

Probabilistic Analysis Of The Risk Parameters Involve In Hiv/Aids: Nonlinear Transmission Dynamics

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Abstract

The main aim of the present study is to analyze the risk parameters involved in HIV transmission. The probability of becoming infected with HIV is formulated in terms of the total number of sexual contacts, the probability that a sexual act is infectious and the prevalence. We show that for many realistic situations the probability of becoming infected by multiple partners is equal to the probability of becoming infected by one partner in a monogamous relationship given that the prevalence is the same in both cases. A different rate of recruitment is considered for different classes of the population. The results obtained can be used to explore the possible effects of HIV transmission. We will also discuss the effect of reducing the total number of sexual contacts and changing the habit to safer sexual behaviour to reduce the risk. Finally, the numerical illustrations are presented to support the analytical results.

Keywords:- *HIV/AIDS transmission, Mathematical Modelling, Probabilistic analysis, Risk parameter, Numerical analysis etc.*

Date of Submission: 03-06-2024

Date of Acceptance: 13-06-2024

I. Introduction

There are some important epidemiological factors which affect the transmission dynamics of HIV. The basic research on the transmission dynamics of the HIV virus in human populations has become a major research topic in mathematical biology. Mathematical modelling help to describe the essential relations between epidemiological factors, such as distributed incubation periods and heterogeneity in sexual activity. They also help to identify what kinds of epidemiological data are needed to make predictions of future trends.

The risk of getting HIV varies widely depending on the type of exposure. HIV virus can be transmitted through blood transfusion, sharing of needle, births etc, as well as sexual behavior. As epidemic continues to grow there is much concern about how individuals should change their sexual behaviour to reduce the risk of becoming infected with the human immunodeficiency virus (HIV). AIDS is different from other sexually transmitted disease (STD) in many respects. Since it does not appear that a vaccination to prevent infection will soon be available, changing one's behaviour to minimize the possibility of infection is the only preventive means. In this study we explore the issue of multiple partners to an individual's risk of becoming infected.

Mathematical models have been developed to study various factors of transmission of STD's. Mathematical analysis of AIDS models can be found in, Anderson and May (1987, 1988). Most models on HIV are developed with the purpose to study certain specific characteristics of the transmission dynamics of the AIDS epidemic, such as the long infectious period and incubation period of the diseases. Valerie (1988) suggested a review on mathematical modelling of the Transmission Dynamics of HIV Infection and AIDS. Sandberg and Awerbuch (1989) depicted mathematical formulation and studies of the risk parameters involved in HIV transmission. Blower and Dowlatabadi (1994) explored sensitivity and uncertainty analysis of complex models of disease transmission, they consider HIV model as an example. Bhawe et al. (1995) shown brief impact of an intervention on HIV, sexually transmitted diseases, and condom use among sex workers in Bombay, India. The probabilities of HIV transmission and pair formation was taken into consideration by Tan and Zhu (1996). A brief study on concurrent partnerships and the spread of HIV have been carried out by Morris and Kretzschmar (1997). Pretty et al. (1999) gave some human bites and the risk of human immunodeficiency virus transmission.

Garnett and Rottingen (2001) considered measuring the risk of HIV transmission. Nagelkerke et al. (2001) studied the effect of interventions on model the HIV/AIDS epidemics in India and Botswana. Michael (2005) suggested a model on sex-work harm reduction. Walker et al. (2005) explored a comparative population dynamics of HIV-1 subtypes B and C, subtype-specific differences in patterns of epidemic growth. Xia (2007) gave a model of HIV infection with consideration of vaccine readiness, drug effectiveness and therapeutically failures. Van-geertruyden and D'Alessandro (2008) considered a silent alliance on modeling of Malaria and HIV. Cassels and Clark (2008) provide tools for social and behavioral science research on mathematical models for

HIV transmission dynamics. Baral (2012) have deep sense of burden about HIV among female sex workers in low-income and middle-income countries, they provided a systematic review and meta-analysis.

Recently Kaur et al. (2014) suggested a mathematical analysis of the transmission dynamics of HIV/AIDS, in their study, they considered the wide role of female sex workers. Patel et al. (2014) estimated per-act HIV transmission risk and give a systematic review on AIDS. Oliveira et al. (2017) applied a data mining approach to analyze the HIV/AIDS surveillance system. Saravanakumar et al. (2020) provide a mathematical model of risk factors in HIV/AIDS transmission dynamics, this is a observational study of female sexual network in India. In the most of the study researchers are investigated the transmission dynamics of HIV/AIDS epidemic with emphasis on the role of female sex workers. And also they focus only the heterosexual transmissions of HIV/AIDS and formulate the mathematical model by dividing the total adult population under consideration into suitable classes. In other words, there may be a relatively small chance of acquiring HIV when engaging in a risk behavior with an infected partner only once; but, if repeated many times; the overall likelihood of becoming infected after repeated exposures is actually much higher.

In the present study a probabilistic approach to analysis the individual's risk of having multiple partners is presented. Numerical experiment is performed to demonstrate the transmission probability of being infected by varying different parameters.

II. Mathematical Analysis

For mathematical modeling of HIV through sexual relationship, the following assumptions are made:

- The risk of acquiring HIV is depends upon the total number of sexual contacts (n).
- The probability that a partner is infected is denoted by (p).
- The probability that one contact with an infected partner will result in infection is (r).

Case – I : For a monogamous relationship, the transmission probability ($PT1$), the probability that an individual becomes infected after n contacts is

$$PT1 = p (1 - (1 - r)^n) \dots\dots\dots(1)$$

Case – II : In other situation, where a person has many partners, but only one contact per partner. In this case p and r are assumed to be the same for all partners. The probability of becoming infected after a total number of n contact is

$$PT2 = 1 - (1 - p r)^n \dots\dots(2)$$

Case – III : Now we consider the general situation. When an individual has m^i contacts with each of n different partners, each of whom may have different infectivity per act (r^i) and a different probability of being infected (p^i), then we can write the probability of transmission as

$$PT3 = 1 - \prod_{i=1}^n (1 - p_i (1 - (1 - r_i)^{m_i})) \dots\dots(3)$$

It is noted that $PT1$, and $PT2$ are special cases of $PT3$. Equation (3) is used to explore the difference in the transmission probability of persons who have the total number of sexual contacts but have acquired a different number of partners. In this case equation (3) can reduce to:

$$PT3 = 1 - 1(1 - p(1 - (1 - r)^m))^n \dots\dots(4)$$

One important application of equation (3) is to simulate the transmission probability as the epidemic progresses.

Case – IV : A person who selects multiple partners from the same pool will be choosing, over time, partners who are more likely to have been infected. We consider the special case where r is constant for all partners but p is allowed to grow exponentially with time. This leads to the following equation for transmission probability

$$PT4 = 1 - (1 - p r_1)(1 - p e^{t_1/\tau} r_2)(1 - p e^{t_2/\tau} r_3)\dots\dots(1 - p e^{t_{(n-1)}/\tau} r_n) \dots(5)$$

where one new partner is selected in each unit of time and there is one contact per partner. τ is doubling time.

III. Numerical Results

We present the numerical results based on investigations done in previous section. We provide numerical results for the probability, the behavior of probability at the different value of parameter are examined. Numerical program is developed in MATLAB. It is observed that the numerical results are quite close to the real situations. We display the Probability count of infected individuals with HIV/AIDS on y-axis (vertical), and the number of

contact on x -axis (horizontal). Equations (1) and (2) give non-linear functions of n , p and r . We examine the relationship between n , p and r and their effect on the transmission probability of transmission dynamics, using computer programme in language c. We compute the transmission probability for realistic values of the parameters. The values of 0.001, 0.01 and 0.1 were used for r . For p we consider a range of values from 0 to 1.

Numerical results for PT1, PT2 are summarized in table 1 – 6. We also examine the ratio PT1 / PT2 which are near to one in all cases. To analysis the results, we have also plotted the graphs for PT1 and PT2 in figures 4.1 to 4.6 for different values of p and r .

For small values of r , $PT1 = PT2 = npr$. For example, for $p = 0.5$, $n = 250$, and $r = 0.001$, $npr = 0.125$ while $PT1 = 0.0476$ and $PT2 = 0.0488$. This can also be seen by Taylor expansion of $PT1$ and $PT2$ with respect to r , for $r_0 = 0$.

The Taylor’s expansion of $PT1$ and $PT2$ with respect to r for $r = 0$,

$$PT1 = npr - \frac{n(n-1)}{2!} pr^2 + \text{terms that have } r \text{ to the third or higher powers.}$$

$$PT2 = npr - \frac{n(n-1)}{2!} p^2 r^2 + \text{terms that have } p \text{ and } r \text{ to the third or higher powers.}$$

A small value of p , such as 0.01, represents the prevalence early in the epidemic, while a larger value, such as 0.8, represents the prevalence late in the epidemic. The leading term in both $PT1$ and $PT2$ is npr . Further more for small values of r , the rest of the terms are negligible. Therefore for small values of r , $PT1 = PT2 = npr$.

IV. Conclusions

Simple mathematical models of the transmission dynamics of human immunodeficiency virus help to clarify some of the essential relations between epidemiological factors, and the overall pattern of the AIDS epidemic. They also help to identify what kinds of epidemiological data are needed to make predictions of future trends.

In our study, we have developed a mathematical model based on stochastic process, for the HIV transmission dynamics by taking risk parameter into consideration. Appropriate scale factors are used for each case to draw the graphs with the number of contact for Probability scale. We have noted that the probability of becoming infected by multiple partners is equal to the probability of becoming infected by one partner in a monogamous relationship given that the prevalence is the same in both cases.

Table 1 The probability of transmission of HIV in the monogamous case ($PT1$) vs the multiple partner case ($PT2$) where $p = 0.5$ and $r = 0.001$.

N	PT1	PT2	PT2/PT1	NPR
10	0.0049	0.0049	1.0022	0.005
20	0.0099	0.0099	1.0047	0.010
50	0.0243	0.0246	1.0122	0.025
80	0.0386	0.0392	1.0197	0.040
90	0.0430	0.0440	1.0222	0.045
100	0.0476	0.0487	1.0247	0.050
150	0.0696	0.0722	1.0372	0.075
200	0.0906	0.0951	1.0497	0.100
250	0.1106	0.1175	1.0622	0.125
500	0.1968	0.2212	1.1241	0.250

Table 2: The probability of transmission of HIV in the monogamous case ($PT1$) vs the multiple partner case ($PT2$) where $p = 0.5$ and $r = 0.01$.

N	PT1	PT2	PT2/PT1	NPR
10	0.04780	0.04889	1.02260	0.05
20	0.09104	0.09539	1.04770	0.10
50	0.19749	0.22168	1.12248	0.25
80	0.27623	0.33035	1.19589	0.40
90	0.29763	0.36309	1.21992	0.45
100	0.31698	0.39423	1.24369	0.50
150	0.38927	0.52852	1.35771	0.75

200	0.43301	0.63304	1.46195	1.00
250	0.45947	0.71439	1.55481	1.25
500	0.49671	0.91842	1.84900	2.50

Table 3: The probability of transmission of HIV in the monogamous case (*PT1*) vs the multiple partner case (*PT2*) where $p = 0.5$ and $r = 0.1$.

N	PT1	PT2	PT2/PT1	NPR
10	0.32566	0.40126	1.23215	0.500
20	0.43921	0.64151	1.46060	1.000
50	0.49742	0.92305	1.85567	2.500
80	0.49989	0.98348	1.96739	4.000
90	0.49992	0.99011	1.98052	4.500
100	0.49998	0.99407	1.98821	5.000
150	0.50000	0.99954	1.99908	7.500
200	0.50000	0.99996	1.99993	10.000
250	0.50000	0.99999	1.99999	12.500
500	0.50000	1.00000	2.00000	25.000

Table 4: The probability of transmission of HIV in the monogamous case (*PT1*) vs the multiple partner case (*PT2*) where $p = 0.05$ and $r = 0.001$.

N	PT1	PT2	PT2/PT1	NPR
10	0.00049	0.00050	1.00428	0.0005
20	0.00099	0.00100	1.00905	0.0010
50	0.00244	0.00249	1.02345	0.0025
80	0.00384	0.00399	1.03798	0.0040
90	0.00430	0.00449	1.04285	0.0045
100	0.00476	0.00498	1.04773	0.0050
150	0.00696	0.00747	1.07237	0.0075
200	0.00906	0.00995	1.09736	0.0100
250	0.01106	0.01242	1.12270	0.0125
500	0.01968	0.02469	1.25454	0.0250

Table 5: The probability of transmission of HIV in the monogamous case (*PT1*) vs the multiple partner case (*PT2*) where $p = 0.05$ and $r = 0.01$.

N	PT1	PT2	PT2/PT1	NPR
10	0.00478	0.00498	1.04347	0.005
20	0.00910	0.00995	1.09313	0.010
50	0.01975	0.02469	1.25045	0.025
80	0.02762	0.03922	1.41979	0.040
90	0.02976	0.04401	1.47877	0.045
100	0.03169	0.04878	1.53895	0.050
150	0.03892	0.07227	1.85663	0.075
200	0.04330	0.09518	2.19822	0.100
250	0.04594	0.11753	2.55795	0.125
500	0.04967	0.22124	4.45422	0.250

Table 6: The probability of transmission of HIV in the monogamous case (*PT1*) vs the multiple partner case (*PT2*) where $p = 0.05$ and $r = 0.1$.

N	PT1	PT2	PT2/PT1	NPR
10	0.03256	0.04889	1.50125	0.050
20	0.04392	0.09539	2.17183	0.100
50	0.04974	0.22168	4.45671	0.250

80	0.04998	0.33035	6.60848	0.400
90	0.04992	0.36309	7.27290	0.450
100	0.04999	0.39423	7.88480	0.500
150	0.05000	0.52852	10.5704	0.750
200	0.05000	0.63304	12.6608	1.000
250	0.05000	0.71439	14.2878	1.250
500	0.05000	0.91842	18.3685	2.500

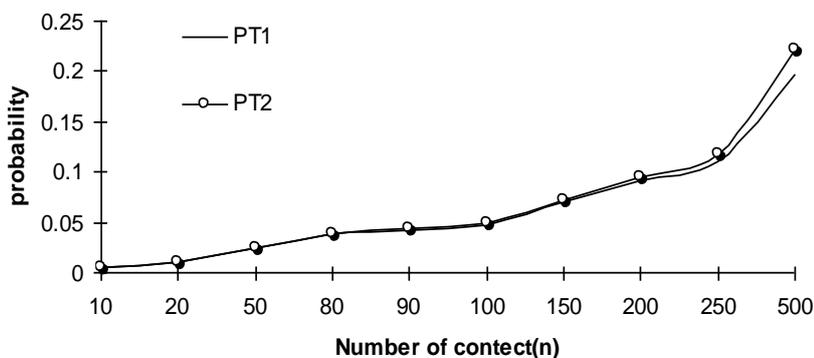


Figure – 1: Calculating PT1 and PT2 when p=0.5 and r=0.001

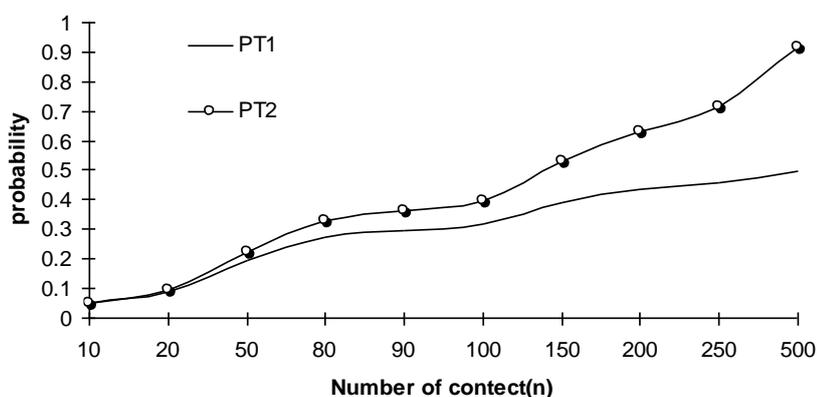


Figure – 2: Calculating PT1 and PT2 when p=0.5 and r=0.01

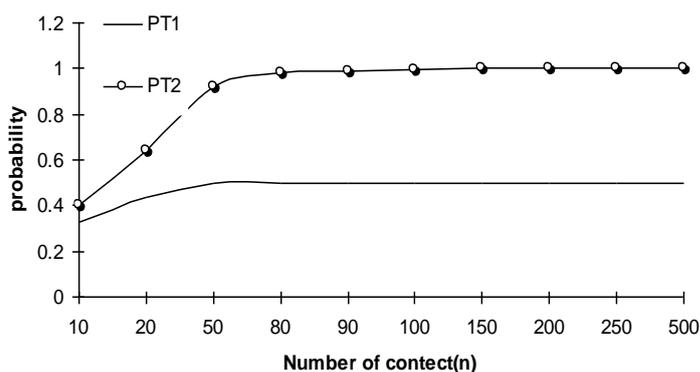


Figure – 3: Calculating PT1 and PT2 when p=0.5 and r=0.01

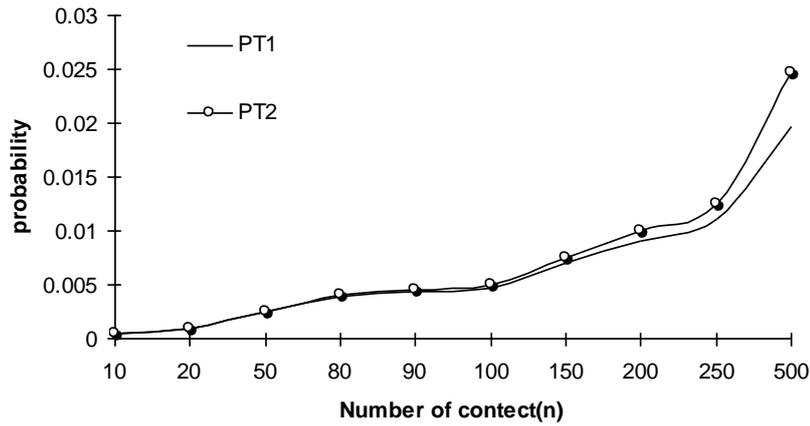


Figure – 4: Calculating PT1 and PT2 when p=0.05 and r=0.001

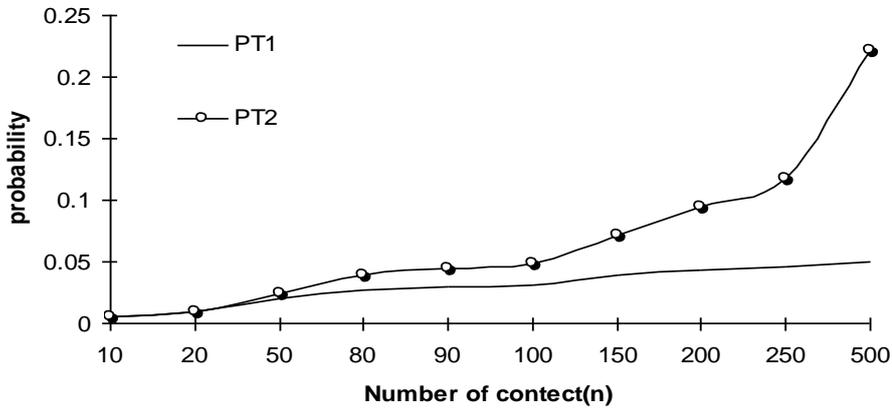


Figure – 5: Calculating PT1 and PT2 when p=0.05 and r=0.01

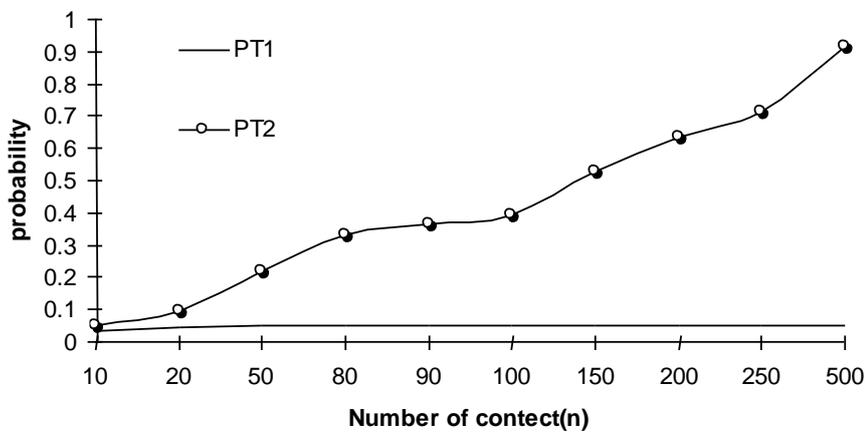


Figure – 6: Calculating PT1 and PT2 when p=0.05 and r=0.1

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