaced by H/100, where H is the hematocrit. Then the volume portion covered by the plasma will be 1-X.

$$\text{Where } \rho_m = X\rho_c + (1-X)\rho_p \text{ and } \eta_m = X\eta_c + (1-X)\eta_p$$

Since the blood vessels are cylindrical form, then the above governing equations have to transform into cylindrical co-ordinates system.

$$\text{We suppose that } x^1 = r, \quad x^2 = \theta, \quad x^3 = z$$

Matrix of metric tensor for cylindrical system is as follow:

$$[g_{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

And matrix of conjugate metric tensor is

$$[g^{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Whereas Christoffel's symbols of 2nd kind are as follows:

$$\left\{ \begin{matrix} 1 \\ 2 \end{matrix} \right\} = -r, \quad \left\{ \begin{matrix} 2 \\ 2 \end{matrix} \right\} = \left\{ \begin{matrix} 2 \\ 1 \end{matrix} \right\} = \frac{1}{r} \quad \text{except of these, all are zero.}$$

Contravariant and physical components of velocity of blood flow will be related as

$$\sqrt{g_{11}} v^1 = v_r \Rightarrow v_r = v^1, \quad \sqrt{g_{22}} v^2 = v_\theta \Rightarrow v_\theta = r v^2, \quad \sqrt{g_{33}} v^3 = v_z \Rightarrow v_z = v^3$$

Further the physical component of $-p_{,j} g^{ij}$ are $-\sqrt{g_{ii}} p_{,j} g^{ij}$

The matrix of physical component of shearing stress – tensor

$\tau^{ij} = \eta_m (e^{ij})^n = \eta_m (g^{ik} v^i_{,k} + g^{jk} v^j_{,k})^n$ will be as follows:

$$\begin{bmatrix} 0 & 0 & \eta_m \left(\frac{dv}{dr} \right)^n \\ 0 & 0 & 0 \\ \eta_m \left(\frac{dv}{dr} \right)^n & 0 & 0 \end{bmatrix}$$

The covariant derivative of τ^{ij} is

$$\tau^{ij}_{;j} = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^j} (\sqrt{g} \tau^{ij}) + \left\{ \begin{matrix} i \\ j \end{matrix} \right\} \tau^{kj}$$

According to above facts the governing tensorial equations can be transformed into cylindrical forms which are as follows:

The equation of continuity –

$$\frac{\partial v}{\partial z} = 0 \quad (2.4)$$

The equation of motion along r, θ , z direction become

$$-\frac{\partial p}{\partial r} = 0, \quad 0 = 0, \quad 0 = -\frac{\partial p}{\partial r} + \frac{\eta_m}{r} \frac{\partial}{\partial r} \left(r \left(\frac{dv}{dr} \right)^n \right) \quad (2.5)$$

Here this fact has been taken in view that in axial flow is symmetric in artery so that $v_\theta = 0$ and v_r, v_z and p do not depend upon. Also the blood flows steadily, i.e.

$$\frac{\partial p}{\partial t} = \frac{\partial v_r}{\partial t} = \frac{\partial v_\theta}{\partial t} = \frac{\partial v_z}{\partial t} = 0$$

On integrating equation (2.4), we get $v_z = v(r)$ (2.6) since v does not depend upon θ

The integrating equation (2.5), we get $p = p(z)$ (2.7) since p does not depend upon θ

Now, with the help of equation (2.6) and (2.7), the equation of motion (2.5) converts in the following form:

$$0 = -\frac{dp}{dz} + \frac{\eta_m}{r} \frac{d}{dr} \left(r \left(\frac{dv}{dr} \right)^n \right) \quad (2.8)$$

III. Analysis and solution

The pressure gradient $-\frac{dp}{dz} = P$ of blood flow in the arteries remote from liver can be supposed to be constant and hence the equation (2.8) takes the following form:

$$\frac{d}{dr} \left[r \left(\frac{dv}{dr} \right)^n \right] = -\frac{P(z)r}{\eta_m} \quad (3.1)$$

Integrating equation [3.1], we get

$$r \left(\frac{dv}{dr} \right)^n = -\frac{P(z)r^2}{2\eta_m} + A(z) \quad (3.2)$$

Since the velocity of blood flow on the axis of the cylindrical arteries is maximum and constant i.e. $r=0, v = v_0$ (constant), we apply this condition on equation [3.2] $A(z) = 0$. Hence the equation (3.2) takes the following form:

$$r \left(\frac{dv}{dr} \right)^n = -\frac{P(z)r^2}{2\eta_m} \Rightarrow -\frac{dv}{dr} = \left(\frac{P(z)r}{2\eta_m} \right)^{1/n} \quad (3.3)$$

Again integrating equation [3.], we get

$$v = -\left(\frac{P(z)}{2\eta_m} \right)^{1/n} \frac{r^{\frac{1}{n}+1}}{\left(\frac{1}{n}+1 \right)} + B(z) \quad (3.4)$$

Now for the value of $B(z)$ we apply the no-slip condition on the stenosis surface given by

$$v = 0 \text{ at } r = R(z), -z_0 \leq z \leq z_0$$

$v = 0$ at $r = R_0, -z_0 \leq z \leq z_0$, we obtain

$$B(z) = \left(\frac{P(z)}{2\eta_m} \right)^{1/n} \frac{nR(z)^{1/n+1}}{n+1}$$

Hence the equation [3.4] takes the following form:

$$v = \left(\frac{P(z)}{2\eta_m} \right)^{1/n} \frac{n}{n+1} \left[R(z)^{\frac{1}{n}+1} - r^{\frac{1}{n}+1} \right] \quad (3.5)$$

This is velocity of blood flow passing through stenosis.

IV. Result and discussion (Bio- Interpretation)

The total flow- flux of blood through the transverse section of the hepatic artery is [17].

$$\begin{aligned} Q &= \int_0^{R(z)} v \cdot 2\pi r dr = \int_0^{R(z)} \left(\frac{P(z)}{2\eta_m} \right)^{1/n} \cdot \frac{1}{n+1} \left(R(z)^{\frac{1}{n}+1} - r^{\frac{1}{n}+1} \right) 2\pi r dr \\ &= \left(\frac{P(z)}{2\eta_m} \right)^{1/n} \cdot \frac{2\pi n}{n+1} \left(\frac{R(z)^{\frac{1}{n}+1} r^2}{2} - \frac{n \cdot r^{\frac{1}{n}+3}}{3n+1} \right) \Big|_0^{R(z)} \\ &= \left(\frac{P(z)}{2\eta_m} \right)^{1/n} \cdot \frac{2\pi n}{n+1} \cdot \frac{(n+1)R(z)^{\frac{1}{n}+3}}{2(3n+1)} \\ Q &= \left(\frac{P(z)}{2\eta_m} \right)^{1/n} \cdot \frac{\pi n R(z)^{\frac{1}{n}+3}}{(3n+1)}, \text{ where } P(z) = -\frac{dp}{dz} \end{aligned}$$

$$Q = \left[\frac{P_i - P_f}{2\eta_m(z_i - z_f)} \right]^{1/n} \cdot \frac{\pi n R(z)^{\frac{1}{n}+3}}{(3n+1)} \quad (4.1)$$

Observation:

Hemoglobin Vs Blood pressure is taken from Medical College Banda (U.P.)

Patient Name:Mr. Santosh, **Age / Sex:**64 Years / Male, Annual No. 1945/2022
Clin: Dr.Karan Rajpoot

Table 1: Clinical blood pressure Vs hemoglobin

S.No.	Date	B.P. (In mm hg)	Hemoglobin (gm/dl)	Hematocrit (H)	B.P. (In Pascal)	Cli. BPD
1	21/09/2022	150/96	10.7	32.1	19998/12798.12	-3599.94
2	24/09/2022	145/92	10.2	30.6	19331.4/12265.44	-3532.98
3	26/09/2022	142/90	9.6	28.8	18664.8/11998.8	-3332.6
4	30/09/2022	147/94	8.8	26.4	19598.04/12532.08	-3532.98

Average Systolic Pressure =19398.06 Pa

Average Diastolic Pressure = 112398.61 Pa

H= Average hematocrit = 29.1

Pi= Pressure in Artery = Average Systolic Pressure =19398.06 Pa

Pf= Pressure in Arterioles = $\frac{S+D}{2}$ = 15898.335 Pa

According to Glenn Elert

(2010)

η_m = Viscosity of mixture = 0.035p.s

According to Gustafson, Daniel R. (1980)

η_p = Viscosity of plasma =0.0015 p.s [18]

Length of common hepatic artery = 0.0347 m

Since $\eta_m = \eta_c X + \eta_p (1 - X)$

$$\text{or, } \eta_m = \eta_c \frac{H}{100} + \eta_p \left(1 - \frac{H}{100}\right) \text{ where } X = \frac{H}{100} \cdot 0.035 = \eta_c \frac{29.1}{100} + 0.0015 \left(1 - \frac{29.1}{100}\right)$$

$\eta_c = 0.116620275$ P.S =Viscosity of cells

Now putting the value of η_c in η_m , we have

$$\eta_m = 0.116620275 \frac{H}{100} + 0.0015 \left(1 - \frac{H}{100}\right)$$

$$\eta_m = 115.120275 \times 10^{-5} H + 0.0015$$

If the 50% stenosed hepatic artery

δ =50% Of radius of hepatic artery =0.0025× 0.5 = 0.00125 m

$R(z)=R_0 - \delta = 0.0025 - 0.00125 = 0.00125$ m

Now from equation (4.1), blood flow flux in hepatic artery is given as

$$Q = \left[\frac{P_i - P_f}{2\eta_m(Z_i - Z_f)} \right]^{1/n} \cdot \frac{\pi n R(z)^{\frac{1}{n}+3}}{(3n + 1)}$$

where $Q = \frac{1000\text{ml}}{\text{min}} = 0.0167 \text{ lit/sec} = 1.67 \times 10^{-5} \text{ m}^3/\text{sec}$

$$1.67 \times 10^{-5} = \left[\frac{19398.06 - 15898.335}{2 \times 0.035 \times 0.0347} \right]^{\frac{1}{n}} \cdot \frac{3.14 \times n \times (0.00125)^{\frac{1}{n}+3}}{(3n + 1)}$$

$$2723.05733 = (1801.01121)^{\frac{1}{n}} \left(\frac{n}{3n + 1} \right)$$

$$f(n) = n \log \frac{n}{3n+1} - 3.43505678n + 3.25551642 = 0$$

Or, $f(x) = x \log \frac{x}{3x+1} - 3.43505678x + 3.25551642 = 0$

$$df(x) = \log \frac{x}{3x+1} + \frac{1}{3x+1} - 3.43505678$$

Solve above equation by Newton- Raphson method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x \cdot \log(x / ((3 \cdot x) + 1)) - (3.43505678 \cdot x) + 3.2555164 = 0$$

Programme:

%% Ingredients

f=@(x) x*log(x/((3*x)+1))-(3.43505678*x)+3.21080489

df=@(x) log(x/((3*x)+1))+1/((3*x)+1)- 3.43505678;

e=10^-4;

x0=0.7;

n=10;

%processing

if df(x0)~=0

```

for i=1:n
x1=x0-f(x0)/df(x0);
fprintf('x%d = %.4f\n',i,x1)
if abs(x1-x0)<e
break
end
if df(x0)==0
disp('Newton Raphson failed')
end
x0=x1;
end
else
disp('Newton Raphson failed');
end

```

Answer:
 >> NR_method
 x1 = 0.8050
 x2 = 0.7907
 x3 = 0.8071
 x4 = 0.8012
 x5 = 0.8012
 or, n = 0.8012

Now again from equation (4.1), we will find pressure drop

$$\Delta P = \left[\frac{(3n + 1)Q}{\pi n R(z)^3} \right]^n \left[\frac{2\eta_m \Delta Z}{R(z)} \right]$$

Substituting values in above equation, we have

$$\Delta P = \left[\frac{(3 \times 0.8012 + 1)(1.67 \times 10^{-5})}{3.14(0.8012)(0.00125)^3} \right]^{0.8012} \left[\frac{2\eta_m(0.0347)}{0.00125} \right]$$

$$\Delta P = (99983.6176)(115.120275 \times 10^{-5}H + 0.0015)$$

$$\Delta P = 115.1014 H + 149.975$$

Putting values of H in above equation (4.3) .We get the following table of blood pressure drop

Table2: Hematocrit Vs Blood pressure drop

Date	21/09/2022	24/09/2022	26/09/2022	28/09/2022
Hematocrit (H)	32.1	30.6	28.8	26.4
Blood pressure drop (ΔP)	3844.73	3672.07	3464.89	3188.65

Figure 2: Graph of hematocrit vs blood pressure drop

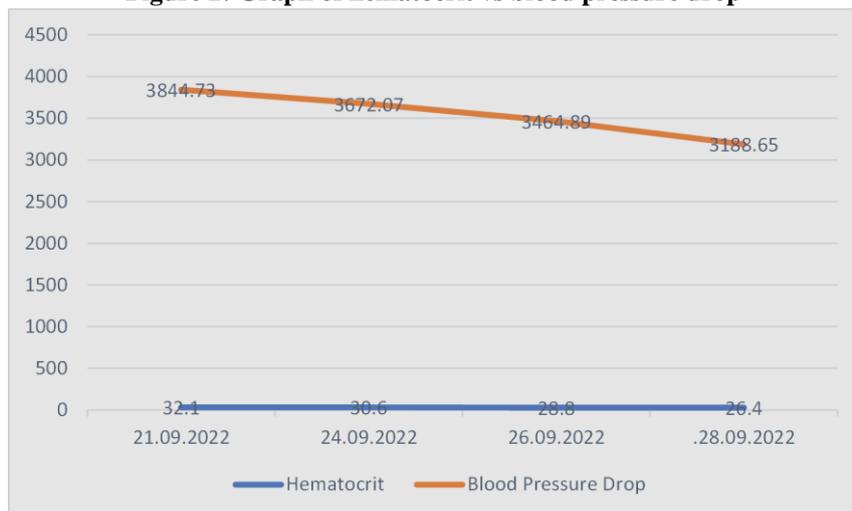
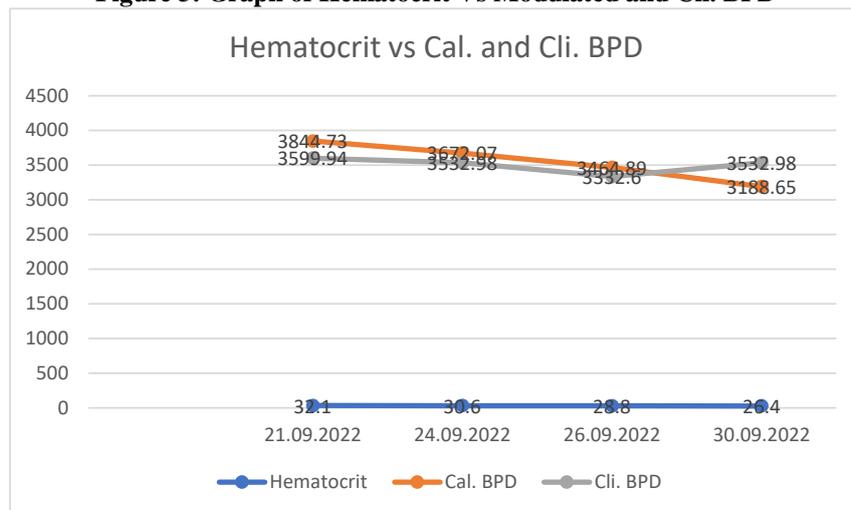


Figure 3: Graph of Hematocrit Vs Modulated and Cli. BPD



V. Observation of graph

The figure 2 graph between blood pressure drop and hematocrit for malaria patient when hepatic artery stenosed 50%, we observed that minimum blood pressure drop 3188.65 on dated 28/09/2022 and maximum blood pressure drop 3844.73 on dated 21/09/2022. At the hematocrit value from 32.1 to 26.4 via 30.6 and 28.8, the blood pressure drop straightly decreases on dated from 21/09/2022 to 28/09/2022 via 24.09/2022 and 26/09/2022. In figure 3 comparative studies of two graphs: graph (i) hematocrit Vs modulated blood pressure drop and graph (ii) hematocrit Vs clinical blood pressure drop.

VI. Conclusion:

The slope of straight line is of absolute value then blood pressure drop depends on hematocrit, when hematocrit increases the blood pressure drop increases and when hematocrit decreases then blood pressure drop also decreases. Here trend of straight line in decreasing sense then we can suggest medicine dose slowly increases. In comparative study of graphs we see that both graphs have nearly same character. So, power law model is verified.

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