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IFGSP-Compactness in Intuitionistic Fuzzy Topological Spaces

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Abstract
In this paper, we introduce intuitionistic fuzzy generalized semi-pre-compactness (IFGSP-compactness) in intuitionistic fuzzy topological spaces. This structure of compactness is based on intuitionistic fuzzy generalized semi-pre-open set. We investigate some of its characterizations and properties. We identify the relations between fuzzy compactness and IFGSP-compactness in intuitionistic fuzzy topological spaces.
***Keywords**: Intuitionistic fuzzy generalized semi-pre-open set, Intuitionistic fuzzy generalized semi-pre-compactness.*

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I. Introduction

Atanassov [1] generalized the concept of fuzzy sets and introduced intuitionistic fuzzy sets, which take into account both the degrees of membership and non-membership subject to the condition that their sum does not exceed 1. Coker [5] subsequently initiated a study of intuitionistic fuzzy topological spaces. Mondal and Samanta [11] gave the concept of intuitionistic gradation of openness of fuzzy sets in X and using this, they defined an intuitionistic fuzzy topological space. Many concepts of fuzzy topological spaces have been extended in intuitionistic fuzzy topological spaces.

The concept of compactness in intuitionistic fuzzy topological spaces is first introduced by Coker [5]. Ramadan et. al. [12] introduced fuzzy almost continuous mapping, fuzzy weakly continuous mapping, fuzzy compactness, fuzzy almost compactness and fuzzy near compactness in intuitionistic fuzzy topological spaces in the sense of Sostak [17]. They investigate the behavior of fuzzy compactness under several types of fuzzy continuous mappings.

In this paper, we introduce IFGSP-compactness in an intuitionistic fuzzy topological spaces. We study their characterizations and properties in intuitionistic fuzzy topological spaces.

II. Preliminaries

Definition 2.1. [1] Let X be a non-empty fixed set. An intuitionistic fuzzy set (IFS) A is an object of the form

$$A = \{\langle x, \mu_{A(x)}, \nu_{A(x)}\rangle : x \in X\},$$

Where, the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership (namely $\mu_{A(x)}$) and the degree of non-membership (namely $\nu_{A(x)}$) of each element x ∈ X to the set A, respectively, and $0 \le \mu_{A(x)} + \nu_{A(x)} \le 1$ for each x ∈ X.

For the sake of simplicity, we shall use the symbol, $A = \{\langle x, \mu_A, \nu_A\rangle\}$ for intuitionistic fuzzy set $A = \{\langle x, \mu A(x), \nu A(x)\rangle : x \in X\}$.

Definition 2.2. [1] Intuitionistic fuzzy sets, $0_\sim$ and $1_\sim$ in X is defined as

$(a)\ 0_\sim = \{\langle x, 0, 1\rangle : x \in X\}$;

$(b)\ 1_\sim = \{\langle x, 1, 0\rangle : x \in X\}$.

Definition 2.3. [1] Let A and B be intuitionistic fuzzy sets of the form
$A = \{\langle x, \mu_{A(x)}, \nu_{A(x)}\rangle : x \in X\}$ and $B = \{\langle x, \mu_{B(x)}, \nu_{B(x)}\rangle : x \in X\}$. Then

$(a)\ A \subseteq B\ iff\ \mu_{A(x)} \le \mu_B(x)\ and\ \nu_{A(x)} \ge \nu_B(x)\ for\ all\ x \in X$;

$(b)\ A = B\ iff\ A \subseteq B\ and\ B \subseteq A$;

$(c)\ A^c\ or\ A' = \{\langle x, \nu_{A(x)}, \mu_{A(x)}\rangle : x \in X\}$;

$(d)\ A \cup B = \{\langle x, \mu_{A(x)} \vee \mu_B(x), \quad \nu_{A(x)} \wedge \nu_B(x)\rangle : x \in X\}$;

$(e)\ A \cap B = \{\langle x, \mu_{A(x)} \wedge \mu_B(x), \quad \nu_{A(x)} \vee \nu_B(x)\rangle : x \in X\}$;

$(f)\ [\,]A = \{\langle x, \mu_{A(x)}, \quad 1 - \mu_{A(x)}\rangle : x \in X\}$;

$(g)\ \langle\,\rangle A = \{\langle x, 1 - \nu_{A(x)}, \nu_{A(x)}\rangle : x \in X\}$.

Definition 2.4. [5] Let $\{A_i : i \in J\}$ be an arbitrary family of intuitionistic fuzzy sets in X, Then

$(a) \cup Ai = \{\langle x, \quad \vee \mu_{A_i}(x), \quad \wedge \nu_{A_i}(x)\rangle : x \in X\}$;
$(b) \cap Ai = \{\langle x, \quad \wedge_{\mu A_i}(x), \quad \vee \nu A_i(x)\rangle : x \in X\}$.

Definition 2.5. [5] An intuitionistic fuzzy topology (IFT) on a non-empty set X is a family $\mathcal{T}$ of IFS in X satisfying the following axioms;

(T_1) $0_{\sim}, 1_{\sim} \in \mathcal{T}$,
$(T2)$ $G_1 \cap G_2 \in \mathcal{T}$ for any $G_1, G_2 \in \mathcal{T}$,
$(T3)$ $\cup G_i \in \mathcal{T}$ $for\ any\ arbitrary\ family$ $\{G_i : i \in J\} \subseteq \mathcal{T}$.
The pair $(X, \mathcal{T})$ is called an intuitionistic fuzzy topological space.

An intuitionistic fuzzy set A is said to be intuitionistic fuzzy open set (IFOS) in $(X, \mathcal{T})$, if $A \in \mathcal{T}$. The compliment of intuitionistic fuzzy open set is intuitionistic fuzzy closed set (IFCS).

Definition 2.6. [5] Let $(X, \mathcal{T})$ be an IFTS and $A = \{< x, \mu_A, \nu_A >\}$ be an intuitionistic fuzzy set in X. Then the fuzzy interior and fuzzy closure of A are defined by

$$(a)\ int(A) = \cup\{G : G\ is\ an\ IFOS\ in\ X\ and\ G \subseteq A\}.$$
$$(b)\ cl(A) = \cap\{K : K\ is\ an\ IFCS\ in\ X\ and\ K \supseteq A\}.$$

The $int(A)$ is just largest intuitionistic fuzzy open subset of X contained in A. Clearly, A is IFOS iff $int(A) = A$. The $cl(A)$ is just smallest intuitionistic fuzzy closed subset of X containing A. Clearly, A is IFCS iff $cl(A) = A$.

III. Intuitionistic Fuzzy Generalized Semi-Pre-Compactness

In this section, we introduce intuitionistic fuzzy generalized semi-pre-compactness in intuitionistic fuzzy topological spaces. This structure of compactness is based on intuitionistic fuzzy generalized semi-pre-open set. We investigate some of its characterizations and properties. We identify the relations between fuzzy compactness and intuitionistic fuzzy generalized semi-pre-compactness in intuitionistic fuzzy topological spaces.

Intuitionistic fuzzy compactness:

In 1997, Coker introduced the intuitionistic fuzzy compactness in intuitionistic fuzzy topological spaces.

Definition 3.1. [5] Let $(X, \mathcal{T})$ be an IFTS. If a family $\{\langle x, \mu_{G_i}, \nu_{G_i} : i \in J\rangle\}$ of intuitionistic fuzzy open sets in X satisfies the condition $\cup \{\langle x, \mu_{G_i}, \nu_{G_i} : i \in J\rangle\} = 1_{\sim}$, then it is called a fuzzy open cover of X.

Definition 3.2. [5] Let $(X, \mathcal{T})$ be an intuitionistic fuzzy topological space. A finite subfamily of a fuzzy open cover $\{\langle x, \mu_{G_i}, \nu_{G_i} : i \in J\rangle\}$ of X, which is also a fuzzy open cover of X is called a finite subcover of $\{\langle x, \mu_{G_i}, \nu_{G_i} : i \in J\rangle\}$.

Definition 3.3. [5] Let $(X, \mathcal{T})$ be an IFTS. A family $\{\langle x, \mu_{K_i}, \nu_{K_i} : i \in J\rangle\}$ of intuitionistic fuzzy closed sets in X satisfies the finite intersection property (FIP) iff every finite subfamily$\{\langle x, \mu_{K_i}, \nu_{K_i} : i = 1, 2, \cdots, n.\rangle\}$ of the family satisfies the condition $\cap_{i=1}^{n} \{\langle x, \mu_{K_i}, \nu_{K_i} : i \in J\rangle\} \neq 0_{\sim}$.

Definition 3.4. [5] An IFTS $(X, \mathcal{T})$ is called fuzzy compact iff every fuzzy open cover of X has a finite subcover.

Definition 3.5. [5] Let $(X, \mathcal{T})$ be an IFTS and A be an intuitionistic fuzzy set in X. If a family $\{\langle x, \mu_{G_i}, \nu_{G_i} : i \in J\rangle\}$ of intuitionistic fuzzy open sets in X satisfies the condition $A \subseteq \cup \{\langle x, \mu_{G_i}, \nu_{G_i} : i \in J\rangle\}$, then it is called a fuzzy open cover of A.

Definition 3.6. [5] Let $(X, \mathcal{T})$ be an intuitionistic fuzzy topological space. A finite subfamily of a fuzzy open cover $\{\langle x, \mu_{G_i}, \nu_{G_i} : i \in J\rangle\}$ of A, which is also a fuzzy open cover of A is called a finite subcover of $\{\langle x, \mu_{G_i}, \nu\_G_i : i \in J\rangle\}$.

Definition 3.7. [5] An intuitionistic fuzzy set $A = \langle x, \mu_A, \nu_A\rangle$ in an IFTS $(X, \mathcal{T})$ is called fuzzycompact iff every fuzzy open cover of A has a finite subcover.

Intuitionistic fuzzy generalized semi-pre-compactness:

Definition 3.8. [15] An intuitionistic fuzzy set A in IFTS $(X, \mathcal{T})$ is said to be

(a) intuitionistic fuzzy semi-pre-open set (IFSP-open set) if there exists an intuitionistic fuzzy pre-open set B such that $B \subseteq A \subseteq cl(B)$.
(b) intuitionistic fuzzy semi-pre-closed set (IFSP-closed set) if there exists an intuitionistic fuzzy pre-closed set C such that $int(C) \supseteq A \supseteq C$.

Definition 3.9. [15] Let $(X, \mathcal{T})$ be an IFTS and A be an intuitionistic fuzzy set. Then

(a) intuitionistic fuzzy semi-pre-interior of A is defined as $IFSP - int(A) = \cup\{G : G\ is\ an\ IFSP - open\ set\ in\ X\ and\ G \subseteq A\}$.

(b) intuitionistic fuzzy semi-pre-closure of A is defined as $IFSP-cl(A) = \cap\{K : K\ is\ an\ IFSP-closed\ set\ in\ X\ and\ A \subseteq K\}$.

Clearly, $IFSP-int(A) = A$ iff A is semi-pre-open and $IFSP-cl(A) = A$ iff A is semi-pre-closed.

Definition 3.10. [15] Let $(X,\mathcal{T})$ be an IFTS and A be an intuitionistic fuzzy set. Then A is said to be

(a) intuitionistic fuzzy generalized semi-pre-open (IFGSP-open set) set if $IFSP-int(A) \supseteq U$, whenever $A \supseteq U$ and U is an intuitionistic fuzzy open set in X.

(b) intuitionistic fuzzy generalized semi-pre-closed (IFGSP-closed set) set if $IFSP-cl(A) \subseteq U$, whenever $A \subseteq U$ and U is an intuitionistic fuzzy open set in X.

Definition 3.11. Let $(X,\mathcal{T})$ be an IFTS. If a family $\{\langle x,\mu_{G_i},\nu_{G_i} : i \in J\rangle\}$ of IFGSP-open sets in X satisfies the condition $\cup\{\langle x,\mu_{G_i},\nu_{G_i} : i \in J\rangle\} = 1_\sim$, then it is called IFGSP-open cover of X.

Definition 3.12. Let $(X,\mathcal{T})$ be an intuitionistic fuzzy topological space. A finite subfamily of a IFGSP-open cover $\{\langle x,\mu_{G_i},\nu_{G_i} : i \in J\rangle\}$ of X, which is also a IFGSP-open cover of X is called a finite subcover of $\{\langle x,\mu_{G_i},\nu_{G_i} : i \in J\rangle\}$.

Definition 3.13. An IFTS $(X,\mathcal{T})$ is called IFGSP-compact iff every IFGSP-open cover of X has a finite subcover.

Definition 3.14. Let $(X,\mathcal{T})$ be an IFTS. A family $\{\langle x,\mu_{K_i},\nu_{K_i} : i \in J\rangle\}$ of IFGSP-closed sets in X satisfies the finite intersection property (in short, FIP) iff every finite subfamily $\{\langle x,\mu_{K_i},\nu_{K_i} : i = 1,2,\cdots,n.\rangle\}$ of the family satisfies the condition $\cap_{i=1}^{n}\{\langle x,\mu_{K_i},\nu_{K_i}\rangle\} \neq 0_\sim$.

Theorem 3.15. An IFTS $(X,\mathcal{T})$ is IFGSP-compact iff every family $\{\langle x,\mu_{K_i},\nu_{K_i} : i \in J\rangle\}$ of IFGSP-closed sets with finite intersection property has a non-empty intersection.

Proof. Let IFTS $(X,\mathcal{T})$ is IFGSP-compact. Suppose, $\{\langle x,\mu_{K_i},\nu_{K_i} : i \in J\rangle\}$ be any family of IFGSP-closed sets in X such that $\cap\{\langle x,\mu_{K_i},\nu_{K_i} : i \in J\rangle\} = 0\sim$. Thus, this implies that,

$$\{\langle x, \wedge\mu_{K_i}, \vee\nu_{K_i} : i \in J\rangle\} = 0\sim,$$
$$\Rightarrow \wedge\{\mu_{K_i}(x) : i \in J\} = 0\ and\ \vee\{\nu_{K_i}(x) : i \in J\} = 1,$$
$$\Rightarrow \cup\{\langle x,\nu_{K_i},\mu_{K_i} : i \in J\rangle\} = 1_\sim.$$

Thus, $\{\langle x,\nu_{K_i},\mu_{K_i} : i \in J\rangle\}$ is a IFGSP-open cover of X. Since, X is IFGSP-compact, so every IFGSP-open cover of X has a finite subcover. Therefore, X has a finite subcover $\{\langle x,\nu_{K_i},\mu_{K_i} : i = 1,2,\cdots,n.\rangle\}$. So, $\cup_{i=1}^{n}\{\langle x,\nu_{K_i},\mu_{K_i} : i \in J\rangle\} = 1_\sim$ this implies that,

$$\{\langle x, \bigvee_{i=1}^{n}\nu_{K_i}, \bigwedge_{i=1}^{n}\mu_{K_i}\rangle\} = 1\sim,$$
$$\Rightarrow \bigvee_{i=1}^{n}\{\mu_{K_i}(x)\} = 0\ and\ \bigwedge_{i=1}^{n}\{\nu_{K_i}(x)\} = 1,$$
$$\Rightarrow \bigcap_{i=1}^{n}\{\langle x,\mu_{K_i},\nu_{K_i}\rangle\} = 0_\sim,$$

which contradicts to our hypothesis. Hence, every family of IFGSP-closed sets with finite intersection property has a non-empty intersection.

Conversely, suppose, every family of IFGSP-closed sets with finite intersection property has a non-empty intersection. Assume that, $\{\langle x,\mu_{K_i},\nu_{K_i} : i \in J\rangle\}$ is any IFGSP-open cover of X, then $\cup\{\langle x,\mu_{K_i},\nu_{K_i} : i \in J\rangle\} = 1_\sim$. Therefore,

$$\cup\{\langle x,\mu_{K_i},\nu_{K_i} : i \in J\rangle\} = 1\sim \Rightarrow \cap\{\langle x,\nu_{K_i},\mu_{K_i} : i \in J\rangle\} = 0_\sim.$$

So $\{\langle x,\nu_{K_i},\mu_{K_i} : i \in J\rangle\}$ is a family of IFGSP-closed sets in X such that $\cap\{\langle x,\nu_{K_i},\mu_{K_i} : i \in J\rangle\} = 0_\sim$. By assumption, we can find a finite subfamily, $\{\langle x,\nu_{K_i},\mu_{K_i} : i = 1,2,\cdots,n.\rangle\}$ such that $\cap_{i=1}^{n}\{\langle x,\nu_{K_i},\mu_{K_i}\rangle\} = 0\sim$, which implies, $\cup_{i=1}^{n}\{\langle x,\mu_{K_i},\nu_{K_i}\rangle\} = 1_\sim$. Thus, $\{\langle x,\mu_{K_i},\nu_{K_i} : i = 1,2,\cdots,n.\rangle\} = 1_\sim$ is a finite subcover of X. Hence, X is IFGSP-compact.

Definition 3.16. Let $(X,\ \mathcal{T})$ be an IFTS and A be an intuitionistic fuzzy set in X. If a family $\{\langle x,\mu_{G_i},\nu_{G_i} : i \in J\rangle\}$ of IFGSP-open sets in X satisfies the condition $A \subseteq \cup\{\langle x,\mu_{G_i},\nu_{G_i} : i \in J\rangle\}$, then it is called a IFGSP-open cover of A.

Definition 3.17. Let $(X,\ \mathcal{T})$ be an intuitionistic fuzzy topological space. A finite subfamily of a IFGSP-open cover $\{\langle x,\mu_{G_i},\nu_{G_i} : i \in J\rangle\}$ of A, which is also a IFGSP-open cover of A is called a finite subcover of $\{\langle x,\mu_{G_i},\nu_{G_i} : i \in J\rangle\}$.

Definition 3.18. An intuitionistic fuzzy set $A = \langle x,\mu_A,\nu_A\rangle$ in an IFTS $(X,\ \mathcal{T})$ is called IFGSP-compact iff every IFGSP-open cover of A has a finite subcover.

Theorem 3.19. Let $(X,\ \mathcal{T})$ be an IFTS. An intuitionistic fuzzy IFGSP-closed subset of an IFGSP-compact space is fuzzy compact relative to X.

Proof. Let A be an IFGSP-closed subset of X. Let $\{\langle x, \mu_{G_i}, \nu_{G_i} : i \in J\rangle\}$ be IFGSP-open cover of A. Then the family $\{\langle x, \mu_{G_i}, \nu_{G_i} : i \in J\rangle\} \cup A'$ is an IFGSP-open cover of X. Since X is IFGSP-compact, there is a finite subfamily $\{\langle x, \mu_{G_i}, \nu_{G_i} : i = 1, 2, \cdots, n.\rangle\}$ of IFGSP-open cover, which also covers X. If this cover contains A' we discard it. Otherwise leave the subcover as it is. Thus, we obtained a finite IFGSP-open subcover of A. So A is IFGSP-compact relative to X.

Theorem 3.20. Let $(X, \mathcal{T})$ be an IFTS. Then $(X, \mathcal{T})$ is IFR-compact iff $(X, \mathcal{T}_{0,1})$ is IFGSP-compact.

Proof. Let $(X, \mathcal{T})$ be IFGSP-compact. Consider, a IFR-open cover $\{[]G_i : i \in J\}$ of X in $(X, \mathcal{T}_{0,1})$. Since, $\{[]G_i : i \in J\} = 1_\sim \Rightarrow \vee\, \mu_G = 1$. By,

$$\nu_{G_i} \le 1 - \mu_{G_i} \Rightarrow \bigwedge \nu_{G_i} \le 1 - \bigvee \nu_{G_i} = 1 - 1 = 0 \Rightarrow \bigwedge \nu_{G_i} = 0$$

We get $\cup\, G_i = 1_\sim$. Since $(X, \mathcal{T})$ is IFGSP-compact, then there exist $G_1, G_2, \cdots, G_n$ such that $\cup_{i=1}^{n} G_i = 1_\sim$. Therefore,

$$\bigcup_{i=1}^{n} G_i = 1_\sim \Rightarrow \bigvee_{i=1}^{n} \mu_{G_i} = 1 \; and \; \bigwedge_{i=1}^{n} \nu_{1-Gi} = 0.$$

Hence, $(X, \mathcal{T}_{0,1})$ is IFGSP-compact.

Conversely, suppose that $(X, \mathcal{T}_{0,1})$ is IFGSP-compact. Consider a IFGSP-open cover $\{G_i : i \in J\}$ of X in $(X, \mathcal{T})$. Since, $\cup\, G_i = 1_\sim$, which implies $\vee\, \mu_{G_i} = 1$ and $\wedge(1 - \mu Gi) = 0$. Since, $(X, \mathcal{T}_{0,1})$ is IFGSP-compact, there exist $G_1, G_2, \cdots, G_n$ such that $\cup_{i=1}^{n}\{[]G_i\} = 1_\sim$. Therefore, $\vee_{i=1}^{n} \mu_{G_i} = 1$ and $\wedge_{i=1}^{n}(1 - \mu Gi) = 0$. Therefore,

$$\mu_{G_i} \le 1 - \nu_{G_i} \Rightarrow 1 = \bigvee_{i=1}^{n} \mu_{G_i} \le 1 - \bigwedge_{i=1}^{n} \nu_{G_i} \Rightarrow \bigwedge_{i=1}^{n} \nu_{G_i} = 0.$$

Thus, $\cup_{i=1}^{n} \mu_{G_i} = 1_\sim$. Hence, $(X, \mathcal{T})$ is IFGSP-compact.

Theorem 3.21. An intuitionistic fuzzy set $A = \langle x, \mu_A, \nu_A\rangle$ in IFTS $(X, \mathcal{T})$ is IFGSP-compact iff for every family $\{\langle x, \mu_{G_i}, \nu_{G_i} : i \in J\rangle\}$ of IFGSP-open sets with properties

$$\mu_A \le \bigvee_{i \in J} \mu_{G_i} \quad and\; 1 - \nu_A \le \bigvee_{i \in J} (1 - \nu_{G_i}),$$

there exist a finite subfamily $\{\langle x, \mu_{G_i}, \nu_{G_i} : i = 1, 2, \cdots, n\rangle\}$ of $\{\langle x, \mu_{G_i}, \nu_{G_i} : i \in J\rangle\}$ such that

$$\mu_A \le \bigvee_{i=1}^{n} \mu_{G_i} \quad and\; 1 - \nu_A \le \bigvee_{i=1}^{n} (1 - \nu_{G_i}).$$

Proof. An intuitionistic fuzzy set $A = \langle x, \mu_A, \nu_A\rangle$ in IFTS $(X, \mathcal{T})$ is IFGSP-compact and $G = \{\langle x, \mu_{G_i}, \nu_{G_i} : i \in J\rangle\}$ be any family of IFGSP-open sets in X satisfies the condition

$$\mu_A \le \bigvee_{i \in J} \mu_{G_i} \quad and\; 1 - \nu_A \le \bigvee_{i \in J} (1 - \nu_{G_i})$$

Then $1 - \nu_A \le 1 - \wedge\{\nu_{G_i} : i \in J\}$, therefore $\nu_A \ge \wedge\{\nu_{G_i} : i \in J\}$, this implies $A \subseteq \{\langle x, \vee\, \mu_{G_i}, \wedge\, \nu_{G_i} : i \in J\rangle\}$. Hence,

$$A \subseteq \cup\, \{\langle x, \mu_{G_i}, \nu_{G_i} : i \in J\rangle\}.$$

By assumption, there exists finite subfamily $\{\langle x, \mu_{G_i}, \nu_{G_i} : i = 1, 2, \cdots, n\rangle\}$ of $\{\langle x, \mu_{G_i}, \nu_{G_i} : i \in J\rangle\}$ which covers A. So,

$$A \subseteq \cup\{\langle x, \mu_{G_i}, \nu_{G_i} : i = 1, 2, \cdots, n\rangle\}.$$

It follows that,

$$A \subseteq \{\langle x, \vee \mu_{G_i}, \wedge\, \nu_{G_i} : i = 1, 2, \cdots, n\rangle\}.$$

Hence,

$$\mu_A \le \bigvee_{i=1}^{n} \mu_{G_i} \quad and\; 1 - \nu_A \le \bigvee_{i=1}^{n} (1 - \nu_{G_i}).$$

Conversely, let $A = \langle x, \mu_A, \nu_A\rangle$ be an intuitionistic fuzzy set in IFTS $(X, \mathcal{T})$ and $\{\langle x, \mu_{G_i}, \nu_{G_i} : i \in J\rangle\}$ be any family of IFGSP-open sets in X satisfies the condition

$$\mu_A \le \bigvee_{i \in J} \mu_{G_i} \quad and\; 1 - \nu_A \le \bigvee_{i \in J} (1 - \nu_{G_i})$$

Thus, $A \subseteq \cup\, \{\langle x, \mu_{G_i}, \nu_{G_i} : i \in J\rangle\}$. Hence, $\{\langle x, \mu_{G_i}, \nu_{G_i} : i \in J\rangle\}$ is a IFGSP-open cover of A. By assumption, there exists a finite subfamily $\{\langle x, \mu_{G_i}, \nu_{G_i} : i = 1, 2, \cdots, n.\rangle\}$ such that

$$\mu_A \leq \bigvee_{i=1}^{n} \mu_{G_i} \quad and\ 1 - \nu_A \leq \bigvee_{i=1}^{n} (1 - \nu_{G_i}) .$$

Therefore,

$$A \subseteq \{\langle x, \vee\mu_{G_i}, \wedge \nu_{G_i} : i = 1, 2, \cdots, n\rangle\}$$

Hence, A is IFGSP-compact.

Theorem 3.22. Let $(X, \mathcal{T})$ be an IFTS. If X is IFGSP-compact, then it is fuzzy compact.

Proof. Suppose, X be IFGSP-compact. Assume contrary that X is not fuzzy compact, then there is atleast one fuzzy open cover $\{\langle x, \mu_{G_i}, \nu_{G_i} : i \in J\rangle\}$ of X not has a finite subcover. i.e $\cup \{\langle x, \mu_{G_i}, \nu_{G_i} : i \in J\rangle\} = 1_\sim$ a fuzzy open cover of X such that $\cup_{i=1}^{n}\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle\} = 1_\sim$. Since, every intuitionistic fuzzy open set is IFGSP-open set. Therefore, a fuzzy open cover $\{\langle x, \mu_{G_i}, \nu_{G_i} : i \in J\rangle\}$ of X becomes IFGSP-open cover of X such that $\cup_{i=1}^{n}\{\langle x, \mu_{G_i}, \nu_{G_i}\rangle\} = 1_\sim$, which is a contradiction. Hence, if X is IFGSP-compact, then it is fuzzy compact.

IV. Conclusion

In the present paper, the term intuitionistic fuzzy generalized semi-pre-compactness (IFGSP-compactness) in intuitionistic fuzzy topological spaces is coined. Its characterizations and properties are investigated. The relations between fuzzy compactness and IFGSP-compactness in intuitionistic fuzzy topological spaces is identified.

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