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A Mathematical Model For Intra-Communal Violence A Case Study Of Uvwie Local Government Area, Delta State.

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Abstract
Intra-communal violence perpetrates across various families and factions in a community and this is strongly supported by the undeniable solidarity felt and exhibited by the violent parties for their respective groups. In this research work, we made some assumptions and constructed a deterministic model to study intra-communal violence. Basic mathematical analyses such as the positivity of solutions, the invariant region, the basic reproduction number, and the stability analysis of the model were performed. The analysis revealed that the violent-free equilibrium is globally asymptotically stable, and that the rate of injustice, the level of insecurity, the level of threat to life and property, the effective contact rate with the aggressive and brutal individuals, and the level of negligence of infrastructural development in the community by the government are positively sensitive parameters of the basic reproduction number. Version 12 of the Mathematica Programming Software was employed as a computational tool throughout the research.
Keywords*: Model, stability analysis, insecurity, threat to life, level of negligence, violent-free equilibrium, globally asymptotically stable*

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I. Introduction

Many part of the world today is been faced with one crisis or the other, which has led to loss of lives and properties. Kumar (2013) opined that the impact of communal violence is severe not only in terms of the loss of life and property but also in terms of the damage it causes to the social fabric. Dominioni, et al(2018), Tsetimi, et al(2021) revealed that ethnic violence and racism diverseness are greatly induced by globalization and migration flows within many nations in the world. Mamo (2021), Okposo, et al(2023) revealed that violence can lead to social instability, dreadful election, and large financial losses, and can be considered as an infectious disease like tuberculosis and COVID-19. In August 2011, several areas of London experienced episodes of large-scale disorder, comprising looting, rioting and violence. Davies, et al (2013), Apanapudor(2007), Aapanapudor and Aderibigbe(2018) presented a mathematical model of the spatial development of the disorder that can be used to examine the effect of varying policing arrangements. One of the capabilities of the model is simulating the general emergent patterns of the events, and focuses on three fundamental aspects: the apparently-contagious nature of participation, the distances travelled to riot locations, and the deterrent effect of policing the system, Izevbizua and Apanapudor(2019), Apanapudor, et al(2023). Their work demonstrated that the spatial configuration of London places some areas at naturally higher risk than others, highlighting the importance of spatial considerations when planning for such events. They also investigated the consequences of varying police numbers and reaction time, which has the potential to guide policy in this area.

Neelo and Absar (2021), Olaosebikan, et al(2015), Mamadu and Apanapudor(2017) revealedthat communal violence in India has become more frequent due to the unwanted interplay between religion and political manipulation, by which India has been affected many times. They further added that communal violence has also been recognized as a significant social and public health problem, resulting in long-term human and economic costs. Neelo and Absar said that the dominant form of communal violence in India has involved the two communities, i.e. Hindus and Muslims.

The case study, Uvwie is a local government in Delta State of Nigeria with different town,densely populated with different infrastructural development. Uvwie shares boundaries with Agbarho to the east, Udu to the south, Ughelli South to the south east, Okpe to the north and Warri to the west. Due to her proximity with Warri, rapid population growth and several road network linking both towns and her environs, it formed a conurbation collectively referred toas Warri by people from other parts of the state and Nigeria at large, though each towns in "Warri conurbation" are under different traditional and political authorities. Congestion in neighboring communities often lead to crisis in physical and health terms, and Uvwie is not out this scenario, Okwonu, et al(2021a), Okwonu,et al(2020). Uvwie is one of the major hubs of economic activities and businesses in Delta State. It is a subgroup of the Urhoboethnic nationality. Its inhabitants are predominantly Christians of different denominations, and some practice a mixture of African traditional religion most notably the Igbe religioncommon amongst Urhobos like most of Southern Nigeria. The local government along with Warri and environs are known nationwide for her unique Pidgin English, Lazarus (2014), Okwonu, et al(2022), Okwonu and Apapnapudor(2019), De la Poza, et al (2016). However, to the best of our knowledge, no one has developed a four-compartment model for intra-communal violence of a local government in Delta State, Nigeria.

Mathematical Model

Mathematical modeling has to do with the use of mathematical ideas and techniques to describe real-world phenomena, investigate important questions about the observed world, test ideas, and make predictions about the real world, Okwonu, et al (2023) Aderibigbe and Apanapudor(2014), Izevbizua and Apanapudor(2019) . The real world finds expressions inengineering, physics, physiology, ecology, wildlife management, chemistry, economics, sports etc. Models are used to guide decision-making, develop policies, Apanapudor, et al(2023), Okwonu, et al (2023) Izevbizua and Apanpudor (2022) or to evaluate specific strategies aimed at reducing intra-communal violence.

In mathematical modeling, mathematical processes are used to transform real-world systems into abstract systems so as to understand, simulate or make predictions about their behavior, Izevbizua and Apanapudor (2020), Izevbizua and Apanapudor (2019), Apanapudor, et al(2020). Mathematical models are useful weapons, not only in infectious disease management and eradication, but also in the effective management of violence and in crime-fighting. Mathematical modeling of violence and its analysis is becoming now popular in societies and systems, Apanapudor, et al(2023b) Apanapudor (2018), Apanapudor, et al (2023a)

Human behaviour is innately nonlinear; hence we assume that the behavioural patterns may best be described by a nonlinear system. Many research works have been conducted in the area of population growth using logistic model and others on domestic violence and its effect. The use of the logistic growth model is widely established in many fields of modeling and forecasting Banks Otoo, et al, (2014), Apanapudor, et al (2023), Ezimadu, et al (2020), Okwonu, et al (2021).Several results forecasting population growth have been obtained from researchers over time Wali, et al(2012). Olson (1999) used a general conditional logistic model to detect linkage between marker loci and common disease with samples of affected sib pairs. Mahapatra and Kant (2005) used a multinomial logistic model to deal with estimation problems and shown that the results of multinomial logistic are more informative and robust compared to the results of binary logistic model. Manjunath and Manjunath(2005), Apanapudor, et al(2023) developed an integrated logistic model using supply chain management system which clearly shows a greater acceptability of logistic model in industry. Otoo, et al (2014), Izevbizua and Apanapudor (2020) used a modelling technique of abusive, susceptible and violence victims, similar to the susceptible, infectious and recovered model in epidemics, for the formulation of the spread of domestic violence as a system of differential equations. They used data from Domestic Violence and Victims Support Unit (DOVVSU) in Tamale and analysed with MATLAB software. The study revealed that the population of Domestic Violence Victims is limited.

Mathematical modeling is very essential in understanding dynamical systems, and many researchers have applied infectious disease model, Okposo, et al (2023) to violence, racism, social media addition, corruption, and other social situations (Delgadillo-Aleman and Ku-Carrillo(2019); Lazaru(2014 dynamics); De la Poza, et al(2016). However, to the best of our knowledge, no one has developed a four-compartment model for intra-communal violence and carried out perception analysis on violent-risk level of a local community in Delta State, Nigeria.

II. Formulation Of The Mathematical Model

The model formation is based on the following assumptions:

i. The considered population is uniformly mixed so that every peaceful resident is equally susceptible to violence.
ii. Violence outbreak occurs over a short time so that birth and natural death rates are ignored.
iii. Additionally, death within the population is only violence-induced
iv. Aggressive and brutal residents become peaceful or irascible at the same rate
v. Brutal residents can at best become irascible but not peaceful.

The residents of the community are divided into the following four mutually exclusive classes: the Peaceful class (P), the Irascible class (I), the Aggressive class (A) and the Brutal class (B). Peaceful individuals get infected with violence and exhibit violent tendencies due to effective interaction with the class of violent residents through the force of infection $\chi = \kappa\beta\varphi\omega\xi\left(\frac{I+\varrho A+\gamma B}{N}\right)$. As a result,a fraction σ of peaceful residents become irascible at the rate $\sigma\chi$ while the remaining fraction$(1-\sigma)$ become aggressive at the rate $(1-\sigma)\chi$. Irascible residents who have exceeded their anger-control limit join the aggressive class at the rate ζdue to intimidation or negative aggressive behavior from aggressive and/or brutal residents. Aggressive residents become brutal at the rate α. As a result of peaceful dialogue and other positive interference, especially from high meaning individuals and government, aggressive individuals become peaceful or irascible at the rates $\tau\delta$ or $(1-\tau)\delta$ respectively, while brutal individuals only become irascible at the rate δ. It is assumed that violence-induced death occurs in all classes at the rate η. Based on the above discussion, the proposed mathematical model is given as follows.

$$\begin{cases} \frac{dP}{dt} = \tau\delta A - (\chi + \eta)P \\ \frac{dI}{dt} = \sigma\chi P + \delta B + (1-\tau)\delta A - (\zeta + \eta)I \\ \frac{dA}{dt} = (1-\sigma)\chi P + \zeta I - (\alpha + \delta + \eta)A \\ \frac{dB}{dt} = \alpha A - (\eta + \delta)B \end{cases} \tag{2.1}$$

Subject to the initial conditions
$P(o) = P_o$, $I(o) = I_o$, $A(o) = A_o$, $B(o) = B_o$

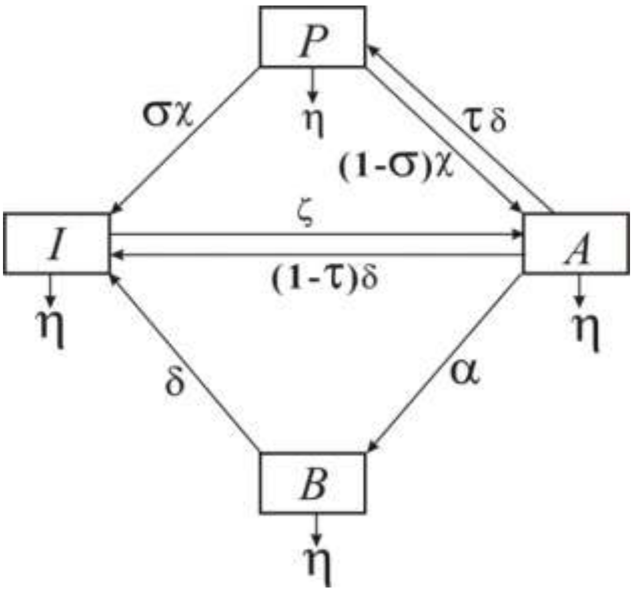

Figure 2.1: Schematic diagram of the Model

The variables and parameters used in this non-linear deterministic model are clearly presented as follows.

Table 2.1. Variable and Parameter Description

Variable/ Parameter	Description
P	The class of peaceful residents
I	The class of irascible residents
A	The class of aggressive residents
B	The class of brutal residents
κ	Effective contact rate with irascible, aggressive and brutal residents
ϱ	Infection coefficient of the aggressive class
γ	Infection coefficient of the brutal class
β	Rate of injustice
φ	Level of insecurity on a scale of $0-1$
ω	Level of threat to life and property on a scale of $0-1$
ξ	Level of negligence of infrastructural development in the community by the government
ζ	Rate at which irascible residents become aggressive
α	Rate at which aggressive residents become brutal
δ	Rate at which aggressive and brutal residents become peaceful or irascible

σ	Proportion of peaceful individuals who become irascible
τ	Proportion of aggressive residents who become peaceful
η	Violent induced death rate

III. Model Analysis

Non-negativity of solution

Theorem 1 (Positivity of Solution):
Suppose $\Gamma = \{(P, I, A, B) \in \mathbb{R}^4 : P(0) > 0, I(0) > 0, A(0) > 0, B(0) > 0\}$, then the solution set $\{P, I, A, B\}$is positive for all$t \geq 0$.

Proof: Observe that from the first equation, in equation (2.1), we obtain

$$\frac{dP}{dt} = \tau\delta A - (\chi + \eta)P,$$

we obtain

$$\frac{dP(t)}{dt} \geq -(\chi + \eta)P.$$

Suppose $P(0) = P_0 \geq 0$, we obtain the solution

$$P(t) \geq P_0 e^{-(\chi+\eta)t} \geq 0$$

Similarly, $I(t) \geq 0, A(t) \geq 0, B(t) \geq 0 \;\; \forall\, t \geq 0$.
This completes the proof.

Theorem 2 (Invariant Region): The set

$$\Gamma = \{(P, I, A, B) \in \mathbb{R}_+^4 : 0 \leq P + I + A + B = N \leq \Lambda/\mu\}$$

is positively-invariant for the model (2.1).

Proof:
The total population size at time t is given by

$$N(t) = P(t) + I(t) + A(t) + B(t).$$

Differentiating this with respect to time, we obtain

$$\frac{dN(t)}{dt} = -\eta N \leq \Lambda - \eta N,$$

where is assumed to be the per capital recruitment rate into the human community. Solving the inequality and solving for $N(t)$ gives

$$N(t) \leq \frac{\Lambda}{\eta} + ce^{-\eta t}.$$

As $t \to \infty$, we obtain

$$N(t) \leq \frac{\Lambda}{\eta}.$$

Therefore, the threshold population level is $\frac{\Lambda}{\mu}$. It follows that the feasible solution set of the model remains in the region: $\Gamma = \left\{(P, I, A, B) \in \mathbb{R}_+^4 : 0 \leq P + I + A + B = N \leq \frac{\Lambda}{\eta}\right\}$. Observe that if the population is higher than the threshold level, the population reduces to the carrying capacity. If $N \leq \frac{\Lambda}{\eta}$, then the solution of the model remains in the invariant region for all $t > 0$. Therefore, the region Γ is positively invariant. This completes the proof.

Basic Reproduction Number

The basic reproduction number of the model is the average number of secondary violence cases caused by a single irascible, aggressive or brutal individual within an entirely peaceful population during his/her infective period. We shall employ the method used by Driessche and Watmough (2002) in finding the expression for R_0. This method is called the next generation matrix approach. Here, we consider the infected classes $X(t) = (I, A, B)$ and based on new infection terms represented by $\mathcal{F}(t)$ and old infection terms represented by $\mathcal{V}(t)$, we rewrite the equations for the infected classes in the form $X'(t) = \mathcal{F}(t) - \mathcal{V}(t)$ where

$$\mathcal{F} = \begin{pmatrix} \sigma\chi P \\ 0 \\ 0 \end{pmatrix} \qquad (2.2)$$

$$\mathcal{V} = \begin{pmatrix} -\delta B - (1-\tau)\delta A + (\zeta + \eta)I \\ -(1-\sigma)\chi P - \zeta I + (\alpha + \delta + \eta)A \\ -\alpha A + (\eta + \delta)B \end{pmatrix}. \qquad (2.3)$$

We now find the Jacobian matrix for each of the matrices $\mathcal{F}$ and $\mathcal{V}$, and evaluate at the disease-free equilibrium, to obtain the matrices F and V respectively.

$$\mathrm{F}=\begin{pmatrix}\beta\kappa\xi\sigma\varphi\omega & \beta\kappa\xi\varrho\sigma\varphi\omega & \beta\gamma\kappa\xi\sigma\varphi\omega\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},$$

$$V=\begin{pmatrix}\zeta+\eta & \delta(-1+\tau) & -\delta\\ -\zeta+\beta\kappa\xi(-1+\sigma)\varphi\omega & \alpha+\delta+\eta+\beta\kappa\xi\varrho(-1+\sigma)\varphi\omega & \beta\gamma\kappa\xi(-1+\sigma)\varphi\omega\\ 0 & -\alpha & \delta+\eta\end{pmatrix}.$$

We now obtain the inverse matrix V^{-1}.

$$V^{-1}=\begin{pmatrix}\mathrm{K}_1 & \mathrm{K}_4 & \mathrm{K}_7\\ \mathrm{K}_2 & \mathrm{K}_5 & \mathrm{K}_8\\ \mathrm{K}_3 & \mathrm{K}_6 & \mathrm{K}_9\end{pmatrix}.$$

where

K_1 **Stability Analysis of the Equilibrium Points**

Intra-communal violence is not a pleasant situation, especially to the residents of the community or the neighboring villages. It is expected that any of such violence should die out within the shortest possible time, especially when the basic reproduction number (R_0) is less than unity, and similarly the disease will become part of the population when the value of the basic reproduction number (R_0)is greater than unity. Therefore it is important to study the stability behaviors of the violence-free equilibrium ($\mathbb{E}_0$) with respect to the basic reproduction number. It is the utmost desire of any goodhearted and well-meaning individual or organization saddled with the responsibility of crisis management within the community to ensure the achievement of the global stability of the violence-free equilibrium point of the said intra-communal violence. To this end, we shall establish that the violence model (2.1) satisfies the conditions for local and global stabilities of violence-free equilibrium points via certain theorems. We start by finding the Jacobian matrix of the above system which is given as.

$$J=\begin{pmatrix}\frac{\partial \mathrm{f}_1}{\partial \mathrm{P}} & \frac{\partial \mathrm{f}_1}{\partial \mathrm{I}} & \frac{\partial \mathrm{f}_1}{\partial \mathrm{A}} & \frac{\partial \mathrm{f}_1}{\partial \mathrm{B}}\\ \frac{\partial \mathrm{f}_2}{\partial \mathrm{P}} & \frac{\partial \mathrm{f}_2}{\partial \mathrm{I}} & \frac{\partial \mathrm{f}_2}{\partial \mathrm{A}} & \frac{\partial \mathrm{f}_2}{\partial \mathrm{B}}\\ \frac{\partial \mathrm{f}_3}{\partial \mathrm{P}} & \frac{\partial \mathrm{f}_3}{\partial \mathrm{I}} & \frac{\partial \mathrm{f}_3}{\partial \mathrm{A}} & \frac{\partial \mathrm{f}_3}{\partial \mathrm{B}}\\ \frac{\partial \mathrm{f}_4}{\partial \mathrm{P}} & \frac{\partial \mathrm{f}_4}{\partial \mathrm{I}} & \frac{\partial \mathrm{f}_4}{\partial \mathrm{A}} & \frac{\partial \mathrm{f}_4}{\partial \mathrm{B}}\end{pmatrix}, \qquad (3.1)$$

where

$\mathrm{f}_1=\tau\delta A-(\chi+\eta)P,$
$\mathrm{f}_2=\sigma\chi P+\delta B+(1-\tau)\delta A-(\zeta+\eta)I,$
$\mathrm{f}_3=(1-\sigma)\chi P+\zeta I-(\alpha+\delta+\eta)A,$
$\mathrm{f}_4=\alpha A-(\eta+\delta)B.$

Theorem 3.1 (Local stability of $\mathbb{E}_0$)

The violence-free equilibrium ($\mathbb{E}_0$) is locally asymptotically stable if the basic reproduction number $\mathrm{R}_0<1$, otherwise it is unstable.

Proof:

We obtain the Jacobian matrix of the system as

$$J_{\mathbb{E}_0}=\begin{pmatrix}-\eta & -\beta\kappa\xi\varphi\omega & \delta\tau-\beta\kappa\xi\varrho\varphi\omega & -\beta\gamma\kappa\xi\varphi\omega\\ 0 & -\zeta-\eta+\beta\kappa\xi\sigma\varphi\omega & \delta-\delta\tau+\beta\kappa\xi\varrho\sigma\varphi\omega & \delta+\beta\gamma\kappa\xi\sigma\varphi\omega\\ 0 & -\beta\kappa\xi(-1+\sigma)\varphi\omega & -\alpha-\delta-\eta-\beta\kappa\xi\varrho(-1+\sigma)\varphi\omega & -\beta\gamma\kappa\xi(-1+\sigma)\varphi\omega\\ 0 & 0 & \alpha & -\delta-\eta\end{pmatrix} \qquad (3.2)$$

Observe that

$Trace(J_{\mathbb{E}_0})=\beta\kappa\xi\varphi\omega(\varrho+\sigma)-(\alpha+2\delta+\zeta+4\eta+\beta\kappa\xi\varrho\sigma\varphi\omega)<0,$

and

$$Det(J_{\mathbb{E}_0})=\eta\Big(\alpha\eta(\delta+\zeta+\eta+\beta\gamma\kappa\xi\sigma\varphi\omega)+(\delta+\eta)\big(\eta(\zeta+\eta+\beta\kappa\xi\varrho\sigma\varphi\omega)+\delta(\eta+\zeta\tau+\beta\kappa\xi\tau\varphi\omega)\big)\Big)$$
$$-\beta\eta\kappa\xi\big(\alpha(\delta+\gamma(\zeta+\eta)+\eta\sigma)+(\delta+\eta)(\delta+(\zeta+\eta)\varrho+\eta\sigma+\delta\sigma\tau)\big)\varphi\omega>0$$

Recall that the basic reproduction number is

$$R_0=\frac{\beta\kappa\xi(\alpha(\delta+\gamma\zeta+\eta)+(\delta+\eta)(\delta+\eta+\zeta\varrho))\sigma\varphi\omega}{\alpha\eta(\delta+\zeta+\eta)+\alpha\beta(\delta+\gamma(\zeta+\eta))\kappa\xi(-1+\sigma)\varphi\omega+(\delta+\eta)(\delta\eta+\zeta\eta+\eta^2+\delta\zeta\tau+\beta\kappa\xi(-1+\sigma)(\delta+(\zeta+\eta)\varrho-\delta\tau)\varphi\omega)}.$$

Therefore,

$$\omega = -\frac{(\eta(\alpha+\delta+\eta)(\delta+\zeta+\eta)+\delta\zeta(\delta+\eta)\tau)R_0}{\beta\kappa\xi\varphi(-(\alpha(\delta+\gamma\zeta+\eta)+(\delta+\eta)(\delta+\eta+\zeta\varrho))\sigma+(-1+\sigma)(\alpha(\delta+\gamma(\zeta+\eta))+(\delta+\eta)(\delta+(\zeta+\eta)\varrho-\delta\tau))R_0)}.$$

Substituting this into the expression for $Det(J_{\mathbb{E}_0})$, we obtain

$$Det(J_{\mathbb{E}_0}) = \frac{\eta(\alpha(\delta+\gamma\zeta+\eta)+(\delta+\eta)(\delta+\eta+\zeta\varrho))\sigma(\eta(\alpha+\delta+\eta)(\delta+\zeta+\eta)+\delta\zeta(\delta+\eta)\tau)(-1+R_0)}{-(\alpha(\delta+\gamma\zeta+\eta)+(\delta+\eta)(\delta+\eta+\zeta\varrho))\sigma+(-1+\sigma)(\alpha(\delta+\gamma(\zeta+\eta))+(\delta+\eta)(\delta+(\zeta+\eta)\varrho-\delta\tau))R_0} > 0.$$

Solving for R_0, we obtain $R_0 < 1$. This completes the proof.

Remarks:

1. Theorem 3.1 implies that as long as the initial sizes of the peaceful individuals, irascible individuals, aggressive individuals and brutal individuals are within the basin of attraction of the violence-free equilibrium, violence can be eradicated from the community. This is when $R_0 < 1$. On the other hand, global stability of the violence-free equilibrium guarantees that that eradication of violence does not depend on the initial sizes of the compartments. Thus, it is important to establish that with $R_0 \leq 1$, the violence-free equilibrium is globally asymptotically stable.
2. In order to investigate the global asymptotic stability of the violence-free equilibrium of the model, an appropriate Lyapunov function can be constructed, Ana and James(1976), and Michael andLiancheng (1999), but we shall employ the method introduced by Carlos and Song (2009). Here, we rewrite the model (2.1) in the form
3. $$\begin{cases} \frac{dX}{dt} = L(X,Z) \\ \frac{dZ}{dt} = M(X,Z), \quad M(X,0) = 0 \end{cases} \tag{3.2}$$

where $X = (P)$denotes the uninfected individuals and $Z = (I, A, B)$ denotes the infected individuals.

4. By equation (3.2), we would denote an equilibrium point of the model by $\mathbb{E} = (X, Z)$. The violence-free equilibrium $(\mathbb{E}_0)$ is thus represented as $\mathbb{E}_0 = (X^*, 0)$ where $X^* = (P)$.
5. If the following two conditions are satisfied, then the violence-free equilibrium is globally asymptomatically stable:

   $C1$: $\quad For \left.\frac{dX}{dt}\right|_{Z=0} = L(X, 0),$

   $X^* = (P, 0,)$ is globally asymptomatically stable.

   $C2$: $\quad \frac{dZ}{dt} = D_Z M(X^*, 0)Z - \widehat{M}(X, Z),$

   where $\widehat{M}(X, Z) \geq 0$ for all $(X, Z) \in \Gamma$.
6. Γ is the region where the model is biologically feasible, and $D_Z M(X^*, 0)$ is known as the Metlzer matrix with nonnegative off-diagonal elements.

Theorem 3.2 (Global stability of the violence-free equilibrium):

The equilibrium point $\mathbb{E}_0 = (X^*, 0)$ of the system (3.2) is globally asymptotically stable if $R_0 < 1$, and conditions (C1) and (C2) are satisfied.

Proof:

First let us introduce the recruitment term Λ in the Peaceful Class. Observe that

$$\frac{dX}{dt} = L(X,Z) = [\Lambda + \tau\delta A - (\chi + \eta)P], \tag{3.3}$$

$$\frac{dZ}{dt} = M(X,Z) = \begin{bmatrix} \sigma\chi P + \delta B + (1-\tau)\delta A - (\zeta+\eta)I \\ (1-\sigma)\chi P + \zeta I - (\alpha+\delta+\eta)A \\ \alpha A - (\eta+\delta)B \end{bmatrix}, \tag{3.4}$$

$$\left.\frac{dX}{dt}\right|_{Z=0} = L(X,0) = [\Lambda - \eta P]. \tag{3.5}$$

Equating the right hand side of equation (3.5) to zero and solving, we see that $X^* = \left(\frac{\Lambda}{\eta}\right)$ is the only equilibrium point. Solving the system of ordinary differential equation given by (3.3) for $P(t)$, we obtain

$$P(t) \leq \frac{\Lambda}{\eta} + \left(P_0 - \frac{\Lambda}{\eta}\right)e^{-\eta t}. \tag{3.6}$$

As $t \to \infty$, we see that $P(t) \to \frac{\Lambda}{\eta}$. This implies global convergence of $X = (P)$. Hence $X^* = \left(\frac{\Lambda}{\eta}, 0,0,0\right)$ is globally asymptotically stable for the system $\frac{dX}{dt}\Big|_{Z=0}$. We now obtain $D_Z M(X^*, 0)Z$.

$$D_Z M(X^*, 0) = \begin{pmatrix} -\zeta - \eta + \beta\kappa\xi\sigma\varphi\omega & \delta - \delta\tau + \beta\kappa\xi\varrho\sigma\varphi\omega & \delta + \beta\gamma\kappa\xi\sigma\varphi\omega \\ \square\zeta - \beta\kappa\xi(-1+\sigma)\varphi\omega & -\alpha - \delta - \eta - \beta\kappa\xi\varrho(-1+\sigma)\varphi\omega & -\beta\gamma\kappa\xi(-1+\sigma)\varphi\omega \\ 0 & \alpha & -\delta - \eta \end{pmatrix}$$

By the condition $(C2)$, we have

$$\widehat{M}(X, Z) = \begin{pmatrix} \sigma(\beta\kappa\xi\varphi\omega(Y + B\gamma + A\varrho) - P\chi) \\ (1-\sigma)(\beta\kappa\xi\varphi\omega(Y + B\gamma + A\varrho) - P\chi) \\ 0 \end{pmatrix}$$

We observe that the condition $\widehat{M}(X, Z) \geq 0$for all $(X, Z) \in \Gamma$ holds. Thus, the condition $(C2)$ is satisfied. Therefore, given $R_0 < 1$, since only $(C1)$ and $(C2)$ are satisfied, then the violence-free equilibrium of the model (1) is globally asymptotically stable. This completes the proof.

Remark: The global stability of the violence-free equilibrium assures us that when the right approach is followed in managing violence/crisis (or maintaining peace) within the community, long-lasting peace can be achieved, no matter the number of brutal individuals, aggressive individuals, or irascible individuals existing in the community at the point in time.

IV. Result And Discussion

In this research work, we have constructed a 4-compartment deterministic model to study intra-communal violence, where we have partitioned the residents of the community into the Peaceful Class, the Irascible Class, the Aggressive Class, and the Brutal Class. We assumed that violence outbreak happens over a short time, so that birth rate and natural death rate were ignored. The mathematical analyses performed on the model, include the positivity of solutions of the model, the invariant region and boundedness of solution. The expression for the average number of secondary violence cases caused by a single irascible, aggressive or brutal individual within an entirely peaceful population during his/her infective period, was obtained via the next generation matrix approach.

V. Conclusion

This research work has constructed a deterministic model to study the violence risk level of the human community. Mathematical analyses have been performed on the model. It was shown that the violence-free equilibrium is both locally and globally asymptotically stable.

References

[1] Aderibigbe, F.M. And Apanapudor, J.S. (2014): On The Extended Conjugate Gradient Method (Ecgm) Algorithm For Discrete Optimal Control Problems And Some Of Its Features, IOSR Journal Of Mathematics

[2] Ahad, N.A.andOkwonu, F.Z.(2020): COVID-19 Outbreak in Malaysia:Investigation on fatality cases,, Journal of advanced Research in Applied Sciencesand Engineering Technology, Vol.20(1), https://doi.org/10.37934/araset.20.1.110

[3] Ana, L. And James, Y. "A.(1976). Deterministic Model For Gonorrhea In A Non-Homogeneous Population," Mathematical Biosciences, Vol. 28, Pp. 221–236

[4] Apanapudor, J.S. (2007): Properties of the Control Operator H in Extended ConjugateGradient Method for Solving Discrete Optimal Control Problems, Journal of theInstitute Of Mathematics and Computer Sc(Mathematics series, Vol. 20, (3), pp. 165-177

[5] Apanapudor, J.S. and Olowu, O. O. (2023): The Fixed Lifetime Inventory Systemwith two Products having the same shelf life, pdf from researchgate.net

[6] Apanapudor, J.S. and Aderibigbe, F.M):On the Justification of the ExtendedConjugate Gradient Method (ECGM) Algorithm for Optimal Control Problems,Quest Journals, Journal Research in Applied mathematics, Vol. 2 issues 5 pp.07-13

[7] Apanapudor, J.S., Okwonu, F.Z.,Maduku, E.O. And Izevbizua, O.(2023): Solvability And Uniqueness Of Solutions Of Two Dimensional Optimization Problems, Fuw Trends In Science And Technology Journal, Www.Ftstjournal.Com, Vol.8, No. 1, Pp. 182 – 187.

[8] Apanapudor, J.S., Okwonu, F.Z.,Maduku, E.O. (2023):

[9] Apanapudor, J.S., Okwonu, F.Z.,Maduku, E.O. (2023):Apanapudor, J.S., Akporido, D.K. and Okwonu, F.Z.(2023): On the GeneralizedMinimum Cost Flow Problem: An Application in Natural Gas Distribution Networks,FUW Trends in Science and Technology Journal, www.ftstjournal.com, Vol. 8.No. 1,pp.

[10] Apanapudor, J.S., Okwonu, F.Z., Ahad, N. A. ,Ogini, N.O. and Yusuf, Z. M.(2023):Perturbation Analysis and the solution of Linear Systems, AIP Conf.Proc. 2896,030001, https://doi.org/10.1063/5.0179165

[11] Apanapudor, J.S. Aderibigbe, F.M. and, Okwonu, F.Z.(2020): An Optimal PenaltyConstant for Discrete Optimal Control Problems, J. Physics, Conf. Series, 1529

[12] Davies T., Fry H., Wilson A. And Bishop S. R. (2013). "A Mathematical Model Of The London Riots And Their Policing". Sci Rep 3, 1303.Https://Doi.Org/10.1038/Srep01303.

[13] De La Poza, E., Jódar, L. And Barreda, S. (2016) "Mathematical Modeling Of Hidden Intimate Partner Violence In Spain: A Quantitative And Qualitative Approach," Abstract And Applied Analysis, Vol. 2016, 8 Pages.

[14] Delgadillo-Aleman, S., Ku-Carrillo, R., Perez-Amezcua, B. And Chen-Charpentier, B. (2019) "A Mathematical Model For Intimate Partner Violence," Mathematical And Computational Applications, Vol. 24, No. 1, P. 29.

[15] Eshagberi G.O. (2019). "The History Of The Uvwie People Of The Niger – Delta". Https://Dutable.Com/2019/10/16/The-History-Of-The-Uvwie-People-Of-The-Niger-Delta-By-Godwin-Onajite-Eshagberi/. Accessed 7th December, 2022.

[16] Gbaramatuvoice (2019). "Again, Five Killed As Effurun Renewed Crises Escalate". Https://Www.Gbaramatuvoicenews.Com/Again-Five-Killed-As-Effurun-Renewed-Crises-Escalate/. Accessed 7th December, 2022.

[17] Hathroubi, S. And Trabelsi, H. (2014) "Epidemic Corruption: A Bio-Economic Homology," European Scientific Journal, Vol. 10, Pp. 228–235.

[18] Izevbizua, O. and Apanapudor, J.S.(2019): Implementing fries Model for thefixed lifetime Inventory System, OPSEARCH, https://doi.org/10.1007/s12597-019-00403-1

[19] Izevbizua, O. And Apanapudor, J.S.(2022): A New Ordering Policy For The Fixed Lifetime Inventory System, Australian Journal Of Basic And Applied Sciences, Https://Doi.Org/10.22587/Ajbas.2022.16.3.1, Vol. 16(3), Pp. 1-7.

[20] Izevbizua, O. and Apanapudor, J.S.(2019): Total Revenue Function for No-Regular Fixed lifetime Inventory System , African Journal of Mathematics andComputer Science Research,Vol. 12(2), pp.24-30

[21] Kizito M. And Tumwiine J. (2018) "A Mathematical Model Of Treatment And Vaccination Interventions Of Pneumococcal Pneumonia Infection Dynamics," Journal Of Applied Mathematics, Volume 2018, Https://Doi.Org/10.1155/ 2018/2539465, Pp 1-15.

[22] Kotola, B.S. And Mekonnen, T.T. (2022). "Mathematical Model Analysis And Numerical Simulation For Codynamics Of Meningitis And Pneumonia Infection With Intervention," Scientific Reports, Vol. 12, No. 1, P. 2639.

[23] Kumar T. K. V. (2013). "5 Communal Violence: A Case Study", Public Events And Police Response: Understanding Public Order Policing In Democratic India, Delhi Online Edition, Oxford Academic Https://Doi.Org/10.1093/Acprof:Oso/9780198090335.003.0005, Accessed 30 Aug. 2022.

[24] Lazarus, S. M. (2014). Mathematical Modeling Of Ethnic Violence Using Differential Equations And Designing Of Violence Risk-Level Determinant Case Study Of Nasarawa South Senatorial, Nasarawa State.

[25] Mahapatra, K. And Kant, S. (2005). "Tropical Deforestation: A Multinomial Logistic Model And Some Country Specific Policy Prescriptions," Forest Policy And Economics, Vol. 7, Pp. 1-24.

[26] Mamo, D.K. (2021). "Modeling The Transmission Dynamics Of Racism Propagation With Community Resilience," Computational Social Networks, Vol. 8, No. 1, Pp. 1–18.

[27] Mamadu, E.J. and Apanapudor, J.S Equation (2017): Variation IterationDecompositin Method for the AnalyticSolution of Gas Dynamic , Transactionof the Nigeria Association of Mathematical Physics, Vol. 4,pp.

[28] Manjunath, S. H. and Manjunath, T. C. (2005) "Development of an integratedlogistic model in an organization for an automotive application problem," J. Theo.Applied. Info. Tech. Available http://www.jatit.org/volumes/research-papers/Vol5No6/2Vol5No6.pdf.

[29] Manpower(2019)."Effurun".https://www.manpower.com.ng/places/city/128202/effurun. Accessed 7 th December,2022.

[30] Mcgrew, K. (2008) "Chapter 5: Corruption and racism in the legalsystem," Counterpoints, vol. 325, pp. 119–147.

[31] Michael Y. L., John R. G., Liancheng W. and Janos K. (1999). "Global dynamics of aSEIR model with varying total population size," Mathematical Biosciences, vol. 160,pp. 191–213.

[32] Mousavi, P. and Pourkiani, M. (2013). "Administrative corruption: ways oftackling the problem," European Online Journal of Natural and SocialSciences: Proceedings, vol. 2, no. 3, p. 178.

[33] Neelo F. and Absar A. (2021). Communal Violence, Mental Health and TheirCorrelates: A Cross-Sectional Study in Two Riot Affected Districts of Uttar Pradeshin India, Journal of Muslim Minority Affairs, 41:3, 510-521, DOI: 10.1080/13602004.2021.1999139.

[34] Okposo, N.I., Addai, E., Apanapudor, J.S., Gomez-Aguilar, J.F.(2023): A study on amonkeypox transmission model within the scope of fractal-fractional derivative withpower-law kernel, European Physical Journal Plus,https://doi.org/10.1140/epip/s13360.023.04334-1

[35] Okwonu, F.Z.andApanapudor, J.S.(2019).: The Statistical Value of Water Transport:A Preliminnary Study of Niger Delta Creek, Data Research Journal,pp.135-145

[36] Okwonu, F.Z.,Ahad, N. A,,Yusof,, Z,A.,Arunaye, F.I., and Apanapudor, J.S.(2021):Application of Shewchart Control Chart to analyze the effect of Income on BodyMass Index and Blood Pressure, Journal of Appplied Mathematics and ComputaionalIntelligence

[37] Okwonu, F.Z.,Ahad, N. A, Okoloko, I.E.,Kamaruddi, S.A., Arunaye, F.I., andApanapudor, J.S.(2022): Robust Hybrid Classification Methods and Applications,Pertanika J. Sc. And Technology, 30(4), pp.2831-2850

[38] Okwonu, F.Z., Apanapudor, J.S. Maduku, E.O. (2023): On Comparative Analysis ofEstimation Procedures, FUW Trends in Science and Technology Journal,www.ftstjournal.com

[39] Okwonu, F.Z.,Ahad, N. A, Apanapudor, J.S. and Arunaye, F.I. (2020): COVID-19Prediction Model (COVID-19-PM) for Social Distancing : The Height Perspective,