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Fuzzy Quasi-Regular Spaces

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Abstract
In thispaper, several characterizations of fuzzy quasi-regular spaces,which are defined bymeans of fuzzy open sets andfuzzyregular closed sets, are established. It is obtained that eachfuzzy set defined in a fuzzy quasi-regular space contains a fuzzy regular closed set and each fuzzy G_δ-set contains a fuzzy closed setin afuzzy quasi-regular space.The conditions under which fuzzy quasi-regular spacesbecome fuzzy weakly bairespacesand fuzzy bairespaces are obtained. It is obtained that fuzzy quasi-regular spaces are not fuzzyhyperconnected spaces.

Keywords :*fuzzyG_δ-set, fuzzy F_σ-set, fuzzyσ-boundary set, fuzzy residual set,Fuzzy regular space, fuzzybairespace, fuzzy weakly baire space.*

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I. Fuzzy Quasi-Regular Spaces

In order to deal with uncertainties, the idea of fuzzy sets, fuzzy set operations was introduced by **L.A. Zadeh** [**19**] in 1965. The potential of fuzzy notion was realized by the researchers and has successfully been applied in all branches of Mathematics. In 1968, **C.L. Chang**[4] introduced the concept of fuzzy topological spaces and his work paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. In classical topology,**John.C. Oxtoby**[**8**] introduced the notion of quasi-regularity and by means of which he produced a productive subclass of the class of Baire spaces which contains all completely metrizable and all Hausdorff locally compact spaces. The condition of quasi-regularity has the flavour of a separation condition [**9**].

In the recent years, there has been a growing trend among many fuzzy topologists to introduce and study various types of fuzzy topological spaces.Motivatedby the works of **John. C. Oxtoby**[**8**] and **A. R. Todd**[**10**], onquasi-regularityin classical topology, the notion of fuzzy quasi-regularity in fuzzy topological spaces was defined by **G.Thangaraj** and **S.Anjalmose** [**1**]. The purpose of this paper is to study several properties and applications of fuzzy quasi-regular spaces.

In section **3**, it isobtained that each fuzzy set definedin a fuzzy quasi-regular space contains a fuzzy regular closedset and each fuzzy G_δ-set in a fuzzy quasi-regular space contains a fuzzy closed set. Also it isestablished thateach fuzzy closed set is contained in a fuzzy regular open set and each fuzzy F_σ-set is contained in a fuzzy regular open set in fuzzy quasi-regular spaces. It is found that each fuzzy residual set containsfuzzy closed set and each fuzzy nowhere denseset is contained in a fuzzy regular open set and each fuzzy first category set is contained in a fuzzy open set in fuzzy quasi-regular spaces. Also it is established thateach fuzzyσ-boundary set is contained in a fuzzy open set and each fuzzy co-σ-boundary set contains a fuzzy open set and each fuzzy open set contains a fuzzy regular open set and a fuzzy somewhere dense set in fuzzy quasi-regular spaces.It is obtained that class of fuzzy F_σ-sets lies between the classes of fuzzy open sets and fuzzy regular closed sets.

In section **4**, the inter-relations between fuzzy regular spaces and fuzzy quasi-regular spaces are established. The conditions under which fuzzy quasi-regular spaces become fuzzy weakly Baire spaces and fuzzy Baire spaces are obtained. It is obtained that fuzzy quasi-regular spaces are not fuzzy hyperconnected spaces.

II. Preliminaries

In order to make the exposition self-contained, some basic notions and results used in the sequel, are given. In this work by(X,T) or simply by x, we will denote a fuzzy topological space due to Chang (1968). Let Xbe a non-empty set and Ithe unit interval [0,1]. A fuzzy set λin X is a mapping from X into I. The fuzzy set$0_X$is defined as $0_X(x) = 0$, for all $x \in X$ and the fuzzy set 1_X is defined as $1_X(x) = 1$, for all$x \in X$.

Definition 2.1 [**4**] **:**Afuzzy topology is a family T of fuzzy sets in X which satisfies the following conditions:
(a). $0_X \in T$and $1_X \in T$.
(b). If$a, b \in T$, then $a \wedge b \in T$.
(c). If $A_i \in T$ for each$i \in J$, then $\vee_i A_i \in T$.
T is called a fuzzy topology for X,and the pair (X,T) is a fuzzy topological space, or fts for short. Every member of T is called a T- openfuzzy set.

Definition 2.2[**4**]**:** Let (X,T) be a fuzzy topological space and λ be any fuzzy set in (X,T). The interior, the closure and the complement of λ are defined respectively as follows:
(i). $\text{int}(\lambda) = \vee \{ \mu / \mu \leq \lambda, \mu \in t\}$;
(ii). $\text{cl}(\lambda) = \wedge\{ \mu / \lambda \leq \mu, 1-\mu \in t \}$.
(iii). $\lambda'(x) = 1 - \lambda(x)$, for all $x \in X$.
For a family$\{ i \in J \}$of fuzzy sets in(X,T), the union $\psi = \vee_i (\lambda_i)$ and the intersection $\delta = \wedge_i (\lambda_i)$, are defined respectively as
(iv). $\Psi(x) = \sup_i\{ \lambda_i(x) / x \in X\}$.
(v). $\Delta(x) = \inf_i\{ \lambda_i(x) / x \in X\}$.

Lemma 2.1[**2**]**:** For a fuzzy set λ of a fuzzy topological space X,
(i). $1-\text{int}(\lambda) = \text{cl}(1-\lambda)$ and (ii). $1-\text{cl}(\lambda) = \text{int}(1-\lambda)$.

Definition2.3:A fuzzy set λin a fuzzy topological space $(X,T)$is called a
(1). fuzzy regular - open set if $\lambda = \text{intcl}(\lambda)$ and
fuzzy regular-closed set if $\lambda = \text{clint}(\lambda)$ [**2**].
(2).fuzzyG_δ-set if $\lambda = \wedge_{i=1}^{\infty} (\lambda_i)$,where $\lambda_i \in T$;
fuzzyF_σ–set if $\lambda = \vee_{i=1}^{\infty}(\mu_i)$, where $1-\mu_i \in T$ [**3**].

Definition 2.4: A fuzzy set λin a fuzzy topological space (X,T), is called a
(i).**fuzzy dense set** if there exists no fuzzy closed set μ in (X,T)Such that$\lambda < \mu < 1$. That is, $cl(\lambda) = 1$, in (X,T) [**11**].
(ii**). fuzzy nowhere dense set** if there exists no non-zero fuzzy open setμ in (X,T)such that $\mu < cl(\lambda)$. That is, $\text{intcl}(\lambda) = 0$, in (X,T)[**11**].
(iii**). fuzzy first category set**if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $(\lambda_i)'$S are fuzzy nowhere dense sets in (X,T). Any other fuzzy set in(X,T) is said to be of fuzzy second category[**11**].
(iv). **fuzzy residual set**if $1-\lambda$ is a fuzzy first category set in (X,T) [**12**].
(v).**fuzzy somewhere dense set** if there exists a non-zero fuzzy open set μ in (X,T) such that $\mu < cl(\lambda)$. That is, $\text{intcl}(\lambda) \neq 0$, in (X,T) [**18**].
(vi). **fuzzy σ-boundary set** if $\lambda = \vee_{i-1}^{\infty}(\mu_i)$, where $\mu_i = \text{cl}(\lambda_i) \wedge (1-\lambda_i)$ and (λ_i)'s are fuzzy regular open sets in (X,T) [**17**].
(vii). **fuzzy co- σ-boundaryset** if $\gamma = \wedge_{i=1}^{\infty}(\gamma_i)$, where $\gamma_i = \text{int}(1-\lambda_i) \vee \lambda_i$ and(λ_i)'s are fuzzy regular open sets in (X,T) [**17**].
(viii). **fuzzyresolvableset** iffor eachfuzzy closed set μ in(X,T), $\text{cl}(\mu \wedge \lambda) \wedge \text{cl}(\mu \wedge (1-\lambda))$is afuzzynowhere dense in(X,T)[**15**].
(ix).**fuzzy simply open set** if bd(λ)is a fuzzy nowhere dense set in (X,T).That is, λis a fuzzy simply open set in (X,T)if $[\text{cl}(\lambda) \wedge \text{cl}(1-\lambda)]$, is a fuzzy nowhere dense set in (X,T) [**14**].

Definition2.5:A fuzzy topological space (X,T) is called a
(i). **fuzzy regular space**if for eachfuzzy open set λin (X,T), $\lambda = \vee_\alpha (\lambda_\alpha)$, where $\text{cl}(\lambda_\alpha) \leq \lambda$and$\lambda_\alpha \in T$, for each$\alpha$ [**5**].
(ii). **fuzzy Baire space**ifint $(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where$(\lambda_i)'$Sare fuzzy nowhere dense sets in (X,T) [**13**].
(iii). **fuzzy weakly Bairespace** if $\text{int}(\vee_{i=1}^{\infty}(\mu_i)) = 0$, where $\mu_i = \text{cl}(\lambda_i) \wedge (1-\lambda_i)$ and (λ_i)'s are fuzzy regular open sets in (X, T) [**17**].
(iv).**fuzzy open hereditarily irresolvable space**if $\text{intcl}(\lambda) \neq 0$, for any non - zero fuzzy set λ defined on X, then$int(\lambda) \neq 0$, in (X,T) [**12**].
(v). **fuzzy hyperconnectedspace** if every non - null fuzzy open subset of $(X,T)$is fuzzy dense in(X,T) [**7**].

Theorem 2.1 [2] : In a fuzzy topological space,
(a).The closure of a fuzzy open set is a fuzzy regular closed set.
(b). The interior of a fuzzy closed set is a fuzzy regular open set.
Theorem 2.2[14] : If λ is a fuzzy simply open set in a fuzzy topologicalspace(X, T), then $\lambda \wedge (1 - \lambda)$ is a fuzzy nowhere dense set in (X, T).
Theorem 2.3 [16] : If λ is a fuzzy residual set in a fuzzytopological space(X, T), then there existsa fuzzyG_δ-set μin(X, T)suchthat $\mu \leq \lambda$.
Theorem 2.4 [17] : If λ is a fuzzy σ-boundary set in a fuzzy topological space (X, T), then $\lambda$is a fuzzy F_σ-setin (X, T).
Theorem 2.5 [17] : If γ is a fuzzy co- σ-boundary set in a fuzzytopological space(X, T), then $1 - \gamma$is a fuzzyσ-boundary set in (X, T).
Theorem 2.6 [13] :Let(X, T) bea fuzzy topological space. Then the following are equivalent:
(1). (X, T) is anfuzzy Bairespace.
(2).$int\ (\lambda) = 0$, for every fuzzy first category set λ in (X, T).
(3). cl (μ) =1, for every fuzzy residual set μ in (X, T).
Theorem 2.7 [17] : Let(X, T) be a fuzzy topological space. Then, the following are equivalent:
(1).$(X, T)$is a fuzzy weakly Baire space.
(2). $int\ (\lambda) = 0$, for every fuzzy σ-boundary set λ in (X, T).
(3). $cl\ (\mu) = 1$, for every fuzzy co- σ-boundary set μ in (X, T).
Theorem 2.8 [17] : If a fuzzy topological space (X, T) is a fuzzy weakly Baireand fuzzy open hereditarily irresolvable space, then (X, T) is a fuzzy Baire space.
Theorem 2.9[5] **:**Let (X, T)be a fuzzy topological space. Then, the following properties are equivalent:
(i). $(X, T)$is fuzzy hyperconnected,
(ii). $1_X$and $0_X$are the only fuzzy regular open sets in X.

Theorem 2.10 [15]: If λ is a fuzzy closed set with $int\ (\lambda) = 0$, in a fuzzy topological space (X, T), then λ is afuzzy resolvable set in (X, T).
Theorem 2.11[17] : If (X, T) is a fuzzy weakly Baire space, then $int\ (\lambda) \wedge int\ (1 - \lambda) = 0$, for any fuzzy set λ defined on X.

III. FuzzyQuasi-Regular spaces

Motivated by the works of John. C. Oxtoby[8] and A.R. Todd [10],onquasi-regularity in classical topology, the notion of fuzzy quasi-regularity in fuzzy topologicalspaces is defined as follows**:**
Defintion 3.1 : A fuzzy topological space $(X, T)$is called a fuzzy quasi-regular space iffor each fuzzy open setλin (X, T), there exists a fuzzy regular closed set μ in (X, T)such that $\mu \leq \lambda$.
Example3.1 : Let$X = \{a, b, c\}$ and $I = [0, 1]$. The fuzzy setsα, β and γ are defined on X as follows**:**
$\alpha : X \to I$ is defined by $\alpha(a) = 0.4$; $\alpha\ (b) = 0.6$; $\alpha\ (c) = 0.4$,
$B : X \to I$ is defined by $\beta(a) = 0.6$; $\beta\ (b) = 0.4$; $\beta\ (c) = 0.6$,
$\gamma : X \to I$ is defined by $\gamma(a) = 0.4$; $\gamma\ (b) = 0.4$; $\gamma\ (c) = 0.6$.
Then,$T = \{0, \alpha, \beta, \gamma, \alpha \vee \beta, \alpha \vee \gamma, \alpha \wedge \beta, 1\}$ is a fuzzy topology onX. By computation, one can find that

$$cl(\alpha) = 1 - \beta\ ; int\ (1 - \alpha) = \beta\ ;$$
$$cl\ (\beta) = 1 - \alpha\ ; int\ (1 - \beta) = \alpha\ ;$$
$$cl\ (\gamma) = 1 - \alpha\ ; int\ (1 - \gamma) = \alpha\ ;$$
$$cl\ (\alpha \vee \beta) = 1 - [\alpha \wedge \beta]\ ; int\ (1 - [\alpha \vee \beta]) = \alpha \wedge \beta\ ;$$
$$cl\ (\alpha \vee \gamma) = 1 - [\alpha \wedge \beta]\ ; int\ (1 - [\alpha \vee \gamma]) = \alpha \wedge \beta\ ;$$
$$cl\ (\alpha \wedge \beta) = 1 - (\alpha \vee \beta)\ .\ int\ (1 - [\alpha \wedge \beta]) = \alpha \vee \beta.$$

The fuzzy regular closed sets in (X, T) are $1 - \alpha$, $1 - \beta$, $1 - (\alpha \vee \beta)$and $1 - (\alpha \wedge \beta)$ and $1 - \beta \leq \alpha\ ; 1 - \alpha \leq \beta\ ;\ 1 - (\alpha \vee \beta) \leq \gamma\ ;\ 1 - (\alpha \wedge \beta) \leq \alpha \vee \beta\ ; 1 - \beta \leq \alpha \vee \gamma$ and $1 - (\alpha \vee \beta) \leq \alpha \wedge \beta$.Thus, for each fuzzy open set$\lambda (= \alpha, \beta, \gamma, \alpha \vee \beta, \alpha \vee \gamma, \alpha \wedge \beta)$,there exists a fuzzy regular closed set μ
$(1 - \alpha, 1 - \beta, 1 - (\alpha \vee \beta), 1 - (\alpha \wedge \beta))in(X, T)$ such that $\mu \leq \lambda$. Hence (X, T) is a fuzzy quasi-regular space.
Example 3.2 : Let$X = \{a, b, c\}$ and $I = [0, 1]$. The fuzzy setsα, β and γ are defined on X as follows**:**
$\alpha : X \to I$ is defined by $\alpha(a) = 0.5$; $\alpha\ (b) = 0.4$; $\alpha\ (c) = 0.4$,
$B : X \to I$ is defined by $\beta(a) = 0.6$; $\beta\ (b) = 0.4$; $\beta\ (c) = 0.6$,
$\gamma : X \to I$ is defined by $\gamma(a) = 0.4$; $\gamma\ (b) = 0.5$; $\gamma\ (c) = 0.4$.

Then, $T = \{ 0, \alpha, \beta, \gamma, \alpha \vee \gamma, \beta \vee \gamma, \alpha \wedge \gamma, 1\}$ is a fuzzy topology on X. By computation, one can find that

$$cl(\alpha) = 1 - (\alpha \vee \gamma) \; ; int\ (1 - \alpha) = \alpha \vee \gamma \; ;$$
$$cl\ (\beta) = 1 - \gamma \; ; int\ (1 - \beta) = \gamma \; ;$$
$$cl\ (\gamma) = 1 - [\beta \vee \gamma] \; ; int\ (1 - \gamma) = \beta \vee \gamma \; ;$$
$$cl\ (\alpha \vee \gamma) = 1 - [\alpha \vee \gamma] \; ; int\ (1 - [\alpha \vee \gamma]) = \alpha \vee \gamma \; ;$$
$$cl\ (\beta \vee \gamma) = 1 - \gamma \; ; int\ (1 - [\beta \vee \gamma]) = \gamma \; ;$$
$$cl\ (\alpha \wedge \gamma) = 1 - (\beta \vee \gamma) . int\ (1 - [\alpha \wedge \gamma]) = \beta \vee \gamma .$$

The fuzzy regular closed sets in (X, T) are $1 - \gamma$, $1 - (\alpha \vee \gamma)$, $1 - (\beta \vee \gamma)$.

Now for the fuzzy open set α, $(1 - \gamma) \nleq \alpha$; $1 - (\alpha \vee \gamma) \nleq \alpha$ and $1 - (\beta \vee \gamma) \nleq \alpha$.

Thus, for the fuzzy open set αin (X, T), there is no fuzzy regular closed set μ $(1 - \gamma,\ 1 - (\alpha \vee \gamma), 1 - (\beta \vee \gamma))$ in (X, T) such that $\mu \leq \lambda$. Hence (X, T) is not a fuzzy quasi-regular space.

Proposition 3.1 : If there existsa fuzzy open setγ such that $cl(\gamma) \leq \lambda$,for each fuzzy open set λ in a fuzzy topological space (X, T), then (X, T) is a fuzzyquasi-regular space.

Proof : Letλ be a fuzzy open set in (X, T).Suppose that $cl\ (\gamma) \leq \lambda$, where $\gamma$is a fuzzy open setin (X, T). By Theorem2.1, $cl\ (\gamma)$is afuzzyregular closed set in (X, T).Let $\mu = cl\ (\gamma)$. Hence, for the fuzzy open setλin (X, T), the existence of a fuzzy regular closed set μ in (X, T) such that $\mu \leq \lambda$ implies that (X, T)isa fuzzy quasi-regular space.

Proposition 3.2 : If δ is a fuzzy closed set in a fuzzy quasi-regular space (X, T), then there exists a fuzzy regular open set α in(X, T)such that $\delta \leq \alpha$.

Proof : Letδ be a fuzzy closed set in (X, T). Then, $1 - \delta$ is a fuzzy open set in (X, T). Since (X, T) is a fuzzy quasi-regular space, there exists a fuzzy regular closed set μin (X, T) such that$\mu \leq 1 - \delta$ .Then, $\delta \leq 1 - \mu$.Let $\alpha = 1 - \mu$. Hence, for the fuzzy closed set δ, there exists a fuzzy regular open set α in (X, T)such that $\delta \leq \alpha$.

Proposition 3.3 : Ifλ is a fuzzy G_δ-set in a fuzzy quasi-regular space (X, T), then there exists a fuzzy closed set θ in (X, T) such that $\theta \leq \lambda$.

Proof : Letλ be a fuzzy G_δ-set in (X, T). Then,$\lambda = \wedge_{i=1}^{\infty} (\lambda_i)$, where $\lambda_i \in T$ .Since(X, T) is a fuzzy quasi-regular space, for the fuzzy open set λ_i , there exists a fuzzy regular closed set μ_i in (X, T) such that$\mu_i \leq \lambda_i$. This implies that $\wedge_{i=1}^{\infty} (\mu_i) \leq \wedge_{i=1}^{\infty} (\lambda_i)$and then $\wedge_{i=1}^{\infty} (\mu_i) \leq \lambda$, in (X, T).Since fuzzy regular closed setsare fuzzyclosedsets in a fuzzy topological space,$\wedge_{i=1}^{\infty} (\mu_i)$is a fuzzy closed set in (X, T).Let $\theta = \wedge_{i=1}^{\infty} (\mu_i)$. Thus, θ is a fuzzy closed set in (X, T) such that $\theta \leq \lambda$.

Corollary3.1 : If μ is a fuzzy F_σ -set in a fuzzy quasi-regular space (X, T), then there exists a fuzzy openset γ in (X, T)such that $\mu \leq \gamma$.

Proof : Letμ be a fuzzy F_σ-set in (X, T). Then, $1 - \mu$ is a fuzzy G_δ-setin (X, T) and by Proposition 3.3, there exists a fuzzy closed set θin (X, T) such that$\theta \leq 1 - \mu$. This implies that $\mu \leq 1 - \theta$, in (X, T). Let $\gamma = 1 - \theta$. Thus, γ is a fuzzy open setin (X, T) such that $\mu \leq \gamma$.

Proposition3.4 : Ifλ is a fuzzy set defined on X in a fuzzy quasi-regular space (X, T), then there exists a fuzzy regular closed set μ in (X, T)such that $\mu \leq \lambda$.

Proof: Letλbe a fuzzyset defined on Xin (X, T). Then, $int\ (\lambda)$isafuzzy open set in (X, T). Since $(X, T)$is afuzzy quasi-regular space,there exists a fuzzy regular closed set μin (X, T) such that $\mu \leq int\ (\lambda)$. Now$int\ (\lambda) \leq \lambda$, implies that $\mu \leq \lambda$, in (X, T).

Corollary3.2 : If$\lambda$is a fuzzy set defined on X in a fuzzy quasi-regular space (X, T), then there exists a fuzzy regular open set δin(X, T) such that$cl\ (\lambda) \leq \delta$.

Proof: For a fuzzy set λ, $cl(\lambda)$is a fuzzy closed set in (X, T) and $1 - cl(\lambda)$is a fuzzy open set in (X, T). By Proposition3.4, there exists a fuzzy regular closed set μ in (X, T) such that $\mu \leq 1 - cl(\lambda)$. Then, $cl(\lambda) \leq 1 - \mu$, in (X, T).Let $\delta = 1 - \mu$. Thus, δ is a regular open set in (X, T)such that $cl\ (\lambda) \leq \delta$.

Proposition3.5 : Ifλ is a fuzzy somewhere dense set in a fuzzy quasi-regular space (X, T), then there exists a fuzzy regular closed set μin (X, T) such that $\mu \leq int\ cl\ (\lambda)$.

Proof: Letλbe a fuzzy somewhere dense set in (X, T). Then, $int\ cl(\lambda) \neq 0$, in (X, T). Now $int\ cl\ (\lambda)$is a open set in (X, T).Since (X, T) is a fuzzy quasi-regular space, there exists a fuzzy regular closed set μin (X, T) such that $\mu \leq int\ cl\ (\lambda)$.

Proposition 3.6 : Ifλ is a fuzzy nowhere dense set in a fuzzy quasi-regular space (X, T), then there exists a fuzzy regular open set δin (X, T) such that $\lambda \leq \delta$.

Proof: Let λ be a fuzzy nowhere dense set in (X, T). Then, $int\ cl(\lambda) = 0$, in (X, T). Now $cl(\lambda)$ is a fuzzy closed set in (X, T). Since (X, T) is a fuzzy quasi-regular space, by Proposition 3.2, there exists a fuzzy regular open set δ in (X, T) such that $cl(\lambda) \leq \delta$. Now $\lambda \leq cl(\lambda)$, implies that $\lambda \leq \delta$, in (X, T).

Proposition 3.7 : If η is a fuzzy first category set in a fuzzy quasi-regular space (X, T), then there exists a fuzzy open set δ in (X, T) such that $\eta \leq \delta$.

Proof: Let η be a fuzzy first category set in (X, T). Then, $\eta = \bigvee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X, T). Since (X, T) is a fuzzy quasi-regular space, by Proposition 3.6, there exists a fuzzy regular open set δ_i in (X, T) such that $\lambda_i \leq \delta_i$. Then $\bigvee_{i=1}^{\infty}(\lambda_i) \leq \bigvee_{i=1}^{\infty}(\delta_i)$. This implies that $\eta \leq \bigvee_{i=1}^{\infty}(\delta_i)$. Since fuzzy regular open sets are fuzzy open sets in a fuzzy topological space, $\bigvee_{i=1}^{\infty}(\delta_i)$ is a fuzzy open set in (X, T). Let $\delta = \bigvee_{i=1}^{\infty}(\delta_i)$. Hence, for the fuzzy first category set η, there exists a fuzzy open set δ in (X, T) such that $\eta \leq \delta$.

Proposition 3.8: If θ is a fuzzy residual set in a fuzzy quasi-regular space (X, T), then there exists a fuzzy closed set β in (X, T) such that $\beta \leq \theta$.

Proof: Let θ be a fuzzy residual set in (X, T). Then, $1 - \theta$ is a fuzzy first category set in (X, T). Since (X, T) is a fuzzy quasi-regular space, by Proposition 3.7, there exists a fuzzy open set δ in (X, T) such that $1 - \theta \leq \delta$. This implies that $1 - \delta \leq \theta$. Let $\beta = 1 - \delta$. Then, β is a fuzzy closed set in (X, T) such that $\beta \leq \theta$.

Proposition 3.9 : If λ is a fuzzy simply open set in a fuzzy quasi-regular space (X, T), then there exists a fuzzy regular open set δ in (X, T) such that $\lambda \wedge (1 - \lambda) \leq \delta$.

Proof : Let λ be a fuzzy simply open set in (X, T). Then, by Theorem 2.2, $\lambda \wedge (1 - \lambda)$ is a fuzzy nowhere dense set in (X, T). Since (X, T) is a fuzzy quasi-regular space, by Proposition 3.6, there exists a fuzzy regular open set δ in (X, T) such that $\lambda \wedge (1 - \lambda) \leq \delta$.

Proposition 3.10 : If λ is a fuzzy residual set in a fuzzy quasi-regular space (X, T), then there exists a fuzzy G_δ-set μ and a fuzzy closed set θ in (X, T) such that $\theta \leq \mu \leq \lambda$.

Proof : Let λ be a fuzzy residual set in (X, T). Then, by Theorem 2.3, there exists a fuzzy G_δ-set μ in (X, T) such that $\mu \leq \lambda$. Since (X, T) is a fuzzy quasi-regular space, for the fuzzy G_δ-set μ by Proposition 3.3, there exists a fuzzy closed set θ in (X, T) such that $\theta \leq \mu$. Then, it follows that $\theta \leq \mu \leq \lambda$.

Corollary 3.3 : If η is a fuzzy first category set in a fuzzy quasi-regular space (X, T), then there exists a fuzzy open set α and a fuzzy G_σ-set β in (X, T) such that $\eta \leq \beta \leq \alpha$.

Proof : Let η be a fuzzy first category set in (X, T). Then, $1 - \eta$ is a fuzzy residual set in (X, T). Since (X, T) is a fuzzy quasi-regular space, by Proposition 3.10, there exists a fuzzy G_δ-set μ and a fuzzy closed set θ in (X, T) such that $\theta \leq \mu \leq 1 - \eta$. This implies that $1 - \theta \geq 1 - \mu \geq 1 - [1 - \eta]$. Let $\alpha = 1 - \theta$ and $\beta = 1 - \mu$. Then, α is a fuzzy open set and β is a fuzzy F_σ-set in (X, T) and $\eta \leq \beta \leq \alpha$, in (X, T).

Proposition 3.11 : If μ is a fuzzy σ-boundary set in a fuzzy quasi-regular space (X, T), then there exists a fuzzy open set γ in (X, T) such that $\mu \leq \gamma$.

Proof : Let μ be a fuzzy σ-boundary set in (X, T). Then, by Theorem 2.4, μ is a fuzzy F_σ-set in (X, T). Since (X, T) is a fuzzy quasi-regular space, by Corollary 3.1, there exists a fuzzy open set γ in (X, T) such that $\mu \leq \gamma$.

Proposition 3.12 : If μ is a fuzzy σ-boundary set in a fuzzy quasi-regular space (X, T), then there exists a fuzzy regular closed set η in (X, T) such that $cl(\mu) \leq \eta$.

Proof : Let μ be a fuzzy σ-boundary set in (X, T). Then, by Proposition 3.11**,** there exists a fuzzy open set γ in (X, T) such that $\mu \leq \gamma$. This implies that $cl(\mu) \leq cl(\gamma)$. By Theorem 2.1, $cl(\gamma)$ is a fuzzy regular closed set in (X, T). Let $\eta = cl(\gamma)$. Thus, for the fuzzy σ-boundary set μ, there exists a fuzzy regular closed set η in (X, T) such that $cl(\mu) \leq \eta$.

Corollary 3.4 : If μ is a fuzzy σ-boundary set in a fuzzy quasi-regular space (X, T), then there exists a fuzzy closed set η in (X, T) such that $\mu \leq \eta$.

Corollary 3.5 : If μ is a fuzzy σ-boundary set in a fuzzy quasi-regular space (X, T), then there exist fuzzy regular closed sets α and η in (X, T) such that $\alpha \leq \mu \leq \eta$.

Proof : Let μ be a fuzzy σ-boundary set in (X, T). Then, by Proposition 3.12**,** there exists a fuzzy regular closed set η in (X, T) such that $cl(\mu) \leq \eta$. Now $\mu \leq cl(\mu)$, in (X, T). Since (X, T) is a fuzzy quasi-regular space, by Proposition **3.4,** for the fuzzy set μ on X, there exists a fuzzy regular closed set α in (X, T) such that $\alpha \leq \mu$.

Thus, for the fuzzy σ-boundary set μ, there exist fuzzy regular closed sets α and η in (X, T) such that $\alpha \leq \mu \leq \eta$.

Proposition3.13 : If θ is a fuzzy co-σ-boundary set in a fuzzy quasi-regular space(X, T), then there exists a fuzzy regular open set δ in (X, T) such that $\delta \leq int\ (\theta)$.
Proof : Letθbe a fuzzy co-σ- boundary set in (X, T). Then, by Theorem 2.5, $1 - \theta$ is a fuzzy σ-boundary setin (X, T). Since (X, T) is a fuzzy quasi-regular space, by Proposition 3.12,there exists a fuzzy regular closed set ηin (X, T)such that $cl(1 - \theta) \leq \eta$. By Lemma 2.1,$cl\ (1 - \theta) = 1 - int(\theta)$, in (X, T). Then, $1 - int\ (\theta) \leq \eta$ and $1 - \eta \leq int(\theta)$. Let $\delta = 1 - \eta$. Hence δ is a fuzzy regular open set in (X, T)such that $\delta \leq int(\theta)$.
Corollary 3.6 : If θ is a fuzzy co-σ-boundary set in a fuzzy quasi-regular space(X, T), then there exists a fuzzy open set δ in (X, T)such that $\delta \leq \theta$.

Proposition3.14 : Ifλ is a fuzzy open set in a fuzzy quasi-regular space(X, T), then there exists a fuzzy regular open set δ in (X, T)such that $\delta \leq \lambda$.
Proof : Letλ be a fuzzy open set in (X, T). Since (X, T) is a fuzzy quasi-regular space, for the fuzzy open setλin (X, T), there exists a fuzzy regular closed set μ in (X, T) such that$\mu \leq \lambda$. Then, $int(\mu) \leq int(\lambda) = \lambda$. Since fuzzy regular closed sets are fuzzy closed sets in a fuzzy topological space, μ is a fuzzy closed set in (X, T). By Theorem2.1, $int(\mu)$ is a fuzzy regular open set in (X, T). Let $\delta = int(\mu)$. Thus, for the fuzzy open set λin(X, T), there exists a fuzzy regular open set δ in (X, T)such that $\delta \leq \lambda$.
Corollary 3.7 : Ifλ is a fuzzy open set in a fuzzy quasi-regular space(X, T), then there exists a fuzzy regular open set δ in (X, T) anda fuzzy regular closed setαin (X, T) such that $\alpha \leq \delta \leq \lambda$.
Proof : Letλ be a fuzzy open set in (X, T). Since (X, T) is a fuzzy quasi-regular space, for the fuzzy open setλ in (X, T), by Proposition3.14, there exists a fuzzy regular open set δ in (X, T) such that $\delta \leq \lambda$. By Proposition **3.4,**for the fuzzy set δon X, there exists a fuzzy regular closed set α in (X, T) such that$\alpha \leq \delta \leq \lambda$.

Proposition 3.15: Ifλ is a fuzzy open set in a fuzzy quasi-regular space(X, T), then there exists a fuzzysomewhere dense set δin (X, T)such that $\delta \leq \lambda$.
Proof : Letλ be a fuzzy open set in (X, T). Since (X, T) is a fuzzy quasi-regular space, for the fuzzy open setλ in (X, T), by Proposition 3.14, there exists a fuzzy regular open set δ in (X, T) such that $\delta \leq \lambda$. Now $int\ cl(\delta) = \delta$, implies that $int\ cl(\delta) \neq 0$and thus δ is a fuzzy somewhere dense set in (X, T).

Proposition 3.16 : Ifλ is a fuzzy open set in a fuzzy quasi-regular space(X, T), then there exists a fuzzysomewhere dense set δand a fuzzy regular closed set αin (X, T) such that$\alpha \leq \delta \leq \lambda$.
Proof : Letλ be a fuzzy open set in (X, T). Since (X, T) is a fuzzy quasi-regular space, for the fuzzy open set λ in (X, T), by Proposition 3.15, there exists a fuzzy somewhere dense set δin (X, T) such that $\delta \leq \lambda$.ByProposition**3.4,** for the fuzzy set δon X, there exists a fuzzy regular closed set αin (X, T) such that$\alpha \leq \delta \leq \lambda$.

Proposition3.17 : If μ is a fuzzy F_σ-set in a fuzzy quasi-regular space (X, T), then there exists a fuzzy open set γand a fuzzy regular closed set αin (X, T) such that $\alpha \leq \mu \leq \gamma$.
Proof : Letμ be a fuzzy F_σ-set in (X, T). Since (X, T) is a fuzzy quasi-regular space, by Corollary3.1,there exists a fuzzy open set γ in (X, T) such that $\mu \leq \gamma$. By Proposition**3.4,** for the fuzzy set μon X, there exists a fuzzy regular closed set α in (X, T) such that$\alpha \leq \mu \leq \gamma$.
Corollary3.8 : If$int(\mu) = 0$, for a fuzzy F_σ-set μ in a fuzzy quasi-regular space (X, T), then 0_X is the fuzzy regular closed set in (X, T)such that $0_X \leq \mu$.
Proof : Letμ be a fuzzy F_σ-set in (X, T). Since(X, T) is a fuzzy quasi-regular space, by Proposition **3.17,** thereexists a fuzzy open set γ and a fuzzy regular closed set αin (X, T) such that $\alpha \leq \mu \leq \gamma$.If $int(\mu) = 0$, then $int(\alpha) = 0$ and this will imply [from $cl\ int(\alpha) = \alpha$] that $cl\ (0) = \alpha$ and then $\alpha = 0$, in (X, T) and 0_X is the fuzzy regular closed set in (X, T) such that $0_X \leq \mu$.

IV. Fuzzy quasi-regular spaces and other fuzzy Topologicalspaces

Proposition 4.1 : If a fuzzy topological space $(X, T)$is afuzzy regular space,then $(X, T)$is a fuzzy quasi-regular space.
Proof: Letλbe a fuzzy open set in (X, T). Since (X, T) is a fuzzy regular space, for the fuzzy open set λin(X, T), $\lambda = \vee_\alpha (\lambda_\alpha)$, where $cl(\lambda_\alpha) \leq \lambda$and $\lambda_\alpha \in T$.By Theorem 2.1, $cl\ (\lambda_\alpha)$ is a fuzzy regular closed set in(X, T). Thus, for the fuzzy open set λin (X, T), there exists a fuzzy regular closed set $cl(\lambda_\alpha)$in (X, T) such that$cl(\lambda_\alpha) \leq \lambda$, implies that (X, T) is a fuzzy quasi-regular space.

Remark : Theconverse of the above Proposition need not be true. That is, a fuzzy quasi-regular space need not be a fuzzy regular space. For, consider the following example **:**

Example 4.1 : Let $X = \{ a, b, c\}$ and $I = [0, 1]$. The fuzzy sets α, β andγ are defined on X as follows **:**

$\alpha : X \to I$isdefined by $\alpha(a) = 0.4$; $\alpha (b) = 0.6$; $\alpha (c) = 0.4$,
$B : X \to I$is defined by $\beta(a) = 0.6$; $\beta (b) = 0.4$; $\beta (c) = 0.6$,
$\gamma : X \to I$isdefined by $\gamma(a) = 0.4$; $\gamma (b) = 0.5$; $\gamma (c) = 0.5$.

Then,$T = \{ 0, \alpha , \beta, \gamma, \alpha\vee\beta, \alpha\vee\gamma, \beta\vee\gamma, \alpha\wedge\beta, \alpha\wedge\gamma , \beta\wedge\gamma , 1\}$ is a fuzzy topology on X. By computation, one can find that

$cl(\alpha) = 1 - \beta$; $int (1 - \alpha) = \beta$;
$cl (\beta) = 1 - \alpha$; $int (1 - \beta) = \alpha$;
$cl (\gamma) = 1 - \gamma$; $int (1 - \gamma) = \gamma$;
$cl (\alpha\vee\beta) = 1 - [\alpha\wedge \beta]$; $int (1 - [\alpha\vee\beta]) = \alpha\wedge \beta$;
$cl (\alpha \vee\gamma) = 1 - [\beta\wedge\gamma]$; $int (1 - [\alpha \vee\gamma]) = \beta\wedge\gamma$;
$cl (\beta\vee\gamma) = 1 - (\alpha \wedge\gamma)$; $int (1 - [\beta\vee\gamma]) = \alpha \wedge\gamma$;
$cl (\alpha\wedge\beta) = 1 - [\alpha\vee \beta]$; $int (1 - [\alpha\wedge\beta]) = \alpha\vee \beta$;
$cl (\alpha \wedge\gamma) = 1 - [\beta\vee\gamma]$; $int (1 - [\alpha \wedge\gamma]) = \beta\vee\gamma$;
$cl (\beta\wedge\gamma) = 1 - (\alpha\vee \gamma)$. $int (1 - [\beta\wedge\gamma]) = \alpha \vee\gamma$.

By computation one can find that the fuzzy regularclosed setsin (X,T) are $1 - \alpha , 1 - \beta , 1 - \gamma$, $1 - [\alpha \vee \beta]$, $1 - (\alpha\vee\gamma)$, $1 - [\beta\vee\gamma]$, $1 - [\alpha\wedge \beta]$, $1 - [\beta \wedge \gamma]and1 - (\alpha \wedge\gamma)$.Also $1 - \beta \leq \alpha$; $1 - \alpha \leq \beta$; $1 - [\beta\vee\gamma] \leq \gamma$; $1 - [\alpha \wedge\beta] \leq \alpha\vee\beta$; $1 - \beta \leq \alpha\vee\gamma$; $1 - (\alpha \wedge\gamma) \leq \beta\vee\gamma$; $1 - [\alpha \vee \beta] \leq \alpha \wedge\beta$; $1 - [\beta \vee\gamma] \leq \alpha \wedge\gamma$ and $1 - [\alpha \vee \beta] \leq \beta \wedge \gamma$. Hence$(X,T)$is a fuzzy quasi-regular space.Now, for thefuzzy open set αin (X,T), $\alpha = (\alpha\wedge \beta)\vee(\alpha \wedge\gamma)\vee(\alpha)$,Where $cl(\alpha \wedge\beta) = 1 - [\alpha \vee \beta] \leq \alpha$; $cl (\alpha \wedge\gamma) = 1 - [\beta \vee\gamma] \leq \alpha$ and $cl(\alpha) = 1 - \beta \leq \alpha$.For the fuzzy open set$\gamma$in (X,T), $cl(\alpha \wedge\beta) = 1 - [\alpha \vee \beta] \leq \gamma$ and$cl (\alpha \wedge\gamma) = 1 - [\beta\vee\gamma] \leq \gamma$. But $\gamma \neq (\alpha \wedge\beta)\vee(\alpha \wedge\gamma)$, in (X,T).Hence (X,T) is not a fuzzyregular space.

The following Propositions give conditions under which fuzzy quasi-regular spaces become fuzzy Baire spaces.

Proposition 4.2: If $int (\beta) = 0$, for each fuzzy F_σ-set β in a fuzzy quasi-regular space(X,T), then (X,T)isa fuzzyBaire space.

Proof : Letλ be a fuzzy first category set in (X,T).Since(X,T) is a fuzzy quasi-regular space, by Corollary3.3, there exists a fuzzy open setα and a fuzzy F_σ-setβ in(X,T)such that $\lambda \leq \beta \leq \alpha$. Then, $int(\lambda) \leq int (\beta)$, in (X,T).By hypothesis, $int (\beta) = 0$and this implies that $int (\lambda) = 0$, in (X,T). Then, byTheorem2.6,(X,T) isa fuzzyBaire space.

Proposition4.3: If each fuzzy G_δ-set is a fuzzy dense set in a fuzzy quasi-regular space (X,T), then$(X,T)$is a fuzzyBairespace.

Proof : Letλ be a fuzzy first category set in (X,T).Since $(X,T)$is a fuzzy quasi-regular space, by Corollary 3.3, there exists a fuzzy open set α and a fuzzy F_σ-set β in (X,T) such that $\lambda \leq \beta \leq \alpha$. Then, $int (\lambda) \leq int (\beta)$, in (X,T). Now β isa fuzzy F_σ-setin (X,T), implies that $1 - \beta$is a fuzzy G_δ-set in (X,T). By hypothesis, $cl (1 - \beta) = 1$, in (X,T). ByLemma 2.1, $1 - int (\beta) = 1$ and$int (\beta) = 0$.This implies that$int (\lambda) = 0$, in (X,T). Then, by Theorem2.6, $(X,T)$is a fuzzyBairespace.

The following Propositions give conditions under which fuzzyquasi-regular spaces become fuzzy weaklyBairespaces.

Proposition 4.4 : If each fuzzy closed set is a fuzzy nowhere dense set in a fuzzy quasi-regular space(X,T), then (X,T) is a fuzzy weaklyBairespace.

Proof : Letλbe a fuzzy σ-boundary set in (X,T). Since (X,T) is a fuzzy quasi-regular space, by Corollary 3.4, then there exists a fuzzy closed set η in (X,T)such that$\lambda \leq \eta$. Then, $int(\lambda) \leq int(\eta)$. By hypothesis,thefuzzy closed set η is a fuzzy nowhere densesetin (X,T) and then $int\, cl(\eta) = 0$. Now $int(\eta) \leq int\, cl(\eta)$, implies that $int(\eta) = 0$, in (X,T). This implies that $int (\lambda) = 0$.Thus, for a fuzzy σ-boundary set λ, $int (\lambda) = 0$, in (X,T).Then, by Theorem 2.7, $(X,T)$is a fuzzy weakly Bairespace.

Corollary4.1: If $int (\lambda) = 0$, for each fuzzy closed set λin a fuzzy quasi-regular space (X,T), then (X,T) isa fuzzyweaklyBaire space.

Proposition 4.5 : If each fuzzy closed set is a fuzzy nowhere dense set in a fuzzy quasi-regular and fuzzy open hereditarily irresolvable space (X, T), then (X, T) is a fuzzy Bairespace.
Proof : The proof followsfrom Proposition **4.4** and Theorem **2.8.**
Proposition4.6 : If a fuzzy topological space(X, T) is a fuzzy hyperconnected, fuzzy open hereditarily irresolvable and fuzzy quasi-regular space,then (X, T) is a fuzzy weaklyBairespace.
Proof : Letλbe a fuzzy closedset in (X, T). Then,$1 - \lambda$ is a fuzzy open set in (X, T). Since (X, T) is a fuzzyhyperconnected space, $1 - \lambda$ is a fuzzy dense set in (X, T) and $cl(1 - \lambda) = 1$andby Lemma 2.1, $1 - int(\lambda) = 1$ and thus$int(\lambda) = 0$, in (X, T). Since $\lambda$is fuzzy closedset in (X, T),$int\, cl(\lambda) = 0$, in $(X, T)$and thus $\lambda$is a fuzzy nowhere dense set in (X, T).Thus, the fuzzy closedset $\lambda$is a fuzzy nowhere dense set in the fuzzy quasi-regular space(X, T). Hence, by Proposition 4.4, $(X, T)$is a fuzzy weaklyBairespace.

Remark : Theconverse of the above proposition need not be true. That is, a fuzzy weaklyBairespace need not be a fuzzy quasi-regular space and a fuzzy hyperconnected space. For, consider thefollowing example**:**
Example 4.2: Let μ_1, μ_2 and μ_3 befuzzy sets of$I = [0,1]$ defined as follows**:**

$$\mu_1(x) = \begin{cases} 0, & 0 \leq x \leq \frac{1}{2}; \\ 2x - 1, & \frac{1}{2} \leq x \leq 1. \end{cases}$$

$$\mu_2(x) = \begin{cases} 1, & 0 \leq x \leq \frac{1}{4}; \\ -4x + 2, & \frac{1}{4} \leq x \leq \frac{1}{2}; \\ 0, & \frac{1}{2} \leq x \leq 1. \end{cases}$$

$$\mu_3(x) = \begin{cases} 0, & 0 \leq x \leq \frac{1}{4}; \\ \frac{1}{3}(4x - 1), & \frac{1}{4} \leq x \leq 1. \end{cases}$$

Clearly $T = \{ 0, \mu_1, \mu_2, \mu_1 \vee \mu_2, 1 \}$ is a fuzzy topology on I.By computation it follows that$l\,(\mu_1) = 1 - \mu_2, cl(\mu_2) = 1 - \mu_1$, $cl\,(\mu_1 \vee \mu_2) = 1$; $int(1 - \mu_1) = \mu_2$, $int(1 - \mu_2) = \mu_1$,$int\,(1 - [\mu_1 \vee \mu_2) = 0$, $cl(\mu_3) = 1 - \mu_2$; $int\,(\mu_3) = \mu_1$; $cl(1 - \mu_3) = 1 - \mu_1$; $int\,(1 - \mu_3) = \mu_2$. Now $int\, cl\,(\mu_1) = int(1 - \mu_2) = \mu_1$; $int\, cl\,(\mu_2) = int\,(1 - \mu_1) = \mu_2$; $int\, cl\,(\mu_1 \vee \mu_2) = 1$; $nt\, cl\,(\mu_3) = int(1 - \mu_2) = \mu_1$; $int\, cl(1 - \mu_3) = int(1 - \mu_1) = \mu_2$. Then, μ_1 and μ_2 are fuzzy regular open sets and thus $1 - \mu_1$and $1 - \mu_2$are fuzzy regular closed sets in(I, T).Now $\delta_1 = \text{cl}\,(\mu_1) \wedge (1-\mu_1) = (1-\mu_2) \wedge (1-\mu_1)$,
$\delta_2 = \text{cl}\,(\mu_2) \wedge (1-\mu_2) = (1-\mu_2) \wedge (1-\mu_1)$ .Then,$\delta = \delta_1 \vee \delta_2$, is a fuzzy σ-boundary set in(I, T) and $int\,(\delta) = int[(1 - \mu_2) \wedge (1 - \mu_1)] = int\,[1 - (\mu_1 \vee \mu_{2]})] = 1 - cl\,(\mu_1 \vee \mu_{2]}) = 1 - 1 = 0$.Hence$(I, T)$ is a fuzzy weaklyBaire space.
Now,for the fuzzy open sets μ_1 , μ_2 and $\mu_1 \vee \mu_2$, $1-\mu_1 \nleq \mu_1$; $1 - \mu_2 \nleq \mu_1$;$1-\mu_1 \nleq \mu_2$; $1 - \mu_2 \nleq \mu_2$; $1-\mu_1 \nleq \mu_1 \vee \mu_2$; $1 - \mu_2 \nleq \mu_1 \vee \mu_2$. This implies that$(I, T)$ is not a fuzzy quasi-regular space. Also, for the fuzzy open set μ_1,cl $(\mu_1) = 1-\mu_2 \neq 1$ implies that (I, T) is not a fuzzy hyperconnected space.

Proposition 4.7 : If λ is a fuzzy closed set with $int(\lambda) = 0$, in a fuzzy fuzzy quasi-regular space (X, T), then λ isa fuzzy resolvableset in the fuzzy weakly Bairespace(X, T).
Proof : The proof follows from Corollary 4.1 and Theorem2.10.

Proposition 4.8: If λ is a fuzzysetdefined on Xin a fuzzy quasi-regularspace (X, T) in which each fuzzy closed set is a fuzzy nowhere dense set, then $int\,(\lambda) \wedge int\,(1 - \lambda) = 0$, in (X, T).
Proof : Let λ be a fuzzy set defined on X in (X, T).By hypothesis, each fuzzy closed set is a fuzzy nowhere dense set in the fuzzy quasi-regular space$(X, T)$and then by Proposition 4.4,(X, T) is a fuzzy weaklyBairespace. ByTheorem2.11, for the fuzzy set λin (X, T),$int\,(\lambda) \wedge int\,(1 - \lambda) = 0$, in$(X, T)$.

Proposition4.9 : If a fuzzy topological space (X, T) is a fuzzy quasi-regular space, then(X, T) is not a fuzzyhyperconnectedspace.

Proof : Let λ be a fuzzy open set in (X,T). Since (X,T) is a fuzzy quasi-regular space, for the fuzzy open set λ in (X,T), by Proposition 3.14, there exists a fuzzyregular open set δ in (X,T) such that $\delta \leq \lambda$. Then, by Theorem 2.12, (X,T) is not a fuzzy hyperconnected space.

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