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Transient Analysis Of A Limited Capacity Queueing Model With Environmental Change And Catastrophic Effects

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Abstract:
In this paper, we undertake the transient analysis of a limited capacity queueing system with two environmental states in the presence of catastrophes. When a catastrophe occur at the service- facility as a Poisson process with rate ξ, the number of customers is instantly reset to zero at certain random times. The change in the environment also affects the state of the queueing system. In other words, the state of the queueing system is a function of environmental change factors.The effects of environmental change and catastrophes are extensively dealt with a system are studied. The steady-state behavior of the queueing system is also derived. Some particular cases of the model with and without catastrophes are also obtained and discussed.

Keywords: *Transient analysis, Catastrophes, Environment, Limited capacity, Steady-state solution.*

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I. Introduction:

In the queueing literature, analytical results for the transient behaviour of queueing models are not as widely available as the steady-state results. The steady-state measures can not give insight into the transient behaviour of the system. The steady-state results are well suited to study the performance measures of the system on a long time scale while the transient solutions are more useful for studying the dynamic behaviour of the system over a finite period. Among several methods, probability generating function technique is one of the techniques that is used to obtain transient solution. Even in the case of an simple M/M/1/N queue, analytical approach to obtain transient behavior is very difficult. In this regard, we have obtained the transient solution of a limited capacity queueing system with two environmental states in the presence of catastrophes.

In the recent past, many authors have introduced a new class of queueing systems with catastrophes. The notion of catastrophe played a very important role in various areas of science and technology, in particular birth and death models. This consists of adding to the standard assumptions the hypothesis that the number of customers is instantly reset to zero at certain random times. The catastrophes occur at the service- facility as a Poisson process with rate ξ. Whenever a catastrophe occurs at the system, all the customers there are destroyed immediately, the server gets inactivated momentarily, and the server is ready for service when a new arrival occurs.

The queuing system with catastrophe was studied by Krishna Kumar and Arivudainambi [13] in 2000 and later 2003 by Crescenzo, et al. [5], where the authors deduced transient probabilities in the M/M/1 queue model with catastrophe. Catastrophic modeling and analysis is important in population genetics ([1, 2, 8, 11] and their references). It is also well known that population processes may be modeled by networks of queues [3] and computer networks with a virus may be modeled by queueing networks with catastrophes [4]. Jain and Kanethia [9] discussed and obtained the transient analysis of a queue with environmental and catastrophic effects. Liu and Liu [18] studied the transient probabilities of an M/PH/1 queue model with catastrophes which is regarded as a generalization of an M/M/1 queue model with catastrophes.

II. Description and Application of the Model:

Consider an M/M/1/N queueing system with two environmental states in the presence of catastrophes. Customers arrive at the service station one by one according to a Poisson stream with arrival rate $\lambda 1$ and 0 of which only one is operative at any instant. There is a single server which provides service to all the arriving customers. Service times are independently and identically distributed exponential random variable with

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parameter $\mu 1$ and $\mu 2$. The customers are served according to the first come, first served rule. Apart from arrival and service processes, the catastrophes occur at the service facilities as a Poisson process with rate ξ. Whenever a catastrophe occurs at the system, all the available customers are destroyed immediately, the server get inactivated momentarily and the server is ready for service only when a new arrival occurs.

This model can be used in a variety of queueing systems found in real-world. In agriculture, if a crop is infected with a particular species of insects due to change in temperature (environment), we may use some chemical agents or compounds to treat such type of insects. The number of bacteria that destroys the crop, in large part, relies on the effectiveness and amount of the chemical reagents used. In other words, the use of the chemical reagents can wipe out the whole of the insects or a part of it. The effect of these chemical reagents on bacteria which make them zero instantaneously can be regarded as the occurrence of a catastrophe.

Now, in the next section we setup the assumptions and definitions of the model. In section 4 we formulate the differential-difference equations governing the queueing system and obtain the transient solution of the queueing model. Some particular cases, steady- state solution and mean queue length is also derived and discussed in section 5-7.

III. Assumptions and Definitions of the Model:

(i) The customers arrive in the system one by one in accordance with a Poisson process at a single service station. The arrival pattern is non-homogeneous, i.e. There may exist two arrival rates, namely $\lambda 1$ and 0 of which only one is operative at any instant.

(ii) The customers are served one by one at the single channel. The service time is exponentially distributed. Further, corresponding to arrival rate $\lambda 1$ the Poisson service rate is $\mu 1$ and the service rate corresponding to the arrival rate 0 is $\mu 2$. The state of the system when operating with arrival rate $\lambda 1$ and service rate $\mu 1$ is designated as E whereas the other with arrival rate 0 and service rate $\mu 2$ is designated as F.

(iii) The Poisson rates at which the system moves from environmental states F to E and E to F are denoted by α and β respectively.

(iv) When the system is not empty, catastrophes occur according to a Poisson process with rate ξ. The effect of each catastrophe is to make the queue instantly empty. Simultaneously, the system becomes ready to accept the new customers.

(v) The queue discipline is first- come-first-served.

(vi) The capacity of the system is limited to M. I.e., if at any instant there are M units in the queue then the units arriving at that instant will not be permitted to join the queue, it will be considered lost for the system.

Define,

$P_n(t)$ = Joint probability that at time t the system is in state E and n units are in thequeue, including the one in service.

$Q_n(t)$ = Joint probability that at time t the system is in state F and n units are in the queue, including the one in service.

$R_n(t)$ = The probability that at time t there are n units in the queue, including the one in service.

Obviously,

$$R_n(t) = P_n(t) + Q_n(t) \tag{1}$$

Let us reckon time t from an instant when there are zero customers in the queue and the system is in the environmental state E so that the initial conditions associated with $P_n(t)$ and $Q_n(t)$ become,

$$P_n(0) = \begin{cases} 1 & ; \quad n = 0 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

$$Q_n(0) = 0 \; ; \quad \text{for all n.} \tag{2}$$

IV. Differential-difference equations governing the system and Transient analysis:

The differential-difference equations governing the system are:

$$\frac{d}{dt}P_0(t) = -(\lambda_1 + \beta + \xi)P_0(t) + \mu_1 P_1(t) + \alpha Q_0(t) + \xi\sum_{n=0}^{M} P_n(t) \; ; \quad n = 0 \tag{3}$$

$$\frac{d}{dt}P_n(t) = -(\lambda_1 + \mu_1 + \beta + \xi)P_n(t) + \mu_1 P_{n+1}(t) + \lambda_1 P_{n-1}(t) + \alpha Q_n(t) \; ; \quad 0 < n < M \tag{4}$$

$$\frac{d}{dt}P_M(t) = -(\mu_1+\beta+\xi)P_M(t) + \lambda_1 P_{M-1}(t) + \alpha Q_M(t)\ ; \qquad n = M \tag{5}$$

$$\frac{d}{dt}Q_0(t) = -(\alpha+\xi)Q_0(t) + \mu_2 Q_1(t) + \beta P_0(t) + \xi\sum_{n=0}^{M} Q_n(t); \qquad n = 0 \tag{6}$$

$$\frac{d}{dt}Q_n(t) = -(\mu_2+\alpha+\xi)\,Q_n(t) + \mu_2\,Q_{n+1}(t) + \beta P_n(t);\ 0 < n < M \tag{7}$$

$$\frac{d}{dt}Q_M(t) = -(\mu_2+\alpha+\xi)Q_M(t) + \beta P_M(t); \qquad n = M \tag{8}$$

Let, the Laplace Transform of f(t) be

$$\bar{f}(s) = \int_0^\infty e^{-st} f(t)\,dt \tag{9}$$

Taking Laplace transform of the equations (3)–(8) and using the initial conditions, we get

$$(s+\lambda_1+\beta+\xi)\overline{P}_0(s) - 1 = \mu_1\overline{P}_1(s) + \alpha\overline{Q}_0(s) + \xi\sum_{n=0}^{M}\overline{P}_n(s) \tag{10}$$

$$(s+\lambda_1+\mu_1+\beta+\xi)\overline{P}_n(s) = \mu_1\overline{P}_{n+1}(s) + \lambda_1\overline{P}_{n-1}(s) + \alpha\overline{Q}_n(t);\ 0<n<M \tag{11}$$

$$(s+\mu_1+\beta+\xi)\overline{P}_M(s) = \lambda_1\overline{P}_{M-1}(s) + \alpha\overline{Q}_M(s) \tag{12}$$

$$(s+\alpha+\xi)\overline{Q}_0(s) = \mu_2\overline{Q}_1(s) + \beta\overline{P}_0(s) + \xi\sum_{n=0}^{M}\overline{Q}_n(s) \tag{13}$$

$$(s+\mu_2+\alpha+\xi)\overline{Q}_n(s) = \mu_2\overline{Q}_{n+1}(s) + \beta\overline{P}_n(s)\ ; \qquad 0<n<M \tag{14}$$

$$(s+\mu_2+\alpha+\xi)\overline{Q}_M(s) = \beta\overline{P}_M(s) \tag{15}$$

Define, the probability generating functions by

$$P(z,s) = \sum_{n=0}^{M}\overline{P}_n(s)z^n \tag{16}$$

$$Q(z,s) = \sum_{n=0}^{M}\overline{Q}_n(s)z^n \tag{17}$$

$$R(z,s) = \sum_{n=0}^{M}\overline{R}_n(s)z^n \tag{18}$$

Where

$$R(z,s) = P(z,s) + Q(z,s) \tag{19}$$

And

$$\overline{R}_n(s) = \overline{P}_n(s) + \overline{Q}_n(s) \tag{20}$$

Multiplying equations (10)–(12) by z^n, summing over the respective ranges of n and using equations (16)–(18), we have.

$$\left[z(s+\lambda_1+\mu_1+\beta+\xi) - \lambda_1 z^2 - \mu_1\right]P(z,s) - \alpha\, z\,Q(z,s) + \mu_1(1-z)\overline{P}_0(s)$$
$$-\lambda_1 z^{M+1}(1-z)\overline{P}_M(s) - z - \xi z\sum_{n=0}^{M}\overline{P}_n(s) = 0 \tag{21}$$

Similarly, from equations (13)–(15) on using equations (16)–(18), we have

$$\beta z\,P(z,s) + [\mu_2 - z(s+\mu_2+\alpha+\xi)]\,Q(z,s) - \mu_2(1-z)\overline{Q}_0(s) + \xi z\sum_{n=0}^{M}\overline{Q}_n(s) = 0 \tag{22}$$

Solving equations (21) and (22), we have

$$P(z,s)=\frac{\begin{array}{l}\alpha\mu_2 z(1-z)\overline{Q}_0(s)-\alpha\xi z^2\sum_{n=0}^{M}\overline{Q}_n(s)-\mu_1(1-z)[\mu_2-(s+\mu_2+\alpha+\xi)]\overline{P}_0(s)\\+\lambda_1 z^{M+1}(1-z)[\mu_2-z(s+\mu_2+\alpha+\xi)]\overline{P}_M(s)+z[\mu_2-z(s+\mu_2+\alpha+\xi)]\\+\xi z\sum_{n=0}^{M}\overline{P}_n(s)[\mu_2-z(s+\mu_2+\alpha+\xi)]\end{array}}{\alpha\beta z^2+[z(s+\lambda_1+\mu_1+\beta+\xi)-\lambda_1 z^2-\mu_1][\mu_2-z(s+\mu_2+\alpha+\xi)]} \qquad (23)$$

$$Q(z,s)=\frac{\begin{array}{l}\beta\mu_1 z(1-z)\overline{P}_0(s)+\mu_2(1-z)\left[z(s+\lambda_1+\mu_1+\beta+\xi)-\lambda_1 z^2-\mu_1\right]\overline{Q}_0(s)\\+\xi z\left[\lambda_1 z^2-z(s+\lambda_1+\mu_1+\beta+\xi)+\mu_1\right]\sum_{n=0}^{M}\overline{Q}_n(s)-\beta\lambda_1 z^{M+2}(1-z)\overline{P}_M(s)\\-\beta z^2-\xi\beta z^2\sum_{n=0}^{M}\overline{P}_n(s)\end{array}}{\alpha\beta z^2+\left[z(s+\lambda_1+\mu_1+\beta+\xi)-\lambda_1 z^2-\mu_1\right]\left[\mu_2-z(s+\mu_2+\alpha+\xi)\right]} \qquad (24)$$

Now from equation (19), we have

$$R(z,s)=\frac{\begin{array}{l}\left[\alpha\mu_2 z+\mu_2\{z(s+\lambda_1+\mu_1+\beta+\xi)-\lambda_1 z^2-\mu_1\}\right](1-z)\overline{Q}_0(s)\\+\left[\{\lambda_1 z^2-z(s+\lambda_1+\mu_1+\beta+\xi)+\mu_1\}-\alpha z\right]\xi z\sum_{n=0}^{M}\overline{Q}_n(s)\\+\left[\{z(s+\mu_2+\alpha+\xi)-\mu_2\}\mu_1+\beta\mu_1 z\right](1-z)\overline{P}_0(s)\\+\left[\lambda_1 z^{M+1}\{\mu_2-z(s+\mu_2+\alpha+\xi)\}-\beta\lambda_1 z^{M+2}\right](1-z)\overline{P}_M(s)+[\mu_2-z(s+\mu_2+\alpha+\xi)]z\\-\beta z^2+\left[\{\mu_2-z(s+\mu_2+\alpha+\xi)\}-\beta z\right]\xi z\sum_{n=0}^{M}\overline{P}_n(s)\end{array}}{\begin{array}{l}-z^2 s^2+s\left[\lambda_1 z^3-z^2(\lambda_1+\mu_1+\mu_2+\alpha+\beta+2\xi)+z(\mu_1+\mu_2)\right]\\+(1-z)\left[-z^2\lambda_1(\alpha+\mu_2+\xi)+z\{\alpha\mu_1+\mu_2(\lambda_1+\mu_1+\beta+\xi)\}-\mu_1\mu_2\right]-z^2\xi(\alpha+\beta+\xi)\end{array}} \qquad (25)$$

The unknown quantities in equation (25) are determined as follows:
Setting z=1, in equations (23) and (24) respectively, we have

$$P(1,s)=\sum_{n=0}^{M}\overline{P}_n(s)=\frac{s+\alpha}{s(s+\alpha+\beta)} \qquad (26)$$

and

$$Q(1,s)=\sum_{n=0}^{M}\overline{Q}_n(s)=\frac{\beta}{s(s+\alpha+\beta)} \qquad (27)$$

Further, relation (25) is a polynomial in z and exists for all values of z, including the three zeros of the denominator. Hence, the remaining unknown quantities $\overline{P}_0(s)$, $\overline{Q}_0(s)$ and $\overline{P}_M(s)$ are obtained by setting the numerator equal to zero and substituting the three zeros α_1, α_2 and α_3 (say) of the denominator (at each of which the numerator must vanish).

The Laplace transform of various state probabilities for the number of units in the queue, including the one in service can be picked up as the co-efficient of the different powers of z in the expansion of equation (25).

V. Particular Case:

Now letting $\alpha\to\infty$, $\beta\to 0$ and setting $\mu_1=\mu_2=\mu$ (say) in relation (25), we have

$$r(z,s)=\frac{(1-z)\mu\overline{R}_0(s)-(1-z)\lambda_1 z^{M+1}\overline{P}_M(s)-z-\xi z/s}{\lambda_1 z^2-z(s+\lambda_1+\mu+\xi)+\mu} \qquad (28)$$

Where

$$\overline{R}_0(s)=\overline{P}_0(s)+\overline{Q}_0(s)$$

$$r(z,s)=\lim_{\beta\to 0}\left[\lim_{\alpha\to\infty} R(z,s)\right]$$

Relation (28) is a polynomial in z and exists for all values of z, including the two zeros of the denominator. Hence, the unknown quantities $\overline{R}_0(s)$ and $\overline{P}_M(s)$ can be evaluated as before.

VI. Steady State Results:

This can at once be obtained by the well-known property of the Laplace transform given below:

$\lim_{t\to\infty} f(t) = \lim_{s\to 0} s\ \bar{f}(s),$ if the limit on the left hand side exists.

Thus if

$$R(z)=\sum_{n=0}^{M} R_n\ z^n$$

Where,

$$R_n=\lim_{s\to 0} s\overline{R}_n(s)$$

Then

$$R(z)=\lim_{s\to 0} s\ R\ (z,s)$$

And $\sum_{n=0}^{M} R_n=\sum_{n=0}^{M} P_n+\sum_{n=0}^{M} Q_n=1$ (29)

By employing this property, we have from equation (25).

$$R(z)=\frac{\begin{array}{l}\mu_2(1-z)\left\{\alpha z+z(\lambda_1+\mu_1+\beta+\xi)-\lambda_1 z^2-\mu_1\right\}Q_0\\+\mu_1(1-z)\left[\beta z-\left\{\mu_2-z\left(\mu_2+\alpha+\xi\right)\right\}\right]P_0+\lambda_1 z^{M+1}(1-z)\left\{\mu_2-z\left(\mu_2+\alpha+\xi\right)-\beta z\right\}P_M\\+\left\{\xi z/(\alpha+\beta+\xi)\right\}\left[\beta\left\{\lambda_1 z^2-z(\lambda_1+\mu_1+\alpha+\beta+\xi)+\mu_1\right\}+(\alpha+\xi)\left\{\mu_2-z(\mu_2+\alpha+\beta+\xi)\right\}\right]\end{array}}{\begin{array}{l}z^3\lambda_1(\mu_2+\alpha+\xi)-z^2\left[\lambda_1(\mu_2+\alpha+\xi)+\left\{\alpha\mu_1+\mu_2(\lambda_1+\mu_1+\beta+\xi)\right\}+\xi(\alpha+\beta+\xi)\right]\\+z\left[\left\{\alpha\mu_1+\mu_2(\lambda_1+\mu_1+\beta+\xi)\right\}+\mu_1\mu_2\right]-\mu_1\mu_2\end{array}} \quad (30)$$

Or, we can write

$$R(z)=\frac{T(z)Q_0+N(z)P_0+L(z)P_M+M(z)}{K(z)} \quad (31)$$

Where T(z), N(z) and L(z) are the co-efficient of Q_0, P_0 and P_M respectively in the numerator of equation (30) and K(z) is the denominator of equation (30).

Equation (31) is a polynomial in z and exists for all values of z, including three zeros of the denominator. Hence Q_0, P_0 and P_M can be obtained by setting the numerator equal to zero. Substituting the three zeros b_1, b_2 and b_3 (say) of the denominator (at each of which the numerator must vanish).

Now, three equations determining the unknown quantities Q_0, P_0 and P_M are:

$$T(b_1)Q_0+N(b_1)P_0+L(b_1)P_M=-M(b_1) \quad (32)$$

$$T(b_2)Q_0+N(b_2)P_0+L(b_2)P_M=-M(b_2) \quad (33)$$

$$T(b_3)Q_0+N(b_3)P_0+L(b_3)P_M=-M\ (b_3) \quad (34)$$

After solving these equations, we have

$$Q_0=\frac{-M(b_1)A_{11}+M(b_2)A_{21}-M(b_3)A_{31}}{A}$$

$$P_0=\frac{M(b_1)A_{12}-M(b_2)A_{22}+M(b_3)A_{32}}{A}$$

$$P_M=\frac{-M(b_1)A_{13}+M(b_2)A_{23}-M(b_3)A_{33}}{A}$$

Where

$$A=\begin{vmatrix} T(b_1) & N(b_1) & L(b_1) \\ T(b_2) & N(b_2) & L(b_2) \\ T(b_3) & N(b_3) & L(b_3) \end{vmatrix}$$

A_{ij} is the co-factor of the $(i, j)^{th}$ element of A.

By putting the values of Q_0, P_0 and P_M in equation (31), we have

$$R(z)=\frac{\begin{gathered}T(z)\left[-M(b_1)A_{11}+M(b_2)A_{21}-M(b_3)A_{31}\right]+N(z)\left[M(b_1)A_{12}-M(b_2)A_{22}+M(b_3)A_{32}\right]\\+L(z)\left[-M(b_1)A_{13}+M(b_2)A_{23}-M(b_3)A_{33}\right]+A\cdot M(z)\end{gathered}}{A\cdot K(z)} \quad (35)$$

VII. Mean Queue Length:

Define,

L_q = Expected number of customers in the queue including the one in service.

Then

$L_q = R'(z)\big|_{z=1}$

Therefore, from equation (35), we have

$$L_q=\frac{\begin{gathered}K(1)\left[T'(1)\{-M(b_1)A_{11}+M(b_2)A_{21}-M(b_3)A_{31}\}+N'(1)\{M(b_1)A_{12}-M(b_2)A_{22}+M(b_3)A_{32}\}\right.\\+L'(1)\{-M(b_1)A_{13}+M(b_2)A_{23}-M(b_3)A_{33}\}+A\,M'(1)\big]-\big[T(1)\{-M(b_1)A_{11}+M(b_2)A_{21}\\-M(b_3)A_{31}\}+N(1)\{M(b_1)A_{12}-M(b_2)A_{22}+M(b_3)A_{32}\}+L(1)\{-M(b_1)A_{13}+M(b_2)A_{23}\\\left.-M(b_3)A_{33}\}+A\cdot M(1)\right]K'(1)\end{gathered}}{A\cdot[K(1)]^2} \quad (36)$$

Where dashes denotes the first derivative with respect to z.

Particular Cases:

Case I: Relation (28), on applying the theory of Laplace transforms gives

$$r(z)=\frac{(1-z)\mu R_0-(1-z)\lambda_1 z^{M+1}P_M-\xi z}{\lambda_1 z^2-z(\lambda_1+\mu+\xi)+\mu} \quad (37)$$

Where

$$r(z)=\lim_{s\to 0} s\,r(z,s)$$

Equation (37) is a polynomial in z and exists for all values of z, including the two zeros of the denominator. Hence R_0 and P_M can be obtained by setting the numerator equal to zero. Substituting the two zeros a_1 and a_2 (say) of the denominator (at each of which the numerator must vanish).

Case II: If $\xi = 0$ (i.e., no catastrophe is allowed in the system), then from equation (37), we have

$$r(z)=\frac{\mu R_0-\lambda_1 z^{M+1}P_M}{\mu-\lambda_1 z} \quad (38)$$

The condition, $\lim_{z\to 1} r(z)=1$ gives

$$\mu R_0-\lambda_1 P_M=\mu-\lambda_1 \quad (39)$$

As r(z) is analytic, the numerator and denominator of equation (38) must vanish simultaneously for $z=\mu/\lambda_1$, which is a zero of its denominator. Equating the numerator of equation (38) to zero for $z=\mu/\lambda_1$ we have

$$R_0=\rho^{-M}P_M, \qquad \rho=\lambda_1/\mu<1 \quad (40)$$

Relation (39) and (40) gives

$$R_0=\frac{1-\rho}{1-\rho^{M+1}}, \quad P_M=\frac{(1-\rho)\rho^M}{1-\rho^{M+1}}$$

Now, from equation (38), we have

$$r(z)=\frac{1-\rho}{1-\rho^{M+1}} \cdot \left[\frac{1-(\rho z)^{M+1}}{1-\rho z}\right] \tag{41}$$

Which is a well known result of the M/M/1 queue with finite waiting space M.

When there is an infinite waiting space, the corresponding expression for r(z) is obtained by letting M tends to infinity in equation (41), If Max(ρ, |z|) < 1.

$$r(z)=\frac{1-\rho}{1-\rho z} \tag{42}$$

Which is again a well- known result of the M/M/1 queue with infinite waiting space.

Case III: In [13], Krishan Kumar and Arivudainambi have studied the transient solution of an M/M/1 queue with catastrophes. They have also obtained the steady- state probabilities and mean & variance of the M/M/1 queue with catastrophes.

When a catastrophe occurs at the service facility i.e. ξ >0, the steady- state distribution $\{p_n;\ n \geq 0\}$ of the M/M/1 queue with catastrophes corresponds to:

$$p_0 = (1-\rho)\ ;\ \ n = 0 \tag{43}$$

$$p_n = (1-\rho)\rho^n\ ;\ \ n = 1, 2, 3, \ldots\ldots \tag{44}$$

where

$$\rho = \frac{(\lambda+\mu+\xi)-\sqrt{\lambda^2+\mu^2+\xi^2+2\lambda\xi+2\mu\xi-2\lambda\mu}}{2\mu} \tag{45}$$

Thus equations (43)-(45) provide the steady- state distribution for the queueing system. Obviously, the steady state distribution exists if and only if $\rho < 1$.

Note: The steady-state probability of this Markov process exists if and only if $\xi > 0$ *or* $\xi = 0$ *and* $\lambda > \mu$. It is also observed that the results of equations (43)-(45) agree with the model discussed above and with [4] by Chao, X.

VIII. Conclusion:

In this paper, we have established a queueing model and obtained the transient solution of the model with environmental change and catastrophic effects. The steady state result and mean queue length of the model is also derived and discussed. We have also obtained some particular cases with and without catastrophes.

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