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Effect Of Preservation In Price Sensitive Inventory For Small Organizations

Dr Subhash Chandra Das
Chandrapur College, Chandrapur, Purba Bardhaman, West Bengal, India-713145

Abstract
The aim of work is to focus on preservation facility. Preservation facility can gain profit for business organisations. Selling price dependent demand and time varying deterioration item is considered in the inventory model. To diminish deterioration rate of the item preservation technology is applied and solved the inventory model. By the consideration of three-parameter Weibull distributed deterioration the problem becomes highly non-linear. Partial backlogging is considered with a rate depending on waiting time of customers. Solutions are made by numerical process applying Gaussian Quantum-behaved Particle Swarm Optimization technique. After obtaining the results, statistical analyses of the results are performed and a fruitful decision arrived.
Key Word: *Inventory; Preservation Technology; Deterioration; Partial Backlogging; Price Sensitive; PSO.*

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I. Introduction

In the retailer business selling price of an item is important to attract customers. Generally, small business organisations are not interested in preservation facility. However, preservation facility is very important in business model. At any time item can be ready to sale, waiting for right time to show the stock and freshness of the product remains longer are some basic facilities of preservation. In this work three-parameter Weibull distributed deterioration inventory model is considered with selling price dependent demand.

Mishra et al. (2017) considered price and stock dependent demand in an inventory model under preservation. Das et al. (2020) investigates price sensitive inventory model under preservation with three parameter Weibull distributed deterioration. Suryani et al. (2021) investigates inventory management model for tolerance in economic order quantity. Duary et al. (2021) considered Weibull deterioration in an inventory model under variable demand. Priyamvada et al. (2021) considered partial backlogging with preservation technology in an inventory model under price and stock dependent demand. Price sensitive inventory model investigates by Singh et al. (2022) under preservation. Tripathi and Sharma (2022) solved an inventory problem with non-instantaneous deterioration under trade credit. Ibina et al. (2023) considered three-parameter Weibull deterioration supply chain inventory model under inventory dependent demand. Mondal et al. (2023) investigates an inventory model with pricing strategies under partial backlogging. Pakhira et al. (2023) considered an inventory model with partial backlogging under memory effect. Preservation strategies considered in an inventory model by Jain and Singh (2024) under deterioration. Patra et al. (2024) considered an inventory model under preservation and power demand.

In this work, selling price dependent demand and waiting time dependent partial backlogging are considered. Three-parameter Weibull distributed non-instantaneous deterioration is considered and a new preservation technology function is applied. Direct market values are collected and formulated an inventory model. Then solved the optimization problem by Gaussian Quantum behaved Particle Swarm Optimization Technique and results are statistically analysed.

II. Model formulation

Notation
The notation used for the proposed model is given below

t_0	Deterioration starting time (in month)
t_1	Zero stock level time (in month) [a decision variable]
T	Cycle length (in month) [a decision variable]
ξ	Preservation cost/item/unit time (in \$)

$D(\cdot)$	Demand of the item
c_p	Purchase cost/item (in \$)
α	Demand controlling parameter
β	Selling price controlling demand parameter
h	Holding cost/item (in \$)
s	Shortage cost/item (in \$)
R	Replenishment cost/replenishment (in \$)
$I(t)$	Inventory level at any time t
a	Deterioration shape parameter
b	Deterioration scale parameter
$\theta(t)$	Deterioration rate at any time t
$\mu(\xi)$	Preservation technology function
c	Preservation cost controlling parameter
η	Partial backlog controlling parameter
Q	Initial order quantity
B	Maximum backlogged item
$Av\Pr of$	Average profit function

Assumption

The following assumptions are considered to developed the proposed production inventory model

i) The inventory model deals with single item and infinite time horizon.

ii) Demand of the item depending on promotional activity and the form is as $D(\cdot)=\alpha-\beta p$, where α and β are constants.

iii) Three parameter Weibull distributed deterioration $\theta(t)$ is considered and the form is as $\theta(t)=ab(t-t_0)^{b-1}$, where a and b are shape parameter and scale parameter of the deterioration rate. Repair or replenishment is not allowed to deteriorated items.

iv) Preservation technology is applied to reduce the deterioration effect which is of the form $\mu(\xi)=\frac{1}{1+c\xi}$, where c is preservation cost controlling parameter. Clearly, $\mu(\xi)$ is decreasing function of ξ and $\mu(\xi)\to 0$ as $\xi\to\infty$.

v) Partially backlogged shortages are allowed and which depends on the waiting time of the customers. The rate of partial backlog is $\frac{1}{1+\eta(T-t)}$.

vi) Lead time negligible with infinite replenishment.

Model description

At the initial time at $t=0$ receives $(Q+B)$ units of items of which B units fulfil the backlogged items immediately of the previous cycle. After fulfilment of backlog items, Q items are satisfied the customers' demand. Clearly inventory level decreases due to customers' demand. After time $t=t_0$ deterioration comes into effect and customers demand will decreases the inventory level. At time $t=t_1$ inventory level reaches to zero then backlog occurs and attains maximum backlog B at time $t=T$. Then the inventory model repeats and so on. The governing differential equation for the model is as

$$\frac{dI(t)}{dt}=-(\alpha-\beta p), \qquad 0\le t\le t_0 \tag{1}$$

$$\frac{dI(t)}{dt}+\mu(\xi)\theta(t)I(t)=-(\alpha-\beta p), \qquad t_0<t\le t_1 \qquad (2)$$

$$\frac{dI(t)}{dt}=-\frac{\alpha-\beta p}{1+\eta(T-t)}, \qquad t_1<t\le T \qquad (3)$$

with the boundary conditions $I(0)=Q$, $I(t_1)=0$ and $I(T)=-B$.

Solving equation (1) and using $I(0)=Q$ can reduced to

$$I(t)=Q-(\alpha-\beta p)t, \qquad 0\le t\le t_0 \qquad (4)$$

Now, solving equation (2) and using $I(t_1)=0$ can be reduced to

$$I(t)=(\alpha-\beta p)e^{-\mu(\xi)a(t-t_0)^b}\int_t^{t_1}e^{\mu(\xi)a(u-t_0)^b}du, \qquad t_0<t\le t_1 \qquad (5)$$

Since, $I(t)$ is continuous at $t=t_0$ and continuity gives

$$Q=(\alpha-\beta p)\left[t_0+\int_{t_0}^{t_1}e^{\mu(\xi)a(t-t_0)^b}dt\right] \qquad (6)$$

Solving equation (3) and using $I(T)=-B$ one can get

$$I(t)=\frac{(\alpha-\beta p)}{\eta}\log\left|1+\eta(T-t)\right|-B, \qquad t_1<t\le T \qquad (7)$$

In equation (7) replacing $t=t_1$ one can get

$$B=\frac{(\alpha-\beta p)}{\eta}\log\left|1+\eta(T-t_1)\right| \qquad (8)$$

Now, different inventory related costs are as

Sales revenue (*SR*)= $\int_0^{t_1}p(\alpha-\beta p)dt+pB$

i.e., $SR=p(\alpha-\beta p)t_1+pB$

Holding cost (*HC*)= $h\int_0^{t_1}I(t)dt$

HC= $hQt_0-h(\alpha-\beta p)\frac{t_0^2}{2}+h(\alpha-\beta p)\int_{t_0}^{t_1}e^{-\mu(\xi)a(t-t_0)^b}\int_t^{t_1}e^{\mu(\xi)a(u-t_0)^b}dudt$

Shortage cost (*SC*)= $s\int_{t_1}^{T}-I(t)dt$

SC= $\frac{s}{\eta}\left\{(\alpha-\beta p)(T-t_1)-B\right\}$

Purchase cost (*PC*)= $c_p(Q+R)$

Preservation cost (*PRC*)= $Q\xi t_1$

Replenishment cost (*RC*)= R

Then, the objective function, i.e, the average cost is given by

$$Av\Pr of=\frac{1}{T}\left[SR-PC-SC-PRC-RC-HC\right]$$

So,

$$Av\Pr of\left(p,t_1,T\right)=\frac{1}{T}\left[p\left(\alpha-\beta p\right)t_1+pB-c_p\left(Q+R\right)-\frac{s}{\eta}\left\{\left(\alpha-\beta p\right)\left(T-t_1\right)-B\right\}-Q\xi t_1-R\right.$$

$$\left.-\left\{hQt_0-h\left(\alpha-\beta p\right)\frac{t_0^2}{2}+h\left(\alpha-\beta p\right)\int_{t_0}^{t_1}e^{-\mu(\xi)a\left(t-t_0\right)^b}\int_{t}^{t_1}e^{\mu(\xi)a\left(u-t_0\right)^b}dudt\right\}\right] \quad (9)$$

Now, the corresponding optimization problem is

$Maximize\ \ Av\Pr of$

subject to $0\le t_1\le T$ and $p>0$.

III. Solution Procedure

Clearly, the objective function is highly non-linear. So, to solve the optimization problem soft computing needed. Here Gaussian Quantum-Behaved Particle Swarm Optimization (GQPSO) technique is used to solve. Coelho (2010) first introduced GQPSO technique for solving optimization problem through numerical process. Sun et al. (2012) improved the Coelho (2010) GQPSO technique by choosing parameters for better solution. In GQPSO technique every solution can freely move in the search space near the previous solution. Let, $x(i)=\left(x_{i1},x_{i2},\cdots,x_{in}\right)$, $i=1,2,\cdots,M$ are M initial solution in the search space where $L_j\le x_{ij}\le U_j$, $\forall j=1,2,\cdots,n$ and L_j,U_j are lower and upper bounds of variable. Then in Particle swarm optimization (PSO) technique firstly finds optimum among the solutions and called it global best solution $x(gbest)=\left(x_{gbest,1},x_{gbest,2},\cdots,x_{gbest,n}\right)$. In GQPSO technique next iterative solution can be found by

$$x^{k+1}(i)=g(i)+\beta\left|M_{best}(i)-x(i)\right|\ln(1/w)\quad \text{if } u\ge 0.5$$

$$x^{k+1}(i)=g(i)-\beta\left|M_{best}(i)-x(i)\right|\ln(1/w)\quad \text{if } u<0.5$$

where parameter β control the expansion or contraction of new solution and w,u are parameter values uniformly distributed in $[0,1]$. Also, the $M_{best}(i)$ and g can be obtain by

$$M_{best}(i)=\frac{1}{M}\sum_{j=1}^{M}x_{ij}^k$$

$$g(i)=\frac{c_1x_{ij}+c_2x_{gbest,j}}{c_1+c_2},\ j=1,2,\cdots,n$$

where c_1 and c_2 are absolute random number generated from Gaussian probability distribution with mean zero and standard deviation one.

IV. Numerical Illustration

To validate two examples are drawn from the market one is for potato seller and another is onion seller. The parameters are sets according to best fitted to the seller. Due non-linearity of the objective functions, these are solved numerically by GQPSO technique. Each problem compiled and runs 30 times with a C++ programming and the results are collected.

Example 1: For the potato seller parameters are considered as

$c_p=\$10.0$, $h=\$2.0$, $s=\$3.0$, $R=\$200.0$, $\alpha=150$, $\beta=1.1$, $t_0=0.02$ Month, $\eta=0.1$, $a=0.8$, $b=3.5$, $c=0.4$ and $\xi=\$1.3$.

Example 2: For the onion seller parameters are considered as

$c_p=\$17.0$, $h=\$3.5$, $s=\$9.0$, $R=\$300.0$, $\alpha=250$, $\beta=2.1$, $t_0=0.01$ Month, $\eta=0.1$, $a=1.4$, $b=2.3$, $c=0.3$ and $\xi=\$1.4$.

For both the example can evaluated without expenses for preservation by setting $\xi=\$0.0$. Also, statistical analyses are performed for the four cases. Table 1 and Table 2 shows obtained results.

Table 1: Results of potato seller

		$Av\,Prof$ (in $)	t_1 (in month)	T (in month)	p (in $)	Q	B
With Preservation	**Mean**	**2067.338721**	**0.81**	**1.27**	**32.83**	**112.59**	**51.02**
	Best	**2079.233160**	0.80	1.19	32.89	116.65	54.97
	Worst	**2051.292880**	0.83	1.29	32.70	110.53	43.51
	Standard Deviation	7.870770	0.009872	0.028256	0.043512	1.686265	2.939971
Without Preservation	**Mean**	**2049.811852**	**0.81**	**1.27**	**32.82**	**125.17**	**51.51**
	Best	**2064.649582**	0.80	1.20	32.89	130.46	55.49
	Worst	**2035.398175**	0.836	1.30	32.64	122.94	43.78
	Standard Deviation	7.842496	0.009460	0.028370	0.071988	1.960703	3.011550

Table 2: Results of onion seller

		$Av\,Prof$ (in $)	t_1 (in month)	T (in month)	p (in $)	Q	B
With Preservation	**Mean**	**4369.302542**	**0.70**	**1.22**	**56.92**	**107.49**	**65.35**
	Best	**4377.396939**	0.70	1.11	56.99	109.55	75.39
	Worst	**4356.767485**	0.71	1.30	56.73	106.38	51.03
	Standard Deviation	4.831560	0.004148	0.051772	0.069131	0.824229	6.522215
Without Preservation	**Mean**	**4337.769479**	**0.70**	**1.22**	**56.92**	**115.32**	**65.70**
	Best	**4346.987912**	0.70	1.10	56.99	116.96	74.78
	Worst	**4326.353952**	0.71	1.29	56.74	114.08	51.45
	Standard Deviation	5.472243	0.003396	0.052015	0.057348	0.843018	6.316395

From Table 1 it is clear that mean average profit for preservation is greater than best average profit for without preservation. In Table 2, worst average profit for preservation is greater than best average profit for without preservation. So, preservation facility enhances the profit for sellers. Also, if preservation is applied then sellers can choose the right time sale their item as to get high selling price for the item.

V. Conclusion

This work deals with the small capital organiser to include preservation facility. For this selling price dependent demand and preservation technology function included inventory model is solved for deteriorating item with three parameter Weibull distributed deterioration rate.

Thus work can extends by considering trade credit, stock dependent demand. Also, one can extend this by considering fuzzy numbers in an inventory model.

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