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A Note On Real Fuzzy Subfields

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Abstract
This Research paper develops some idea and novelty relating to real fuzzy subfields according to present complex scenario. It has been explained idea of fuzzy subfield and duly discussed numerous algebraic properties. We have also cited that any fuzzy subfield of the field of rational is also rational number itself. The same result holds for real numbers provided the membership function is continuous. We also showed that a lower level subset is subfields of a field. Fuzzy logic allows for the inclusion of vague human assessments in computing problems. Also, it provides an effective means for conflict resolution of multiple criteria and better assessment of options. New computing methods based on fuzzy logic can be used in the development of intelligent systems for decision making, identification, pattern recognition, optimization and control.

Keywords: *Fuzzy Subfields, Membership functions, fuzzy set, anti L-fuzzy subfield.*

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I. Introduction

A field is an algebraic structure which plays a significant role in number theory, algebra, and many other areas of mathematics. Fields serve as development notions in various mathematical domains. The fuzzy set theory is founded on the doctrine of concerning relative graded membership basing human mechanism of recognition as well as perception Fuzzy logic is one of the most important branches in mathematics. It was originally developed as a mathematical theory for modelling of the unpredictability. The term Fuzzy refers to something imprecise. or vague. The computer cannot easily understand such cases. Thus, it cannot produce an exact result of True or False. But a Fuzzy Logic algorithm makes systems more intelligent and helps them understand the problems where there may be other answers than true or false. There are lots of applications of fuzzy logic in our daily life. It has been used in numerous applications such as facial pattern recognition**,** air conditioners, washing machines, vacuum cleaners, antiskid braking systems, and transmission systems, control of subway systems and unmanned helicopters, knowledge-based systems for multiobjective optimization of power systems. Fuzzy logic is extremely useful for many people involved in research and development including engineers (electrical, mechanical, civil, chemical, aerospace, agricultural, biomedical, computer, environmental, geological, industrial, and mechatronics), mathematicians, computer software developers and researchers, natural scientists (biology, chemistry, earth science, and physics), medical researchers, social scientists (economics, management, political science, and psychology), public policy analysts, business analysts, and jurists. Indeed, the applications of fuzzy logic, once thought to be an obscure mathematical curiosity, can be found in many engineering and scientific works. Fuzzy logic has been used in numerous applications such as facial pattern recognition, air conditioners, washing machines, vacuum cleaners, antiskid braking systems, transmission systems, control of subway systems and unmanned helicopters, knowledge-based systems for multiobjective optimization of power systems, weather forecasting systems, models for new product pricing or project risk assessment, medical diagnosis and treatment plans, and stock trading. Fuzzy logic has been successfully used in numerous fields such as control systems engineering, image processing, power engineering, industrial automation, robotics, consumer electronics, and optimization. This branch of mathematics has instilled new life into scientific fields that have been dormant for a long time. Thousands of researchers are working with fuzzy logic and producing patents and research papers.

In 1965, famous mathematician Zadeh [1], the man behind the fuzzy logic is widely known as the father of mathematical framework of fuzzy logic was a professor of electrical engineering in the University of California at Berkeley. He published his famous research paper about fuzzy sets and subsequently many mathematician have been applied various aspects of theory and application of fuzzy set. Fuzzy subfield and linear subspace have been developed [2]. Gradually fuzzy sets related to several algebraic structures such as non-associative ring [3] and time series [4]. Malik and Mordeson [5] studied fuzzy subfield and its basic properties .The development about fuzzy algebraic structure may be viewed in [6, 7, 8].After the introduction of fuzzy sets by some great mathematician. Several researchers explored on the generalization of the concept of fuzzy sets [9, 10]. The idea about of fuzzy subgroups, anti-fuzzy subgroups, fuzzy fields and fuzzy linear spaces was introduced by Biswas

R [13,14]. Presently several mathematicians has been working constantly development of the fuzzy logic. Later the concept of fuzzy subgroups, anti-fuzzy subgroups, fuzzy fields and fuzzy linear spaces was introduced by great mathematician [15]. Several mathematicians have been working constantly development of the fuzzy logic. There are numerous applications of real fuzzy subfield in daily life. It has been used to solve real-life world problem when human may face many uncertain situations. The fuzzy logical model of perception (FLMP) paradigm/fuzzy model have been expanded to include perception and also inspected by Jati R.K et.al.[16].

We will extended the discussion of real fuzzy sets to developing a new concept of real fuzzy subfield by adding a second dimension in membership function of fuzzy set. Many field theory problems can be handled by real fuzzy set. This theory will be useful source of motivation for mathematician in future research work.

II. Preliminaries:

We recall first the elementary notion of fuzzy sets and fuzzy subfield which play a key role for our further analysis. Definition (2.1): Let us consider fuzzy set A, A = {(x,μ_F (x))| x $\in$ X} where μ_F (x) is called the membership function for the fuzzy set A. X is referred to as the universe of discourse. The membership function associates each element x $\in$ X with a value in the interval [0, 1]. In fuzzy sets, each elements is mapped to [0, 1] by membership function, that is μ_F : X $\in$ [0, 1], where [0, 1] means real numbers between 0 and 1 (including 0, 1).

Definition (2.1)
Let a set A is given set. Then membership function can be used to define a set A is given by μ_A (x) =1 if x $\in$ A = 0 if x $\notin$ A

Definition (2.2)
Let X be any set and L be a lattice could be [0, 1].Then a fuzzy set A in X characterized by a membership function μ_A : X $\rightarrow$ L. Let X be a field and F be a fuzzy subset of X with membership function μ_F. Then F is called a fuzzy subfield of X if

(i) μ_F (x+y) $\geq$ min(μ_F (x), μ_F (y))
(ii) μ_F(-x) $\geq$ μ_F (x)
(iii) μ_F(xy) $\geq$ min(μ_F (x), μ_F (y))
(iv) μ_F(x^{-1}) $\geq$ μ_F (x)
(v) μ_F(0)=1
(vi) μ_F(1)=1

Definition (2.3)
Let (F, +, ·) be a field. A fuzzy subset A of F is said to be a fuzzy subfield of F if the following conditions are satisfied:
(i) A (x–y) ≥ min (A(x), A(y)), for all x and y in F,
(ii) A (xy) ≥ min (A(x), A(y)), for all x and y in F,
(iii) A(x^{-1}) ≥A(x), for all x in F− {0} where 0 is the additive identity element of F

Definition (2.4):
Let X be a non-empty set and L be a complete lattice. A L-fuzzy subset A of X is a function A**:** X → L.

Definition (2.5):
Let (F, +, ·) be a field. A L-fuzzy subset A of F is said to be an anti L-fuzzy subfield of F if the following conditions are satisfied.
(i) A(x+y) A(x) ≤A(y), for all x and y in F,
(ii) A (-x) ≤ A(x) for all x in F,
(iii) A (xy) ≤A(x) $\vee$ A(y), for all x and y in F,
(iv)A (x^{-1}) ≤A(x), for all x in F− {0}, where 0 is the identity elements.

Definition (2.6):
Let A be a fuzzy subset of X. For a in L, then lower level subset of A is the set
A_a = {x $\in$ X: A(x) ≤ a}.

Theorem-1

If F is a fuzzy subfield of the field of real numbers with membership function μ_F and if μ_F is continuous, then μ_F**(x)** =1 holds for all $x \in F$.

Proof: Let X be any real number. Then there exists a sequence of rational numbers $\{x_n\}$ such that $\lim_{n\to\infty} x_n = x$

Since μ_F is continuous.

So $\mu_F(x) = \mu_F(\lim_{n\to\infty} x_n)$

$= \lim_{n\to\infty} \mu_F(x_n)$

$= \lim_{n\to\infty} \mu_F(1)$ ($\because$ F is a fuzzy subfield)

$= \lim_{n\to\infty} 1$ ($\because$ F is a fuzzy subfield**)**

Theorem -2.

If is a fuzzy subfield of rational numb**er** with membership function μ_F,

Then μ_F (a)-1=0 for all $a \in F$.

Proof: Since from definition of membership function for all rational number x

μ_F (a)=μ_F (−a)

μ_F =μ_F (a^{-1}) for $a \neq 0$.

For any positive integer k.

μ_F (a)= μ_F (1+1+1+…+1) =μ_F (1)

k- times

Put a= $\frac{m}{n}$ for any rational number.

Since $\mu_F (1) \geq \mu_F \left(\frac{m}{n}\right)$

$\mu_F (1) \leq \mu_F (mn^{-1})$

$= \mu_F \left(\frac{m}{n}\right)$

so $\mu_F (1) = \mu_F \left(\frac{m}{n}\right)$.

Since from definition of fuzzy subfields

$\mu_F (1) = \mu_F (0)$

$\Rightarrow$ μ_F (a)=1 for all $a \in Q$

$\Rightarrow \mu_F$ (a)-1=0, for all $a \in Q$.

Theorem-3

Let A be an anti L-fuzzy subfield of a field (F, +.).Then for a in L such that $a \geq A(0)$, $a \geq A(1)$, A_a is a subfield of F where 0 and 1 are Identity elements of F.

Proof:

Since $A(x) \leq a$ and $A(y) \leq a$ for all x, y are in A_a .

Now $A(x-y) \leq A(x) \vee A(y \leq a \vee a = a$.

So $x-y \in A_a$

Also $A(xy^{-1}) \leq A(x) \vee A(y) \leq a \vee a = a$

Therefore A_a is a subfield of F.

III. Conclusion:

The present articles provide an overview of various aspects of fuzzy subfield and its application in different field. The various properties of fuzzy subfield over rational and real number were reviewed. If same properties applies on complex number system result holds or not left for researcher interest. Therefore more research in this field could be done for beneficial of researcher as well as mathematics society.

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