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Approximate Solution Of A Covid-19 Mathematical Model By Using Numerical Methods

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Abstract
A new COVID-19 SIR model introduced by Rahim ud Din and Ibrahem A. Algehyne describes the interaction between susceptible S(t), Infected I(t) and Recovered R(t) populations. This model is represented in nonlinear ordinary differential system of equations. In this paper, numerical simulations are calculated by Matlaband compared with the results by numerical fourth order Runge-Kutta (RK4) and Nonstandard Finite Difference Scheme (NSFD).
Keywords: *SIR; Covid; NFDS; RK4.*

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I. INTRODUCTION

COVID-19, caused by the novel coronavirus SARS-CoV-2, emerged in late 2019 and quickly became a global pandemic, affecting millions of people worldwide [18,3,6,24]. Mathematical models have played a crucial role in understanding, predicting, and managing the spread of the virus. These models provide a quantitative framework for epidemiologists, healthcare professionals, and policymakers to make informed decisions and implement effective control measures [7,5,18].

One of the fundamental mathematical models used to study the spread of infectious diseases like COVID-19 is the SIR model (Susceptible-Infectious-Recovered) [8,14,4]. This model divides the population into compartments of susceptible individuals, infectious individuals, and recovered individuals. It tracks how these populations change over time through a system of differential equations, considering parameters such as the transmission rate and recovery rate [25,13,9,11].

These models are continually updated with real-world data, improving their accuracy and usefulness in the ongoing fight against COVID-19.[21]

II. MODEL FORMULATION

The "Mathematical Analysis of COVID-19 by Using SIR Model with Convex Incidence Rate" is a research or analysis that extends the classic SIR (Susceptible-Infectious-Recovered) model used for modeling disease spread, particularly the spread of COVID-19. In this extended model, the incidence rate of infection is described as a convex function of the number of infectious individuals. This means that the rate of disease transmission increases with the number of infectious individuals, but at a decreasing rate [4].

The system of differential equations provided represents a modified SIR (Susceptible-Infectious-Recovered) model for modeling the spread of an infectious disease [15,22].

The Model
The differential equations governing the model are as follows [2]:

$$\begin{cases} \frac{dS}{dt} = b - k\big(1 - \alpha S(t)I(t)\big) - \alpha k\beta S(t)I(t) - \square S(t), \\ \frac{dI}{dt} = k\big(1 - \alpha S(t)I(t)\big) + \alpha k\beta S(t)I(t) - (d_0 + \gamma + \square)I(t), \\ \frac{dR}{dt} = \gamma I(t) - \square R(t). \end{cases} \quad (1)$$

S(t): The number of susceptible individuals at a given time.
I(t): The number of infectious individuals at the same time.
R(t): The number of individuals who have recovered or been removed from the infected group.

The key components of this extended SIR model are as follows:
b: represents the birth rate of new susceptible individuals into the population.

k: represents a constant that controls how quickly individuals transition from being susceptible to infected.
α:represents a parameter that influences the interaction between susceptible and infectious individuals.
$\boldsymbol{d_0}$: represents the disease-induced death rate among infected individuals.
***γ*:** represents the recovery rate of infected individuals.
μ: represents the natural death rate.

This set of differential equations describes how the populations of susceptible, infectious, and recovered individuals change over time in response to interactions and transitions between these compartments. The model considers births, deaths, disease transmission, recovery, and natural mortality rates.

To analyze and solve this system, we would typically use mathematical techniques such as numerical methods or computer simulations. The specific parameter values and initial conditions would need to be determined based on the characteristics of the disease you are modeling.

Solving this system of differential equations analytically can be challenging, especially for nonlinear systems like the one you've presented. Typically, such systems are solved numerically using software tools like MATLAB [15].

III. NUMERICAL RESULTS AND ANALYSIS

The initial values and the parameters used to solve the system are summarized in the following table. They were taken from**www.worldometers.info/coronavirus/country/pakistan,** 2 February to 20 September, 2021.

Parameters	**Physical description**	**Numerical value**
S(t)	Susceptible compartment	220 in millions
I(t)	Infected compartment	0 in million
R(t)	Recovered compartment	0 in million
d0	Death due to corona	0.02
μ	Natural death	0.0062
b	Birth rate	10.7
β	Protection rate	0.9, 0.0009
k	Constant rate	0.00761
α	Isolation rate	0.9, 0.0009
γ	Recovery rate	0.0003

Table 1: Parameters and their values [19].

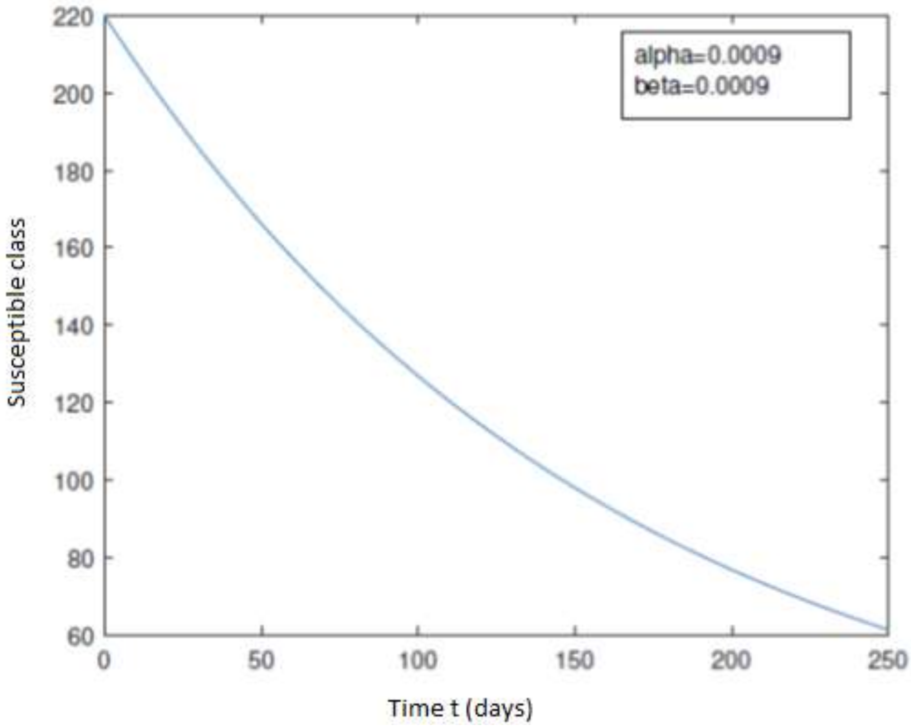

Figure 1: Dynamical behaviour of susceptible population of the considered model using Matlab

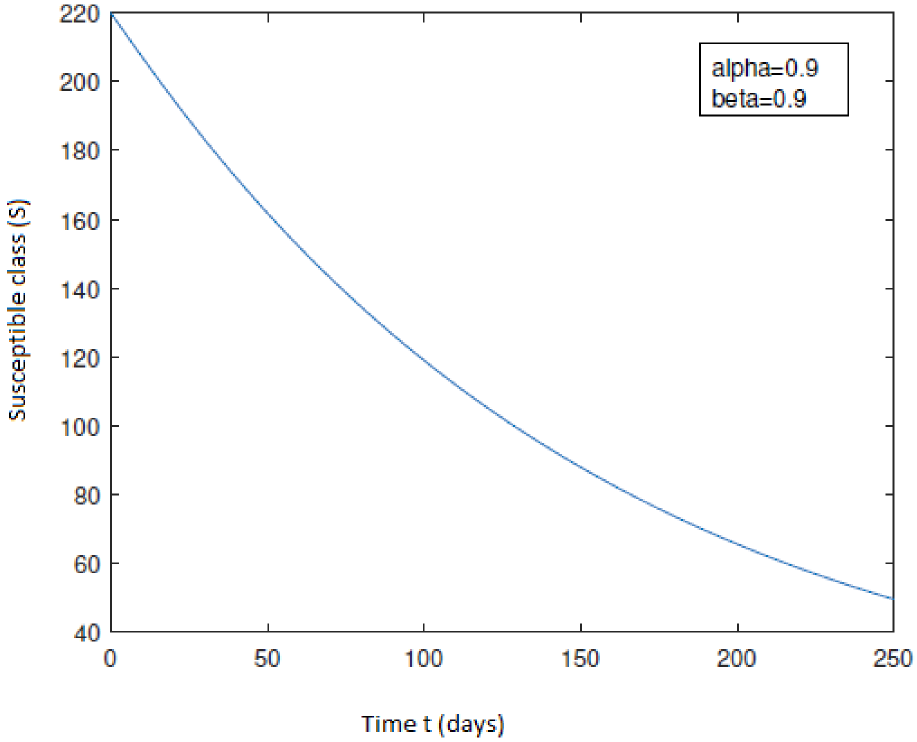

Figure 2: Dynamical behaviour of susceptible population of the considered model using Matlab

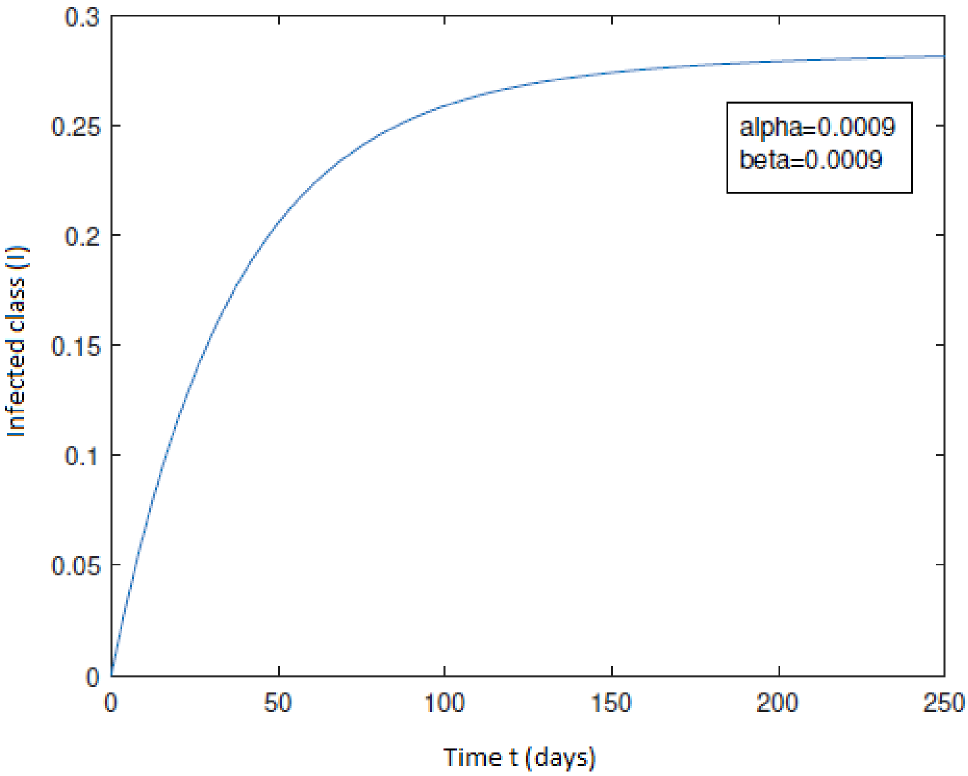

Figure 3: Dynamical behaviour of infected population of the considered model using Matlab

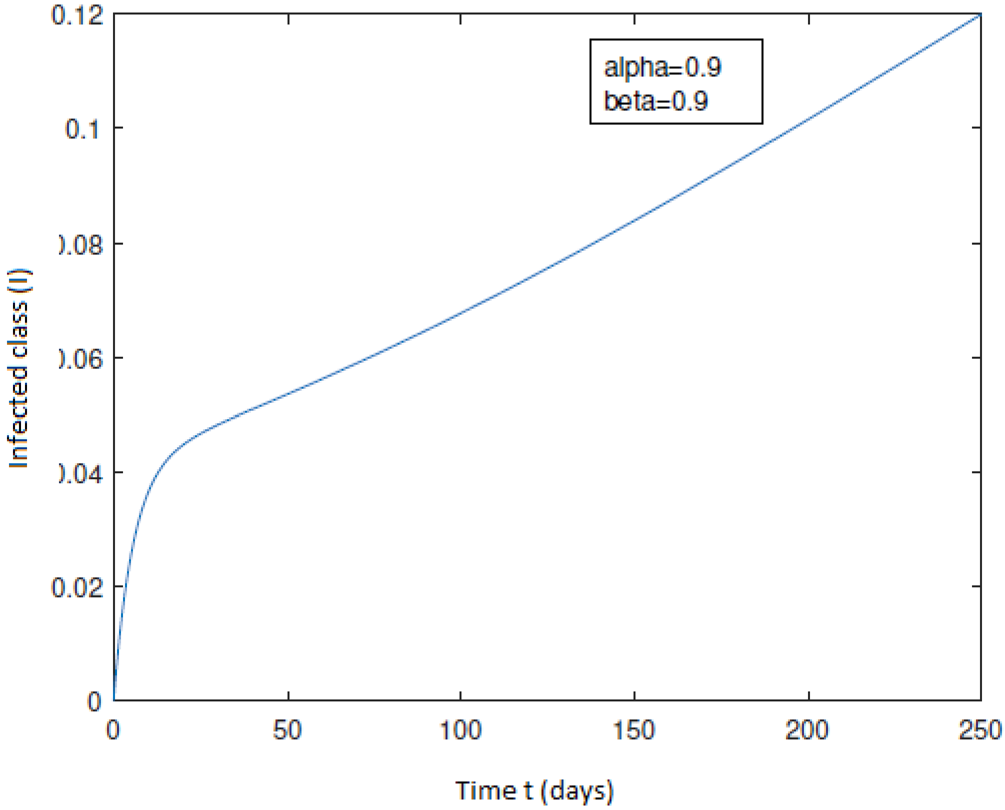

Figure 4: Dynamical behaviour of infected population of the considered model using Matlab

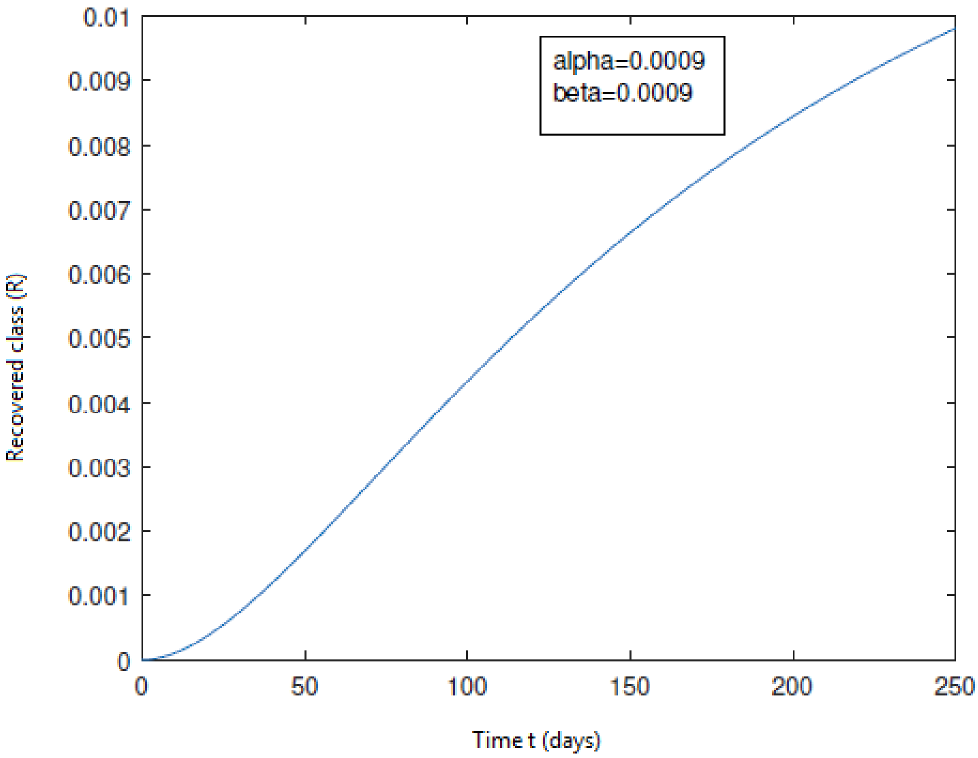

Figure 5: Dynamical behaviour of recovered population of the considered model using Matlab

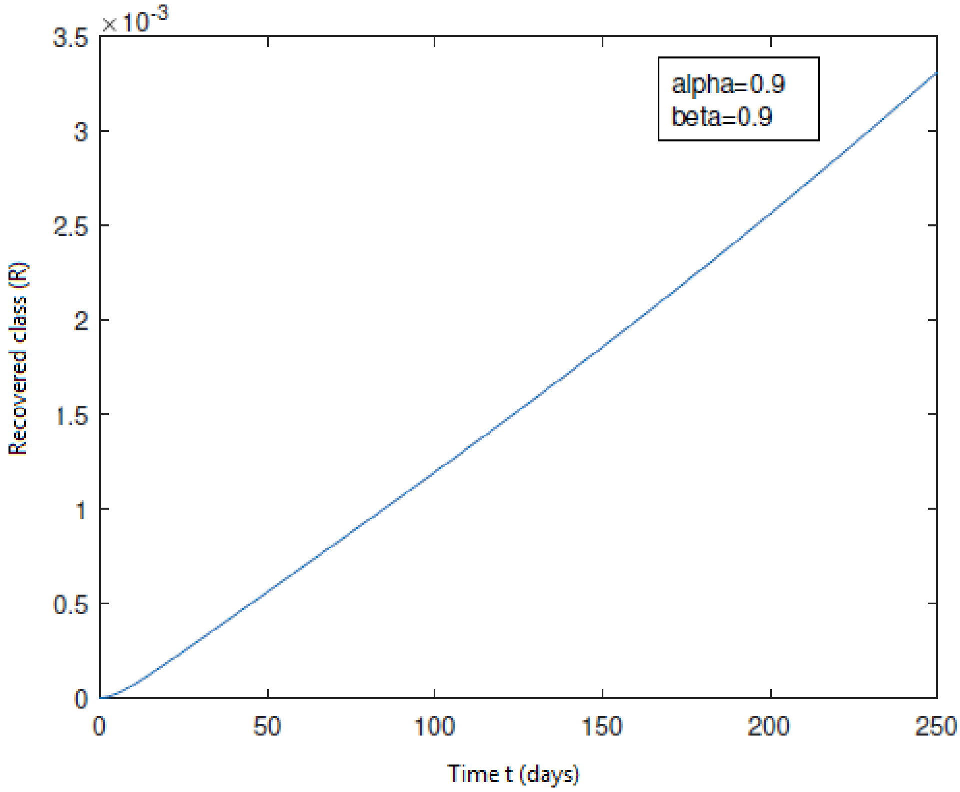

Figure 6: Dynamical behaviour of recovered population of the considered model using Matlab

The choice of method depends on the specific characteristics of your problem, including the nature of the nonlinearities, boundary conditions, geometry, and desired level of accuracy. It's essential to carefully analyze your problem and consider the strengths and weaknesses of different methods before selecting the most suitable one for your application. Additionally, computer software and numerical libraries are often available to implement these methods efficiently.

IV. RUNGE KUTTA 4TH ORDER (RK4)

The Fourth-Order Runge-Kutta method, often abbreviated as RK4, is a numerical technique used for solving ordinary differential equations (ODEs). It's a popular and widely used method because of its accuracy and reliability. RK4 is particularly effective for solving initial value problems, where you have an ODE and an initial condition specifying the value of the function at a particular point.

The 4th-order Runge-Kutta method is a numerical technique used to solve ordinary differential equations of the form [10]:

$$\begin{cases} \frac{dy}{dx} = f(x,y), \\ y(0) = y_0. \end{cases} \quad (2)$$

The 4th-order Runge-Kutta method is based on the following elements:

$$y_{i+1} = y_i + (a_1K_1 + a_2K_2 + a_3K_3 + a_4K_4)h,$$

where knowing the value of$y = y_i$atx_i, we can find the value of $y = y_{i+1}\ at\ x_{i+1}$, and $h = x_{i+1} - x_i$. Equation (4.1) is approximated using the first five terms of the Taylor series:

$$y_{i+1} = y_i + \frac{dy}{dx}|_{x_i,y_i}(x_{i+1} - x_i) + \frac{1}{2!}\frac{d^2y}{dx^2}|_{x_i,y_i}(x_{i+1} - x_i)^2 + \frac{1}{3!}\frac{d^3y}{dx^3}|_{x_i,y_i}(x_{i+1} - x_i)^3$$
$$\frac{1}{4!}\frac{d^4y}{dx^4}|_{x_i,y_i}(x_{i+1} - x_i)^4. \quad (3)$$

Given that $\frac{dy}{dx} = f(x,y)$ and $x_{i+1} - x_i = h$:

$$y_{i+1} = y_i + hf(x_i,y_i) + \frac{h^2}{2!}f'(x_i,y_i)| + \frac{h^3}{3!}f''(x_i,y_i) + \frac{h^4}{4!}f'''(x_i,y_i).(4)$$

One of the most popular solutions used is:

$y_{i+1} = y_i + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)h,$ (5)
such that:

$$K_1 = f(x_i, y_i),$$
$$K_2 = f\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}K_1 h\right),$$
$$K_3 = f\left(x_i + \frac{1}{2}h, y_i + \frac{1}{2}K_2 h\right),$$
$$K_4 = f(x_i + h, y_i + K_3 h).$$

V. APPLICATION OF THE FOURTH ORDER RUNGE KUTTA METHOD

We have performed calculations using Matlab for two cases with h = 1 and α = β = 0.0009 and h = 1 and α = β = 0.9. Here are the results:

t	S_i	I_i	R_i
0	220.00	0.00	0.00
1	218.74	0.0075	0.0000
2	217.49	0.0148	0.0000
3	216.24	0.0219	0.0000
...	...	...	...
246	60.431	0.2815	0.0097
247	60.157	0.2815	0.0097
248	59.884	0.2816	0.0098
249	59.613	0.2816	0.0098
250	59.344	0.2816	0.0098

Table 2: RK4 for h = 1 and α = β = 0.0009

t	S_i	I_i	R_i
0	220.00	0.00	0.00
1	218.74	0.0070	0.0000
2	217.49	0.0128	0.0000
3	216.25	0.0178	0.0000
...	...	...	...
246	61.058	0.1071	0.0030
247	60.784	0.1073	0.0030
248	60.512	0.1076	0.0031
249	60.242	0.1079	0.0031
250	59.973	0.1082	0.0031

Table 3: RK4 for h = 1 and α = β = 0.9:

These tables show the values of S_i, I_i,, and R_i, at different time points (*t*) for the given parameter values and step sizes. The Fourth-Order Runge-Kutta method was used to approximate the solutions for the SIR model.

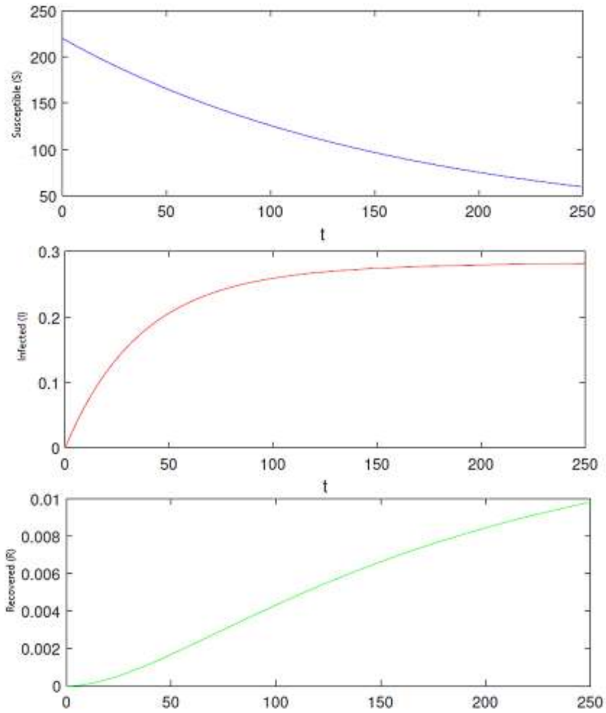

Figure 7: Approximate Analytical Solution Using RK4 for α = β = 0.0009 and h = 1

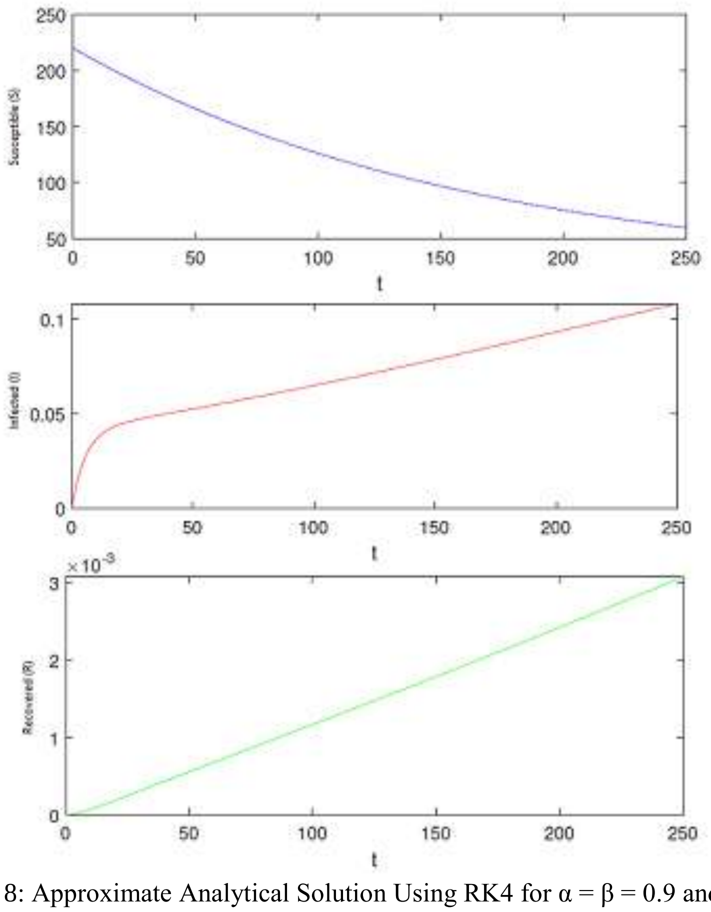

Figure 8: Approximate Analytical Solution Using RK4 for α = β = 0.9 and h = 1

VI. NONSTANDARD FINITE DIFFERENCE METHOD (NSFD)

The Nonstandard Finite Difference (NSFD) method is a numerical technique used for solving differential equations, particularly partial differential equations (PDEs). It belongs to the class of finite difference methods, which are used to approximate solutions to differential equations by discretizing the spatial and/or temporal domains. What sets NSFD apart from standard finite difference methods is its use of nonstandard discretization schemes, which can provide advantages in certain situations.

The nonstandard finite difference method (NSFD) is applied to a small system of three nonlinear equations of the form:

$$\frac{dU}{dt} = AU + G(U),$$

where *A* is a constant matrix, *U* is a vector, and *G(U)* contains nonlinear terms, provided that there is a repeated eigenvalue of A. NSFD is unique in that the effect of nonlinearity can be added or removed without the need to interrupt calculations and use a separate linear method [12].

Mickens developed a set of modeling rules to guide the incorporation of essential physical properties of differential equations into NSFD numerical schemes [1,17,16].

Mickens has developed a set of modeling rules to guide the incorporation of essential physical properties of differential equations into NSFD (Non-Standard finite difference schemes).[16,12]

	ED	Mikens
ED1	$\frac{du}{dt} = -\lambda\, u$	$\frac{u_{k+1} - u_k}{h} = -\lambda\, u_k$
ED2	$\frac{du}{dt} = -u^2$	$\frac{u_{k+1} - u_k}{h} = -u_k u_{k+1}$
ED3	$\frac{du}{dt} = -u^3$	$\frac{u_{k+1} - u_k}{h} = -\frac{2u_{k+1}^2\, u_k^2}{u_{k+1} + u_k}$

Table 4: NSFD Mickens

These equations represent different orders of derivatives with respect to time (t) using the NSFD approach. They involve various terms at consecutive time steps and constants such as λ and h .

VII. APPLICATION OF NSFD TO THE SYSTEM

In the application of the NSFD method to the system (3.1), the following difference equations are obtained:

$$\begin{aligned} S_{j+1} &= S_j + h[b - k\left(1 - \alpha S_j(t) I_j(t)\right) - \alpha k \beta S_j(t) I_j(t) - \square S_j(t)], \\ I_{\{j+1\}} &= I_j + h[k\left(1 - \alpha S_j(t) I_j(t)\right) + \alpha k \beta S_j(t) I_j(t) - (d_0 + \gamma + \square) I_j(t)], \\ R_{j+1} &= R_j + h[\gamma I_j(t) - \square R_j(t)]. \end{aligned} \quad (6)$$

In the application of the nonstandard finite difference method to the system (1), calculations were performed usingMatlab for the following case:

t	S_i	I_i	R_i
0	220.00	0.00	0.00
1	218.7354	0.0076	0.0000
2	217.4786	0.0150	0.0000
3	216.2297	0.0222	0.0000
...	...	...	...
246	60.2209	0.2815	0.0097
247	60.9470	0.2816	0.0098
248	59.6749	0.2816	0.0098
249	59.4044	0.2816	0.0098
250	59.1356	0.2817	0.0098

Table 5: NSFD method with a time step h=1 and parametersα=β=0.0009.

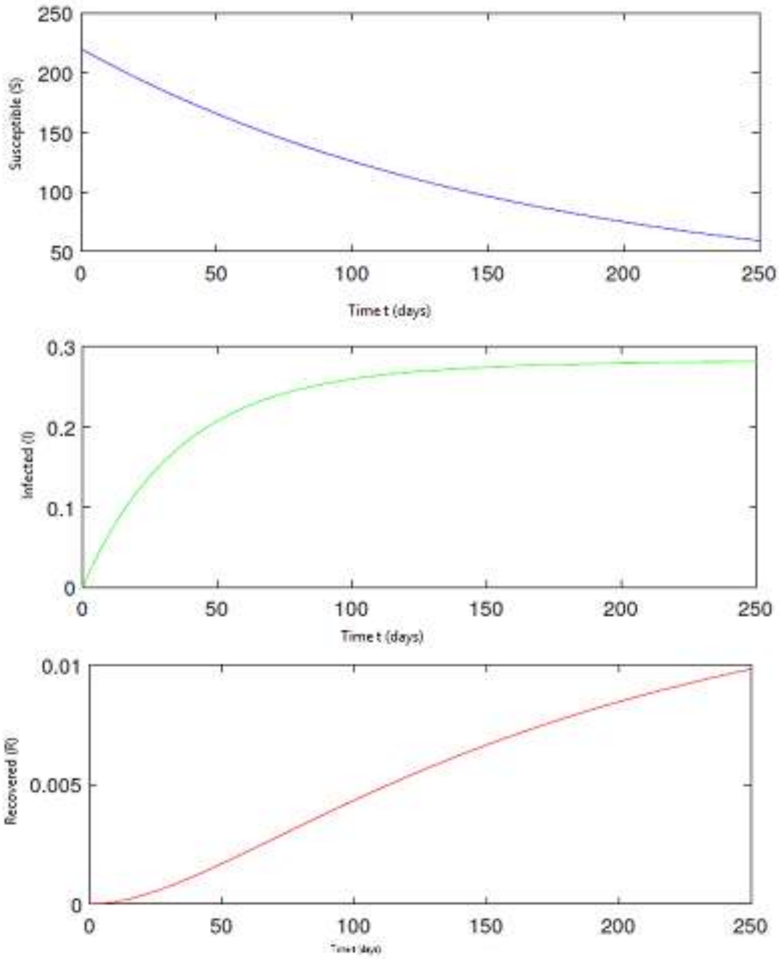

Figure 9: Approximate Analytical Solution Using NSFD for α = β = 0.0009 and h = 1

These results represent the values of the compartments S, I, and R at different time points (t) when using the nonstandard finite difference method with the specified parameters.

t_i	S_i	I_i	R_i
0	220	0	0
1	218.7354	0.0076	0
2	217.4798	0.0139	0
3	216.2328	0.0191	0
4	215.9944	0.0233	0
...	...	...	...
246	60.8485	0.1073	0.0030
247	60.5751	0.1076	0.0030
248	60.3034	0.1079	0.0031
249	60.0334	0.1082	0.0031
250	59.7650	0.1085	0.0031

Table 6: NSFD method with a time step h=1 and parametersα=β=0.9.

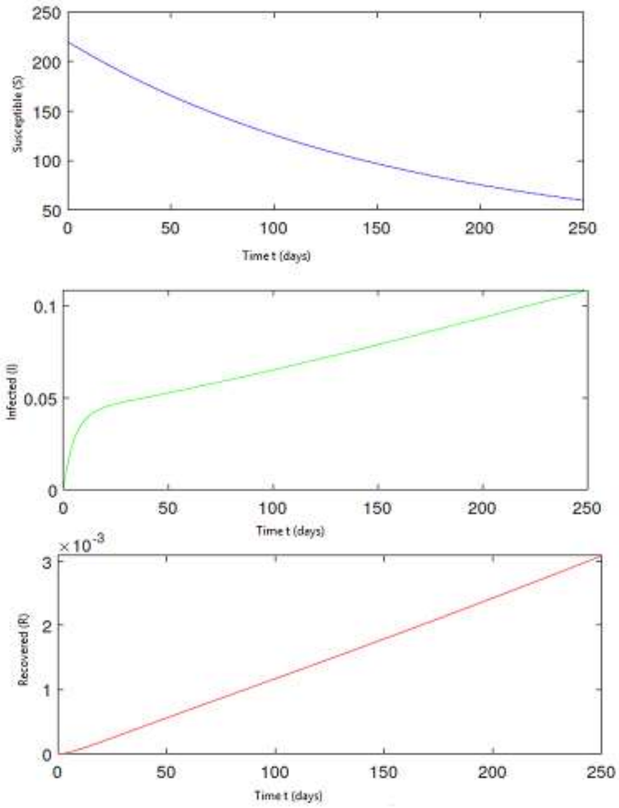

Figure 10: Approximate Analytical Solution Using NSFD for α = β = 0.9 and h = 1

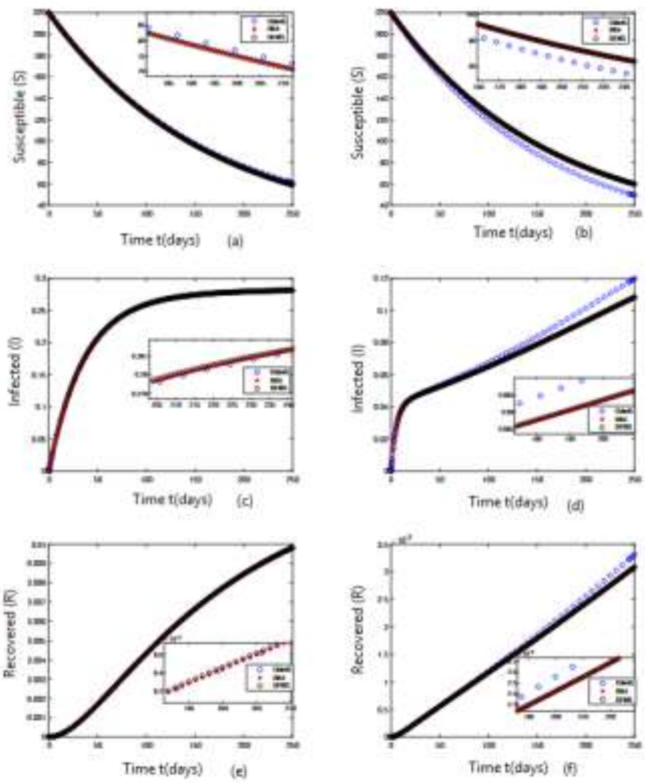

Figure 11: Dynamic behavior of susceptible, infected, and recovered populations using the three methods ode45, RK4, and NSFD for α = β = 0.0009, h = 1 in (*a*), (*c*), and (e), And for $\alpha = \beta$ = 0.9, h = 1 in (*b*), (*d*), and *(f)*.

VIII.DISCUSSION

Figures (2), (4), (6), (8) "RK4," and (10) "NSFD" represent the solutions for *S*, *I*, and *R* obtained through numerical methods. As observed, susceptibility decreased, and infection increased during the first four months. However, after that period, the infection rate also slowed down, while the recovery rate from infection was faster. This simulation was performed with protection and isolation parameters set to $\alpha = \beta = 0.9$.

Now, by decreasing the protection and isolation rates to $\alpha = \beta = 0.0009$, we plotted figures (1), (3), (5), (7) "RK4," and (9) "NSFD." In this case, susceptibility decreased more rapidly, the infection rate increased at a higher pace, and the recovery rate became slower.

From these simulations, it is evident that protection and isolation rates play a significant role in controlling the spread of infection. Figure (11) shows that the applied numerical methods are convergent when the protection and isolation rates decrease ($\alpha = \beta = 0.0009$ in (*a*), (*c*), and (*e*)). Conversely, when the protection and isolation rates are increased, the curves slightly diverge during the first 50 days ($\alpha = \beta = 0.9$ in (*b*), (*d*), and (*f*)).

IX.CONCLUSION

In this work, we presented a mathematical model of COVID-19 propagation of the SIR type. Our model illustrates that COVID-19 spreads through contact and describes how quickly susceptible individuals become infected. These new infections are the cause of the epidemic. For this reason, this paper is dedicated to implementing a mathematical model of the coronavirus that includes a class of isolation. We have numerically and graphically demonstrated that by controlling the contact rate, it is possible to control the current disease.

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