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On Fuzzy C-Almost P-Spaces

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Abstract
In this paper, the concept of fuzzy C-almost P-Space is introduced and studied. The conditions under which fuzzy P -spaces and fuzzy almost P -spaces become fuzzy C-almost P-spaces are obtained. It is established that fuzzy C-almost P-spaces are neither fuzzy hyperconnected spaces nor fuzzy open hereditarily irresolvable spaces. Fuzzy regular Oz -spaces are fuzzy C-almost P-spaces and fuzzy Oz and fuzzy P-spaces, are fuzzy C-almost P-spaces.

Keywords : *Fuzzy G_δ -set, fuzzy σ -nowhere dense set, fuzzy σ - boundary set, fuzzy hyperconnected space, fuzzy Oz-space, fuzzy P-space, fuzzy almost P-space, fuzzy open hereditarily irresolvable space.*

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I. Introduction

The concept of fuzzy sets as a new approach for modelling uncertainties was introduced by **L.A. Zadeh** [**22**] in the year 1965 . The potential of fuzzy notion was realized by the researchers and has successfully been applied in all branches of Mathematics. In 1968, **C.L. Chang** [**3**] introduced the concept of fuzzy topological space. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. **A. K. Mishra** [**8**] introduced the concept of P-spaces as a generalization of ω_μ–additive spaces of **R. Sikorski** [**10**]. Almost P-spaces in classical topology was introduced by **A.I.Veksler** [**21**] as P ’-spaces and was also studied further by **R. Levy** [**6**] and **C.L. Kim** [**5**].

The concept of P-spaces in fuzzy setting was introduced by **G.Balasubramanian** [**12**]. Fuzzy almost P-spaces was introduced and studied by **G.Thangaraj** and **C.Anbazhagan** in [**17**]. In the recent years, there has been a growing trend to introduce and study various types of fuzzy topological spaces. In this paper, the concept of fuzzy C-almost P-Space is introduced and studied. It is obtained that fuzzy σ -nowhere dense sets , fuzzy σ - boundary sets are fuzzy somewhere dense sets in fuzzy C-almost P-spaces. A condition under which a fuzzy almost P -space becomes a fuzzy C-almost P- space , is obtained. It is established that fuzzy C-almost P-spaces, are neither fuzzy hyperconnected spaces nor fuzzy open hereditarily irresolvable spaces. Fuzzy regular Oz -spaces are fuzzy C-almost P-spaces and fuzzy Oz and fuzzy P-spaces, are fuzzy C-almost P-spaces.

II. Preliminaries

Some basic notions and results used in the sequel, are given in order to make the exposition self - contained. In this work by (X,T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). Let X be a non-empty set and I the unit interval [0,1]. A fuzzy set λ in X is a mapping from X into I. The fuzzy set 0_X is defined as $0_X(x) = 0$, for all $x \in X$ and the fuzzy set 1_X is defined as $1_X(x) = 1$, for all $x \in X$.

Definition 2.1 [**3**] **:** A fuzzy topology is a family T of fuzzy sets in X which satisfies the following conditions :

(a). $0_X \in T$ and $1_X \in T$

(b). If A, B ∈ T, then $A \wedge B \in T$,

(c). If $A_i \in T$ for each $i \in J$, then $\vee_i A_i \in T$.

T is called a fuzzy topology for X, and the pair (X,T) is a fuzzy topological space, or fts for short. Members of T are called fuzzy open sets of X and their complements fuzzy closed sets.

Definition 2.2 [**3**] **:** Let (X,T) be a fuzzy topological space and λ be any fuzzy set in (X,T). The interior , the closure and the complement of λ are defined respectively as follows :

(i). $\text{int}(\lambda) = \vee \{ \mu / \mu \leq \lambda, \mu \in T \}$;

(ii). $\text{cl}(\lambda) = \wedge \{ \mu / \lambda \leq \mu, 1-\mu \in T \}$.

(iii). $\lambda'(x) = 1 - \lambda(x)$, for all $x \in X$.

For a family $\{ \lambda_i / i \in J \}$ of fuzzy sets in (X,T), the union $\psi = \vee_i (\lambda_i)$ and intersection $\delta = \wedge_i (\lambda_i)$, are defined respectively as

(iv). $\psi(x) = \sup_i \{ \lambda_i(x) / x \in X \}$

(v). $\delta(x) = \inf_i \{ \lambda_i(x) / x \in X \}$.

Lemma 2.1 [**1**] **:** For a fuzzy set λ of a fuzzy topological space X,

(i). $1 - \text{int}(\lambda) = \text{cl}(1-\lambda)$ and (ii). $1 - \text{cl}(\lambda) = \text{int}(1-\lambda)$.

Definition 2.3 : A fuzzy set λ in a fuzzy topological space (X,T) is called a

(i). fuzzy regular- open set in (X, T) if $\lambda = \text{int cl}(\lambda)$;

fuzzy regular – closed set in (X, T) if $\lambda = \text{cl int}(\lambda)$ [**1**].

(ii). fuzzy G_δ -set in (X, T) if $\lambda = \wedge_{i=1}^{\infty} (\lambda_i)$, where $\lambda_i \in T$ for $i \in I$; fuzzy F_σ -set in (X, T) if $\lambda = \vee_{i=1}^{\infty} (\lambda_i)$, where $1 - \lambda_i \in T$ for $i \in I$ [**2**].

(iii). fuzzy dense set if there exists no fuzzy closed set μ in (X,T) such that $\lambda < \mu < 1$. That is, $\text{cl}(\lambda) = 1$, in (X,T) [**13**] .

(iv). fuzzy nowhere dense set if there exists no non-zero fuzzy open set μ in (X,T) such that $\mu < \text{cl}(\lambda)$. That is, $\text{int cl}(\lambda) = 0$, in (X,T) [**13**].

(v)**.** fuzzy somewhere dense set if there exists a non-zero fuzzy open set μ in (X,T) such that $\mu < \text{cl}(\lambda)$. That is, $\text{int cl}(\lambda) \neq 0$, in (X,T) [**14**] and $1 - \lambda$ is called a fuzzy complement of fuzzy somewhere dense set in (X, T) and is denoted as fuzzy *cs* dense set in (X, T) [**18**].

(vi). fuzzy first category set if $\lambda = \vee_{i=1}^{\infty} (\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X,T). Any other fuzzy set in (X,T) is said to be of fuzzy second category [**13**].

(vii) .fuzzy residual set if $1 - \lambda$ is a fuzzy first category set in (X,T) [**16**].

(viii). fuzzy σ-boundary set if $\lambda = \vee_{i=1}^{\infty} (\mu_i)$, where $\mu_i = \text{cl}(\lambda_i) \wedge (1 - \lambda_i)$ and (λ_i)'s are fuzzy regular open sets in (X,T) [**9**] .

(ix). fuzzy regular G_δ -set in (X,T) if $\lambda = \wedge_{i=1}^{\infty} \text{int}(\lambda_i)$, where $1 - \lambda_i \in T$; fuzzy regular F_σ -set in (X,T) if $\lambda = \vee_{i=1}^{\infty} \text{cl}(\mu_i)$, where $\mu_i \in T$ [**11**].

Definition 2.4 : A fuzzy topological space (X,T) is called a

(i). fuzzy P- space if each fuzzy G_δ-set in (X,T) is fuzzy open in (X,T) [**12**].

(ii). fuzzy almost P-space if for each non- zero fuzzy G_δ-set λ in (X,T), $\text{int}(\lambda) \neq 0$ in (X,T) [**17**].

(iii). fuzzy open hereditarily irresolvable space if $\text{int cl}(\lambda) \neq 0$, then $\text{int}(\lambda) \neq 0$, for any non-zero fuzzy set λ in (X,T) [**15**].

(iv). fuzzy Oz-space if each fuzzy regular closed set is a fuzzy G_δ -set in (X,T) [**19**].

(v). fuzzy regular Oz-space if each fuzzy regular closed set λ in (X,T) is a fuzzy regular G_δ-set in (X,T) [**20**].

(vi). fuzzy hyperconnected space if every non -null fuzzy open subset of (X,T)

is fuzzy dense in (X,T) [**7**].
(vii). fuzzy extremally disconnected space if the closure of every fuzzy open set of (X,T) is fuzzy open in (X,T) [**4**].

Theorem 2.1 [9 **] :** If λ is a fuzzy residual set in a fuzzy topological space (X,T), then there exists a fuzzy G_δ-set μ in (X,T) such that $\mu \leq \lambda$.

Theorem 2.2 [18] : If λ is a fuzzy somewhere dense set in a fuzzy topological space (X,T), then there exists a fuzzy regular closed set η in (X,T) such that $\eta \leq \mathrm{cl}(\lambda)$.

Theorem 2.3 [9] : If λ is a fuzzy σ-boundary set in a fuzzy topological space (X,T) , then λ is a fuzzy F_σ -set in (X,T).

Theorem 2.4 [1] : In a fuzzy topological space ,
(a). The closure of a fuzzy open set is a fuzzy regular closed set.
(b). The interior of a fuzzy closed set is a fuzzy regular open set.

Theorem 2.5 [17] : A fuzzy topological space (X,T) is a fuzzy almost P-space, if and only if the only fuzzy F_σ -set λ such that $\mathrm{cl}(\lambda) = 1$ in (X,T) is 1_X.

Theorem 2.6 [19] : If a fuzzy topological space (X,T) is a fuzzy Oz and fuzzy P-space, then (X,T) is a fuzzy extremally disconnected space.

Theorem 2.7 [20] : If δ is a fuzzy G_δ-set in a fuzzy regular Oz –space (X,T), then $\mathrm{cl\,int}(\delta)$ is a fuzzy G_δ-set in (X,T).

III. FUZZY C- ALMOST P- SPACES

Definition 3.1 : A fuzzy topological space (X,T) is called a fuzzy C-almost P-Space if for each fuzzy G_δ-set λ in (X,T) , $\mathrm{cl\,int}(\lambda)$ is a fuzzy G_δ-set in (X,T).

Example 3.1 : Let $X = \{a, b, c\}$. Let $I = [0, 1]$. The fuzzy sets α, β and γ are defined on X as follows :
$\alpha : X \to I$ is defined by $\alpha(a) = 0.5$; $\alpha(b) = 0.5$; $\alpha(c) = 0.6$,
$\beta : X \to I$ is defined by $\beta(a) = 0.5$; $\beta(b) = 0.6$; $\beta(c) = 0.5$,
$\gamma : X \to I$ is defined by $\gamma(a) = 0.6$; $\gamma(b) = 0.4$; $\gamma(c) = 0.5$,
Then, $T = \{0, \alpha, \beta, \gamma, \alpha \vee \beta, \alpha \vee \gamma, \beta \vee \gamma, \alpha \wedge \beta, \alpha \wedge \gamma, \gamma \vee [\alpha \wedge \beta], \alpha \vee \beta \vee \gamma, 1\}$ is a fuzzy topology on X. By computation, one can find that

$\mathrm{cl}(\alpha) = 1$;	$\mathrm{int}(1 - \alpha) = 0$;
$\mathrm{cl}(\beta) = 1 - (\alpha \wedge \gamma) = \beta$;	$\mathrm{int}(1 - \beta) = \alpha \wedge \gamma$;
$\mathrm{cl}(\gamma) = 1$;	$\mathrm{int}(1 - \gamma) = 0$;
$\mathrm{cl}(\alpha \vee \beta) = 1$;	$\mathrm{int}(1 - [\alpha \vee \beta]) = 0$;
$\mathrm{cl}(\alpha \vee \gamma) = 1$;	$\mathrm{int}(1 - [\alpha \vee \gamma]) = 0$;
$\mathrm{cl}(\beta \vee \gamma) = 1$;	$\mathrm{int}(1 - [\beta \vee \gamma]) = 0$;
$\mathrm{cl}(\alpha \wedge \beta) = 1 - (\alpha \wedge \beta) = \alpha \wedge \beta$;	$\mathrm{int}(1 - [\alpha \wedge \beta]) = \alpha \wedge \beta$;
$\mathrm{cl}(\alpha \wedge \gamma) = 1 - \beta = \alpha \wedge \gamma$;	$\mathrm{int}(1 - [\alpha \wedge \gamma]) = \beta$;
$\mathrm{cl}(\alpha \vee \beta \vee \gamma) = 1$;	$\mathrm{int}(1 - [\alpha \vee \beta \vee \gamma]) = 0$;

Now $1 - \beta = \beta \wedge \gamma \wedge (\alpha \wedge \beta) = \alpha \wedge \gamma$;
$1 - (\alpha \wedge \beta) = \alpha \wedge (\alpha \vee \gamma) \wedge [\gamma \vee (\alpha \wedge \beta)] = \alpha \wedge \beta$;
$1 - (\alpha \wedge \gamma) = (\alpha \vee \beta) \wedge (\beta \vee \gamma) \wedge (\alpha \vee \beta \vee \gamma) = \beta$.
Then, $1 - \beta$, $1 - (\alpha \wedge \beta)$ and $1 - (\alpha \wedge \gamma)$ are fuzzy G_δ -sets in (X,T). On computation, $\mathrm{cl\,int}(1 - \beta) = \mathrm{cl}(\alpha \wedge \gamma) = 1 - \beta$,
$\mathrm{cl\,int}(1 - [1 - (\alpha \wedge \beta)]) = \mathrm{cl}(\alpha \wedge \beta) = 1 - (\alpha \wedge \beta)$,
$\mathrm{cl\,int}(1 - [1 - (\alpha \wedge \gamma)]) = \mathrm{cl}(\alpha \wedge \gamma) = 1 - \beta$.

Hence for each fuzzy G_δ-set λ ($= 1 - \beta$, $1 - (\alpha \wedge \beta)$, $1 - (\alpha \wedge \gamma)$) in (X,T) , $\mathrm{cl\,int}(\lambda)$ is a fuzzy G_δ-set in (X,T) implies that (X,T) is called a fuzzy C-almost P-Space.

Proposition 3.1 : If μ is a fuzzy F_σ - set in a fuzzy C-almost P-Space (X,T), then $\mathrm{int\,cl}(\mu)$ is a fuzzy F_σ - set in (X,T).

Proof : Let μ be a fuzzy F_σ - set in (X,T) and then $1 - \mu$ is a fuzzy G_δ-set in (X,T). Since (X,T) is a fuzzy C-almost P-Space, $\mathrm{cl\,int}(1 - \mu)$ is a fuzzy G_δ-set in (X,T). By Lemma **2.1**, $\mathrm{cl\,int}(1 - \mu) = 1 - \mathrm{int\,cl}(\mu)$ and thus $\mathrm{int\,cl}(\mu)$ is a fuzzy F_σ - set in (X,T).

Proposition 3.2 : If μ is a fuzzy F_σ - set in a fuzzy C-almost P-space (X,T), then μ is a fuzzy somewhere dense set in (X,T).

Proof : Let μ be a fuzzy F_σ - set in (X,T). Since (X,T) is a fuzzy C-almost P-Space, by Proposition **3.1,** int cl (μ) is a fuzzy F_σ - set in (X,T) and this implies that int cl(μ) ≠ 0 and thus μ is a fuzzy somewhere dense set in (X,T).

Corollary 3.1 : If λ is a fuzzy G_δ-set in a fuzzy C-almost P-Space (X,T), then cl int (λ) ≠ 1, in (X,T).

Proof : Let λ be a fuzzy G_δ-set in (X,T). Then, 1 − λ is a fuzzy F_σ - set in (X,T). Since (X,T) is a fuzzy C-almost P-Space, by Proposition **3.2,** 1 − λ is a fuzzy somewhere dense set in (X,T) and then int cl (1 − λ) ≠ 0. This implies that 1 − cl int(λ) ≠ 0 and thus cl int (λ) ≠ 1, in (X,T).

Remark : In the above Corollary **3.1,** int cl (1 − λ) ≠ 0 in the fuzzy C-almost P-Space (X,T), implies that int cl (1 − λ) = θ and then θ is a fuzzy open set in (X,T) and 1 − cl int (λ) = θ , implies that cl int (λ) = 1 – θ, in (X,T).

Proposition 3.3 : If μ is a fuzzy F_σ - set in a fuzzy C-almost P-Space (X,T), then there exists a fuzzy regular closed set η in (X,T) such that η ≤ cl (μ).

Proof : Let μ be a fuzzy F_σ - set in (X,T). Since (X,T) is a fuzzy C-almost P-Space, by Proposition **3.2,** μ is a fuzzy somewhere dense set in (X,T). Then, by Theorem **2.2,** there exists a fuzzy regular closed set η in (X,T) such that η ≤ cl(μ).

Corollary 3.2 : If λ is a fuzzy G_δ-set in a fuzzy C-almost P-Space (X,T), then there exists a fuzzy regular open set δ in (X,T) such that int (λ) ≤ δ .

Proof : Let λ be a fuzzy G_δ-set in (X,T). Then, 1 − λ is a fuzzy F_σ - set in (X,T). Since (X,T) is a fuzzy C-almost P-Space, by Proposition **3.3,** there exists a fuzzy regular closed set η in (X,T) such that η ≤ cl (1 − λ). Then, η ≤ 1 − int (λ). This implies that int (λ) ≤ 1 − η . Let δ = 1 – η. Hence for the fuzzy G_δ-set λ , there exists a fuzzy regular open set δ in (X,T) such that int (λ) ≤ δ .

Proposition 3.4 : If λ is a fuzzy residual set in a fuzzy C-almost P- space (X,T), then there exists a fuzzy G_δ-set η in (X,T) such that η ≤ cl int (λ).

Proof : Let λ be a fuzzy residual set in (X,T). By Theorem **2.1,** there exists a fuzzy G_δ-set μ in (X,T) such that μ ≤ λ . This implies that cl int (μ) ≤ cl int (λ). Since (X,T) is a fuzzy C-almost P-Space, cl int (μ) is a fuzzy G_δ-set in (X,T). Let η = cl int (μ) . Thus, there exists a fuzzy G_δ-set η in (X,T) such that η ≤ cl int (λ).

Corollary 3.3 : If δ is a fuzzy first category set in a fuzzy C-almost P-space (X,T), then there exists a fuzzy F_σ - set θ in (X,T) such that int cl (δ) ≤ θ.

Proof : Let δ be a fuzzy first category set in (X,T). Then, 1 − δ is a fuzzy residual set in (X,T). Since (X,T) is a fuzzy C-almost P-Space, by Proposition **3.4,** there exists a fuzzy G_δ-set η in (X,T) such that η ≤ cl int (1 − δ). This implies that η ≤ 1 − int cl(δ) and int cl (δ) ≤ 1 – η. Let θ = 1 – η. Hence for the fuzzy first category set δ , there exists a fuzzy fuzzy F_σ - set θ in (X,T) such that int cl (δ) ≤ θ.

Proposition 3.5 : If η is a fuzzy σ -nowhere dense set in a fuzzy C-almost P- space (X,T), then η is a fuzzy somewhere dense set in (X,T).

Proof : Let η be a fuzzy σ -nowhere dense set in (X,T). Then, η is a fuzzy F_σ - set in (X,T) with int (η) = 0. Since (X,T) is a fuzzy C-almost P-Space, by Proposition **3.2,** the fuzzy F_σ - set η is a fuzzy somewhere dense set in (X,T).

Proposition 3.6 : If η is a fuzzy σ - boundary set in a fuzzy C-almost P- space (X,T), then η is a fuzzy somewhere dense set in (X,T).

Proof : Let η be a fuzzy σ - boundary set in (X,T). Then, by Theorem **2.3,** η is a fuzzy F_σ - set in (X,T) . Since (X,T) is a fuzzy C-almost P-Space, by Proposition **3.2,** the fuzzy F_σ-set η is a fuzzy somewhere dense set in (X,T).

Remark : In view of the above Propositions **3.5** and **3.6**, one will have the following result : “ *Fuzzy σ -nowhere dense sets , fuzzy σ - boundary sets are fuzzy somewhere dense sets in fuzzy C-almost P-spaces* .”

Proposition 3.7 : If λ is a fuzzy G_δ-set in a fuzzy C-almost P-Space (X,T), then int (λ) $\neq$ 0, in (X,T).

Proof : Let λ be a fuzzy G_δ-set in (X,T). Since (X,T) is a fuzzy C-almost P-Space, by remarks of Corollary **3.1,** cl int (λ) = 1 – θ, where 1 – θ is a fuzzy closed set in (X,T). This implies that int (λ) $\neq$ 0, in (X,T).

IV. SOME RELATIONSHIPS BETWEEN FUZZY C-ALMOST P-SPACES AND OTHER TOPOLOGICAL SPACES

Proposition 4.1: If a fuzzy topological space (X,T) is a fuzzy C-almost P-space, then (X,T) is not a fuzzy open hereditarily irresolvable space.

Proof : Let η be a fuzzy σ - nowhere dense set in (X,T). Then, η is a fuzzy F_σ - set in (X,T) with int (η) = 0. Since (X,T) is a fuzzy C-almost P-Space, by Proposition **3.5,** η is a fuzzy somewhere dense set in (X,T) and thus int cl(η) $\neq$ 0 . But int (η) = 0 in (X,T), implies that (X,T) is not a fuzzy open hereditarily irresolvable space.

Proposition 4.2: If a fuzzy topological space (X,T) is a fuzzy C-almost P-space, then (X,T) is not a fuzzy hyperconnected space.

Proof : Let λ be a fuzzy G_δ-set in (X,T). By Corollary **3.1,** for the fuzzy G_δ-set λ in the fuzzy C-almost P-Space (X,T), cl int (λ) $\neq$ 1, in (X,T). Hence for the fuzzy open set int (λ), cl [int (λ)] $\neq$ 1, in (X,T) implies that (X,T) is not a fuzzy hyperconnected space.

Proposition 4.3 : If each fuzzy regular closed set is a fuzzy G_δ-set in a fuzzy P-space (X,T), then (X,T) is a fuzzy C-almost P-space.

Proof : Let λ be a fuzzy G_δ-set in (X,T). Since (X,T) is a fuzzy P-space, λ is a fuzzy open set in (X,T) .Now cl int (λ) = cl (λ) and by Theorem 2.4, cl (λ) is a fuzzy regular closed set in (X,T). By hypothesis, the fuzzy regular closed set cl (λ) is a fuzzy G_δ-set and thus cl int (λ) is a fuzzy G_δ-set in (X,T). Hence (X,T) is a fuzzy C-almost P-space.

Proposition 4.4 : If a fuzzy topological space (X,T) is a fuzzy Oz and fuzzy P-space, then (X,T) is a fuzzy C-almost P-space.

Proof : Let λ be a fuzzy G_δ-set in (X,T). Since (X,T) is a fuzzy P-space, λ is a fuzzy open set in (X,T) .Now cl int (λ) = cl (λ) and by Theorem 2.4, cl (λ) is a fuzzy regular closed set in (X,T). Since (X,T) is a fuzzy Oz-space, the fuzzy regular closed set cl (λ) is a fuzzy G_δ -set and thus cl int (λ) is a fuzzy G_δ-set in (X,T). Hence (X,T) is a fuzzy C-almost P-space.

Remark : In view of Proposition **4.4** and Theorem **2.6**, one will have the following relations:

Proposition 4.5 : If a fuzzy topological space (X,T) is a fuzzy C-almost P-space, then (X,T) is a fuzzy almost P-space.

Proof : Let λ be a fuzzy G_δ-set in (X,T). Since (X,T) is a fuzzy C-almost P-Space, by Proposition **3.7,** int (λ) $\neq$ 0, in (X,T). Hence (X,T) is a fuzzy almost P-space.

Remark 4.1: The converse of the above proposition need not be true. That is, a fuzzy almost P-space need not be a fuzzy C-almost P-space.

The following proposition gives a condition under which fuzzy almost P-spaces become fuzzy C-almost P-spaces.

Proposition 4.6 : If a fuzzy topological space (X,T) is a fuzzy Oz and fuzzy almost P-space, then (X,T) is a fuzzy C-almost P-space.

Proof : Let λ be a fuzzy G_δ-set in (X,T). Since (X,T) is a fuzzy almost P-space, int (λ) $\neq$ 0, in (X,T). By Theorem **2.4,** cl [int (λ)] is a fuzzy regular closed set in (X,T). Since (X,T) is a fuzzy Oz-space, the fuzzy regular closed set cl int (λ) is a fuzzy G_δ-set in (X,T). Hence (X,T) is a fuzzy C-almost P-space.

Remark 4.2 : It is observed from Propositions **4.1, 4.2** and **4.5** that fuzzy C-almost P-space are not fuzzy hyper-connected and not fuzzy open hereditarily irresolvable spaces even though they are fuzzy almost P-spaces.

Proposition 4.7 : If a fuzzy topological space (X,T) is a fuzzy regular Oz –space, then (X,T) is a fuzzy C-almost P-space.

Proof : Let λ be a fuzzy G_δ-set in (X,T). Since (X,T) is a fuzzy regular Oz –space, by Theorem 2.6, cl int (δ) is a fuzzy G_δ-set in (X,T). Hence (X,T) is a fuzzy C-almost P-space.

Proposition 4.8 : : If a fuzzy topological space (X,T) is a fuzzy C-almost P-space, then the only fuzzy F_σ-set λ such that $cl(\lambda) = 1$ in (X,T) is 1_X.

Proof : The proof follows from Proposition **4.5** and Theorem **2.5.**

V. Conclusion

In this paper the notion of fuzzy C-almost P-Space is introduced and studied. The conditions under which fuzzy P-spaces and fuzzy almost P-spaces become fuzzy C-almost P-spaces are obtained. It is obtained that fuzzy closures of fuzzy F_σ-sets contain fuzzy regular closed sets and fuzzy interiors of fuzzy G_δ-sets are contained in fuzzy regular open sets in fuzzy C-almost P-spaces. It is established that fuzzy C-almost P-spaces are neither fuzzy hyperconnected spaces nor fuzzy open hereditarily irresolvable spaces. Fuzzy regular Oz-spaces are fuzzy C-almost P-spaces and fuzzy Oz and fuzzy P-spaces, are fuzzy C-almost P-spaces.

References

[1]. K. K. Azad, On Fuzzy Semi Continuity, Fuzzy Almost Continuity And Fuzzy Weakly Contiunity, J. Math. Anal. Appl, 82 (1981), 14 – 32.

[2]. G.Balasubramanian , Maximal Fuzzy Topologies, Kybernetika, 31(5) (1995), 459 – 464.

[3]. C. L . Chang, Fuzzy Topological Spaces, J. Math. Anal., 24, (1968), 182 – 190.

[4]. B. Ghosh, Fuzzy Extremally Disconnected Spaces, Fuzzy Sets And Systems Vol.46, No. 2 (1992), 245 – 250.

[5]. C. L. Kim, Almost P-Spaces, Commun . Korean Math. Soc., Vol. 18, No.4 (2003) ,695 – 701.

[6]. R. Levy , Almost P-Spaces, Canad. J. Math., Xxix(2) (1977), 284 – 288.

[7]. Miguel Caldas, Govindappa Navalagi, And Ratnesh Saraf , On Fuzzy Weakly Semi-Open Functions, Proyecciones, Vol.21, No.1, (2002), 51 – 63.

[8]. A. K. Mishra, A Topological View Of P-Spaces, Gen. Topology Appl., 2(4) (1972), 349 – 362.

[9]. R. Palani, Contributions To The Study On Some Aspects Of Fuzzy Baire Spaces, Ph. D. Thesis, Thiruvalluvar University, Tamilnadu, India, 2017.

[10]. R. Sikorski , Remarks On Spaces Of High Power, Fund. Math., 37(1950), 125-136.

[11]. S. Soundara Rajan, A Study On Fuzzy Volterra Spaces, Ph.D. Thesis, Thiruvalluvaruniversity, Tamilnadu, India, 2015

[12]. G. Thangaraj And G.Balasubramanian , On Fuzzy Basically Disconnected Spaces, J. Fuzzy Math., Vol. 9, No.1 (2001), 103 – 110.

[13]. G. Thangaraj And G.Balasubramanian, On Somewhat Fuzzy Continuous Functions , J. Fuzzy Math, Vol.11, No. 2 (2003), 725 – 736.

[14]. G.Thangaraj , Resolvability And Irresolvability In Fuzzy Topological Spaces, News Bull. Cal. Math. Soc., Vol. 31, No.4 – 6 (2008),11 – 14.

[15]. G.Thangaraj And G. Balasubramanian , On Fuzzy Resolvable And Fuzzy Irresolvable Spaces, Fuzzy Sets, Rough Sets And Multi Valued Operations And Appl., Vol.1, No.2 (2009), 173 – 180.