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SDC Labeling For Cycle Of Join Zero Divisor Graphs

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Abstract:
A sum divisor cordial labeling of a graph G with vertex set V(G) is a bijection f from V(G) to {1,2, 3, …, |V(G)|} such that an edge uv is assigned the label 0 if 2 divides f(u)+f(v) and the label 1 otherwise, if it satisfies the condition $|e_f(0) - e_f(1)| \leq 1$ *Then the graph with a sum divisor cordial labeling is called a sum divisor cordial graph. In this paper we introduce the concept of the sum divisor cordial labeling for cycle of join zero divisor graphs.*

***Keywords**: Cycle, Zero divisor graphs, cordial graphs, cycle of graphs, Sum divisor cordial labeling, Sum divisor cordial graphs.*

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I. Introduction

Let $G = (V(G), E(G))$ be a simple, finite and undirected graph. Different types of graphs labeling techniques have been investigated and over to 2000 papers have been published in this area. A graph labeling is an assignment of labels traditionally represented by integers to edges or vertices or both. Beck [6] introduced Zero-divisor graph. Then Anderson and Livingston [7] modified it. Tamizh Chelvam et al [13] proposed graph labeling related to Zero-divisors in a commutative ring. obtained certain labeling for certain class of Zero-divisor graphs corresponding to finite rings. Let R be a commutative ring with non-zero identity, $Z(R)$ it's set of all zero devisors in R and

$Z^*(R) = Z(R) \setminus \{0\}$. the Zero-divisor graph of R is the simple undirected graph Γ(R) with vertex set $Z^*(R)$ and two distinct vertices x and y are adjacent if x y =0. [11] S. Sajana and D. Bharathi studied the Intersection graph of zero-divisors of a finite commutative ring. [12] D. Eswara Rao and D. Bharathi studied the Total zero divisor graph of a commutative ring. All graphs consider in this paper are finite, simple and undirected. This graph product was introduced by Frucht and Harary [3]. After that A. Lourdusamy [10] introduced sum divisor cordial labeling of graphs. Here, we are interested in the sum divisor cordial (SDC) labeling for cycle of join zero divisor graphs. Based on these concepts we give several results on SDC labeling for Zero-divisor graphs. Here we study the cycle of join Zero-divisor graphs.

II. Preliminaries:

Definition 2.1

Let $G = (V(G), E(G))$ be a simple graph and $f\colon V(G) \to \{1, 2, ..., V(G)|\}$ be the bijection. For each edge uv, assign the label 0, if 2/f(u)+f(v) and the label 1 otherwise. The function f is a sum divisor cordial (SDC) labeling if it satisfies the condition $|e_f(0) - e_f(1)| \leq 1$. where $e_f(0)$ is the number of edges labeled with 0 and $e_f(1)$ is the number of edges labeled with 1.Then a graph which admits sum divisor cordial (SDC)labeling is called as a sum divisor cordial (SDC) Graph.

Definition 2.2

The join of two graphs G_1 and G_2 is denoted by $G_1 + G_2$ and whose vertex set is $V(G_1 + G_2) = V(G_1) \cup V(G_2)$ and edge set is
$E(G_1 + G_2) = E(G_1) \cup E(G_2) \cup \{uv: u \in V(G_1), v \in V(G_2)\}$.

Definition 2.3

A Graph G is said to be complete if every pair of its distinct vertices are adjacent.

Definition 2.4

A bipartite graph is a graph whose vertex set V (G) can be partitioned into two subsets V_1 and V_2 such that every edge of G has one end in V_1 and the other end in V_2. (V_1, V_2) is called a bipartition of G.

Definition 2.5

A complete bipartite graph is a bipartite graph with bipartition $(V_1\ V_2)$ such that every vertex of V_1 is joined to all the vertices of V_2. It is denoted by $K_{m,n}$, where $|V_1| = m$ and $|V_2| = n$.
A star graph is a complete bipartite graph $K_{1, n}$.

Definition 2.6

The complement $\bar{G}$ of the graph G is the graph with vertex set V (G) and two vertices are adjacent in $\bar{G}$ if and only if they are not adjacent in G.

Definition 2.7

For a cycle C_n, each vertex of C_n is replaced by connected graphs $G_1, G_2, \ldots, G_n$ and is known as cycle of graphs. We shall denote it by C $(G_1, G_2, \ldots, G_n)$. If we replace each vertex by a graph G, i.e. G_1= G=$G_{2=}$ …, G_n, such cycle of a graph G is denoted by C (n.G).

Definition 2.8

A cycle graph is a graph that contains a single cycle, or a closed chain of vertices connected by edges.
A cycle graph with n vertices is denoted by C_n.

Definition 2.9

Let R be a finite ring. An element a$\neq$ 0 ∈ R is called a zero divisor if there exists a non-zero element b ∈ R such that
a. b = 0 or b. a = 0.

Definition 2.10

A star graph is a Tree with a single internal node and multiple leaves.

Definition 2.11

Cordial labeling is a way to assign labels to vertices and edges using only the numbers 0 and 1. The labels are assigned so that the number of vertices and edges labeled 0 and 1 differ by at most 1.

III. Main Results

In this section, we obtain cycle of join zero-devisor graphs which admits a SDC labeling.

Theorem 3.1. For any prime number p>2, cycle of join zero-devisor graphs $\Gamma(Z_{2p}) + \Gamma(Z_4)$ where n≥3 is a SDC graph.

Proof: Let G = C (n. $(\Gamma(Z_{2p}) + \Gamma(Z_4))$, ∀ n≥3
Let the vertex set and edge set of the graph G are
V(G)= $\{u_{1,1}, u_{1,2}, \ldots u_{1,p}, x_1, u_{2,1}, u_{2,2}, \ldots, u_{2,p}, x_2, \ldots, u_{n,1}, u_{n,2}, \ldots u_{n,p}\}$, ∀ n≥3
E(G)= $\{u_{j,i}u_{j,p}, u_{j,i}x_j, u_{j,p}x_j, u_{1,p}u_{2,p}, u_{2,1}u_{3,1}, \ldots, u_{n,p}u_{1,1}; 1 \le i \le p-1, 1 \le j \le n\}$, ∀ n≥3

Therefore |V(G)|= n(p+1) and
|E(G)|= n(2p-1) +n = 2np

Define f: V(G) → {1,2, 3, …, n(p+1)}, ∀ n ≥ 3 by
f $(u_{1,p})$ =1, f $(u_{2,p})$ =p+2, f $(u_{3,p})$ = 2p+3, …, f $(u_{n,p})$ = (n-1) p + n, ∀ n≥3
f $(u_{1,i})$ = i+1 for $1 \le i \le p-1$,
f $(u_{2,i})$ = p+2+i for $1 \le i \le p-1$,
.
.
.
f$(u_{n,i})$ = (n-1) p + n +i for $1 \le i \le p-1$

f(x_1) =p+1, f (x_2) =2p+2, …, f(x_n)= np+n.

Then the induced edge labeling is as follows f^*: E(G) → {0, 1} is given by

$$f^*(u_{j,i}u_{j,p}) = \begin{cases} 0 \ \textit{if} \ i \ \textit{is even and} \ 1 \le i \le p-1, 1 \le j \le n \\ 1 \ \textit{if} \ i \ \textit{is odd and} \ 1 \le i \le p-1, 1 \le j \le n; \end{cases}$$

$$f^*(u_{j,i}x_j) = \begin{cases} 0 \ \textit{if} \ i \ \textit{is odd and} \ 1 \le i \le p-1, 1 \le j \le n \\ 1 \ \textit{if} \ i \ \textit{is even and} \ 1 \le i \le p-1, 1 \le j \le n; \end{cases}$$

$f^*(u_{j,p}x_j)$ =1, $f^*(u_{j,p}u_{j+1,p})$ =0, $1 \le j \le n-1$
$f^*(u_{n,p}u_{1,p})$ =0.
we note that
$e_f(0)$ = n(p-1) +n=np and $e_f(1)$ = n(p)
then $|e_f(0) - e_f(1)| \le 1$
Hence G is a SDC graph. □

Example 3.2:
A SDC labeling for G= C (4. ($\Gamma(Z_{2p}) + \Gamma(Z_4)$)), is given in figure 1 where n=4, p=3
V(G) = $\{u_{1,1}, u_{1,2}, u_{1,3}, x_1, u_{2,1}, u_{2,2}, u_{2,3}, x_2, u_{3,1}, u_{3,2}, u_{3,3}, x_3, u_{4,1}, u_{4,2}, u_{4,3}\}$, ∀ n≥3
E(G)= $\{u_{j,i}u_{j,3}, u_{j,i}x_j, u_{j,3}x_j, u_{1,3}u_{2,3}, u_{2,p}u_{3,3}, \ldots, u_{4,p}u_{1,3}; 1 \le i \le 2, 1 \le j \le 4\}$,
Therefore, | V(G)|=16 and | E (G) |= 24
Here $e_{f(0)}$ =12 and $e_{f(1)}$ =12
Hence $|e_f(0) - e_f(1)| \le 1$ then C (4. ($\Gamma(Z_{2p}) + \Gamma(Z_4)$)), is a SDC graph.

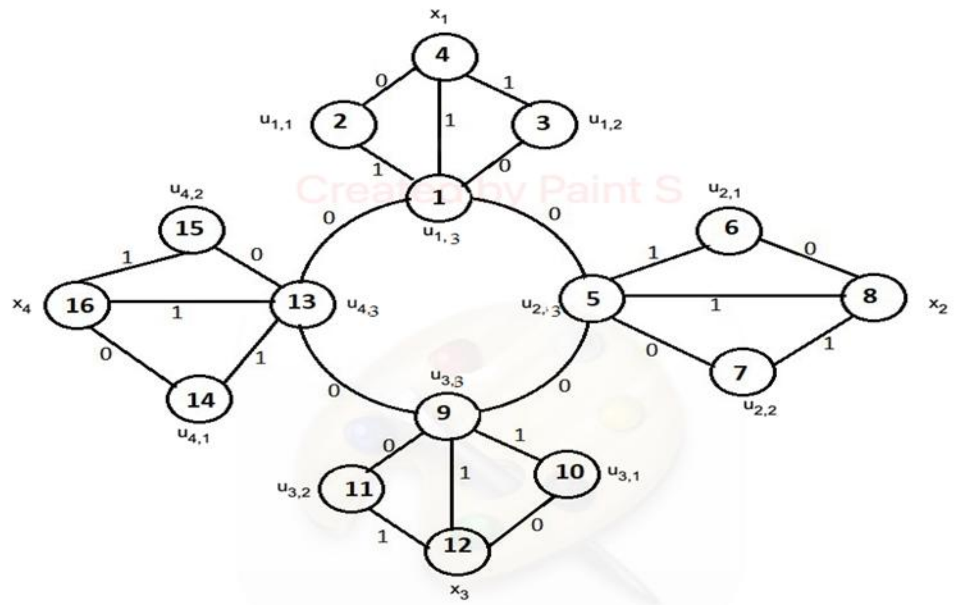

Figure 1. C (4. (Γ(Z_{2p}) + Γ(Z_4)))

Theorem 3.3. For any prime number p>2, cycle of join zero-devisor graphs $\Gamma(Z_{2p}) + \Gamma(Z_6)$ where n≥3 is a SDC graph.
Proof: Let G = C (5. ($\Gamma(Z_{2p}) + \Gamma(Z_6)$))
Let the vertex set and edge set of the graph G are
V(G)= $\{u_{1,1}, u_{1,2}, \ldots u_{1,p}, x_1, y_1, z_1, u_{2,2}, \ldots u_{2,p}, x_2, y_2, z_2, u_{n,1}, u_{n,2}, \ldots u_{n,p}, x_n, y_n, z_n\}$
and E(G)= $\{u_{j,i}u_{j,p}, u_{j,i}x_j, u_{j,p}x_j, u_{j,i}y_j, u_{j,p}y_j, u_{j,p}z_j, x_jy_j, y_jz_j, u_{1,p}u_{2,p}, u_{2,p}u_{3,p}, \ldots, u_{n,p}u_{1,1}; 1 \le i \le p-1, 1 \le j \le n\}$

Therefore |V(G)|=n(p+3) and
|E(G)|=n(4p+1) +n =4np+2n =2n(2p+1)

Define the vertex labeling
f: V(G) → {1, 2, 3…n(p+3)} by
f ($u_{1,i}$) = i+1 for $1 \le i \le p-1$,
f ($u_{2,i}$) =p+4+i for $1 \le i \le p-1$,
f ($u_{3,i}$) = 2p+7+i for $1 \le i \le p-1$,
.
.
.
f($u_{n,i}$)=(n-1)p+(3n-2)+i for $1 \le i \le p-1$,
f ($u_{1,p}$) = 1, f ($u_{2,p}$) =p+4, f ($u_{3,p}$) = 2p+7, …, f ($u_{n,p}$) =(n-1) p+(3n-2),
f(x_1) = p+1, f(x_2) =2p+4, f(x_3) = 3p+7, …, f(x_n)= np+3n-2,

$f(y_1) = p+3$, $f(y_2) = 2p+6$, $f(y_3) = 3p+9$, …, $f(y_n) = np+3n$,
$f(z_1) = p+2$, $f(z_2) = 2p+5$, $f(z_3) = 3p+8$, …, $f(z_n) = np+3n-1$.

Then the induced edge labeling is as follows $f^*: E(G) \rightarrow \{0, 1\}$ is given by

$$f^*(u_{j,i}u_{j,p}) = \begin{cases} 0, & if\ i\ is\ even\ and\ 1 \le i \le p-1, 1 \le j \le n \\ 1, & if\ i\ is\ odd\ and\ 1 \le i \le p-1, 1 \le j \le n; \end{cases}$$

$$f^*(u_{j,i}x_j) = \begin{cases} 0, & if\ i\ is\ odd\ and\ 1 \le i \le p-1, 1 \le j \le n \\ 1, & if\ i\ is\ even\ and\ 1 \le i \le p-1, 1 \le j \le n; \end{cases}$$

$$f^*(u_{j,i}y_j) = \begin{cases} 1, & if\ i\ is\ even\ and\ 1 \le i \le p-1, 1 \le j \le n \\ 0, & if\ i\ is\ odd\ and\ 1 \le i \le p-1, 1 \le j \le n; \end{cases}$$

$$f^*(u_{j,i}z_j) = \begin{cases} 1, & if\ i\ is\ even\ and\ 1 \le i \le p-1, 1 \le j \le 3 \\ 0, & if\ i\ is\ odd\ and\ 1 \le i \le p-1, 1 \le j \le 3; \end{cases}$$

$f^*(u_{j,p}x_j) = 1$, $f^*(u_{j,p}y_j) = 1$, $f^*(u_{j,p}z_j) = 0$, $f^*(x_jy_j) = 0$, $f^*(y_jz_j) = 1$, $f^*(u_{j,p}u_{j+1\,p}) = 0$, $1 \le j \le n-1$

$f^*(u_{n-1,p}u_{1,p}) = 0$
we observe that $e_f(0) = 2np+n = (2p+1)n$ and $e_f(1) = n(2p+1)$ then $|e_f(0) - e_f(1)| \le 1$
Hence G is a SDC graph. □

Example 3.4.
A SDC labeling of the $C(5.\Gamma(Z_6) + \Gamma(Z_6))$ is given in figure 2 where n=5, p=3.
Let $G = C(5.\Gamma(Z_6) + \Gamma(Z_6))$
$V(G) = \{u_{1,1}, u_{1,2}, u_{1,3}, x_1, y_1, z_1, u_{2,1}, u_{2,2}, u_{2,3}, x_2, y_2, z_2, u_{3,1}, u_{3,2}, u_{3,3}, x_3, y_3, z_3, u_{4,1}, u_{4,2}, u_{4,3}, x_3, y_3, z_3, u_{5,1}, u_{5,2}, u_{5,3}, x_5, y_5, z_5\}$, and
$E(G) = \{u_{j,i}u_{j,3}, u_{j,i}x_j, u_{j,3}x_j, u_{j,i}y_j, u_{j,3}y_j, u_{j,3}z_j, x_jy_j, y_jz_j, u_{1,3}u_{2,3}, u_{2,3}u_{3,3}, \ldots, u_{5,3}u_{1,1}; 1 \le i \le 2, 1 \le j \le 5\}$
Therefore, $|V(G)| = 30$ and $|E(G)| = 70$
Here $e_{f(0)} = 35$ and $e_{f(1)} = 35$
Hence $|e_f(0) - e_f(1)| \le 1$ then $C(5.(\Gamma(Z_6) + \Gamma(Z_6))$ is a SDC graph.

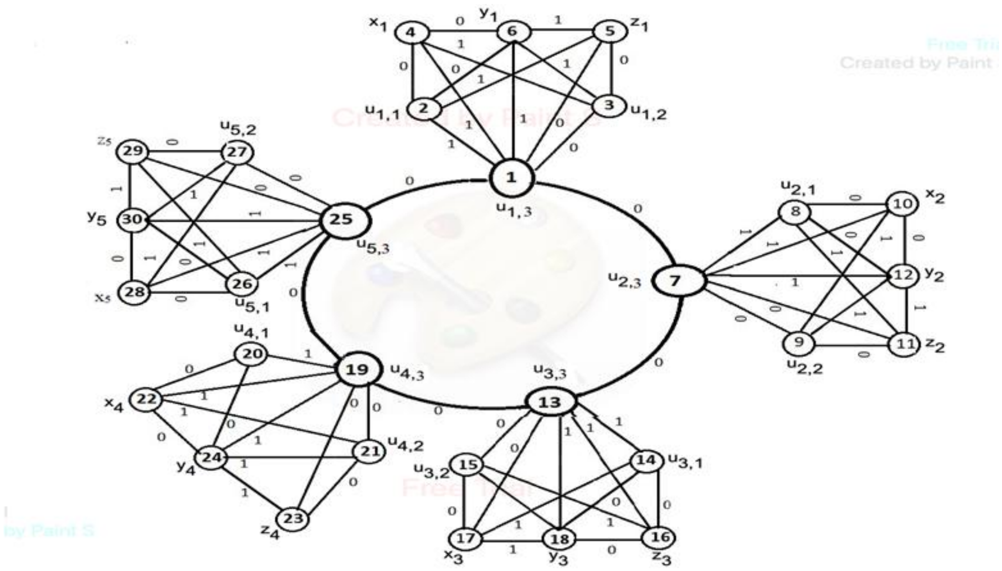

Figure 2. C (5. Γ(Z₆) + Γ(Z₆))

Theorem 3.5. For any prime number p>2, cycle of join zero-devisor graphs $\Gamma(Z_{2p}) + \Gamma(Z_9)$ where $n \ge 3$ does not admit SDC labeling.
□

Example 3.6:
A SDC labeling of the C (4. ($\Gamma(Z_6) + \Gamma(Z_9)$)) is given in figure 3 where n=4, p=3.
Let G= C (4. ($\Gamma(Z_6) + \Gamma(Z_9)$))
V(G) = {$u_{1,1}$, $u_{1,2}$, $u_{1,3}$, x_1, y_1, $u_{2,1}$, $u_{2,2}$, $u_{2,3}$, x_2, y_2, $u_{3,1}$, $u_{3,2}$, $u_{3,3}$, x_3, y_3, $u_{4,1}$, $u_{4,2}$, $u_{4,3}$, x_4, y_4} and
E(G) = {$u_{j,i}u_{j,3}$, $u_{j,i}x_j$, $u_{j,3}x_j$, $u_{j,i}y_j$, $u_{j,3}y_j$, x_jy_j, $u_{1,3}$ $u_{2,3}$, $u_{2,3}u_{3,3}$, …, $u_{4,3}$ u_{11} ; $1 \leq i \leq 2, 1 \leq j \leq 4$}
Therefore, | V (G) |=20 and | E (G) |= 40
Here $e_{f(0)}$ =16 and $e_{f(1)}$ =24
Hence $|e_f(0) - e_f(1)| \nleq 1$ then C (4. ($\Gamma(Z_6) + \Gamma(Z_6)$)) is not a SDC graph.

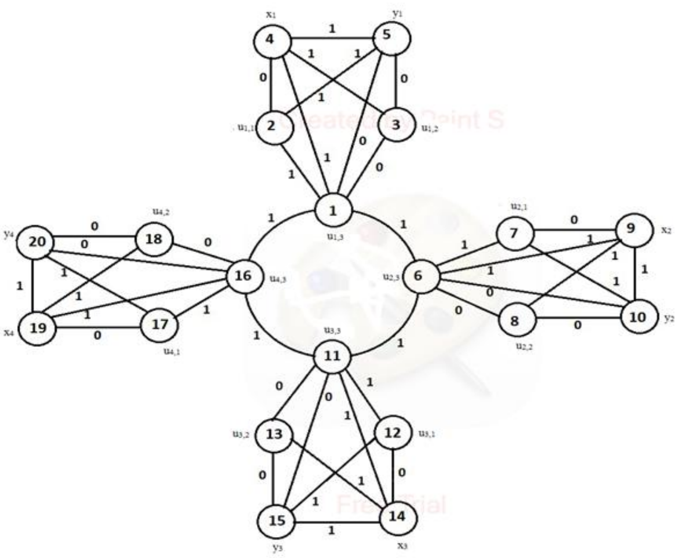

Figure 3. C (4. ($\Gamma(Z_6) + \Gamma(Z_9)$))

Theorem 3.7. For any prime number p >2, cycle of $\Gamma(Z_{p^2})$ does not admit the SDC labeling.
□

Example 3.8:
A SDC labeling of the C (4. $\Gamma(Z_{5^2})$) is given in figure 4 where n=4, p=5.
Let G= C (4. $\Gamma(Z_{5^2})$),
V(G) = {$u_{1,1}$, $u_{1,2}$, $u_{1,3}$, $u_{1,4}$, $u_{2,1}$, $u_{2,2}$, $u_{2,3}$, $u_{2,4}$, $u_{3,1}$, $u_{3,2}$, $u_{3,3}$, $u_{3,4}$, $u_{4,1}$, $u_{4,2}$, $u_{4,3}$, $u_{4,4}$} and
E(G) = {$u_{j,1}$ $u_{j,2}$, $u_{j,1}$ $u_{j,3}$, $u_{j,4}$ $u_{j,2}$, $u_{j,4}$ $u_{j,3}$, $u_{j,2}$ $u_{j,3}$, $u_{j,1}$ $u_{j,4}$; $1 \leq j \leq 4$}
Therefore, | V (G) |=16 and | E (G) |= 28
Here $e_{f(0)}$ =12 and $e_{f(1)}$ =16
Hence $|e_f(0) - e_f(1)| \nleq 1$ then C (4. ($\Gamma(Z_{5^2})$)) is not a SDC graph.

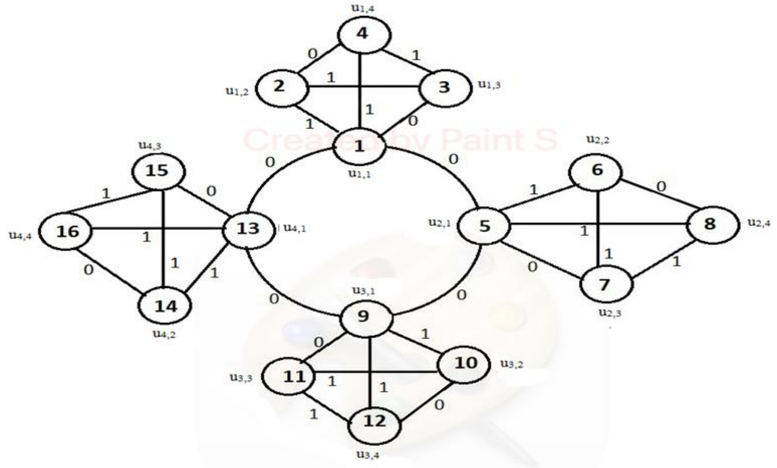

Figure 4. C (4.$\Gamma(Z_{5^2})$)

Theorem 3.9. For any prime number p>2, cycle of join graph, C (3. ($\overline{\Gamma(Z_{p^2})}$+ Γ ($Z_4$))) is a SDC graph.

□

Theorem 3.10. For any prime number p>2, cycle of join graph, C (3. ($\overline{\Gamma(Z_{p^2})}$ + Γ (Z_6)) is a SDC graph.

Proof: Let G = C (3. ($\overline{\Gamma(Z_{p^2})}$ + Γ (Z_6))

Then vertex set of the graph G is

V(G)= $\{u_{1,1}, u_{1,2}, \ldots, u_{1,p-1}, x_1, y_1, z_1, u_{2,1}, u_{2,2}, \ldots, u_{2,p-1}, x_2, y_2, z_2, u_{3,1}, u_{3,2}, \ldots, u_{3,p-1}, x_3, y_3, z_3\}$

And the edge set of the graph G is

E(G) = $\{u_{j,i}x_j, u_{j,i}y_j, u_{j,i}z_j, x_jy_j, y_jz_j, u_{1,1}u_{2,1}, u_{2,1}u_{3,1}, u_{3,1}u_{1,1}; 1 \le i \le p-1 \text{and } 1 \le j \le 3\}$

Therefore |v(G)|= 3p+6 and

|E(G)|= 9p

Define the vertex labeling f: V(G)→ {1, 2, …3p+6}

By f ($u_{1,i}$) = i for $1 \le i \le p-1$,

f ($u_{2,i}$) =p+2+i for $1 \le i \le p-1$,

f ($u_{3,i}$) = 2p+4+i for $1 \le i \le p-1$,

f(x_1) = p, f(y_1) = p+2, f(z_1) = p+1

f(x_2) = 2p+2, f(y_2) =2p+4, f(z_2) =2p+3, f(x_3) = 3p+4, f(y_3) =3p+6, f(z_3) =3p+5.

then the induced edge labeling is given by f^*: E(G) → {0, 1} is given by

$$f^*(u_{j,i}x_j)=\begin{cases} 0 \text{ if } i \text{ is odd and } 1 \le i \le p-1, 1 \le j \le 3 \\ 1 \text{ if } i \text{ is even and } 1 \le i \le p-1, 1 \le j \le 3, \end{cases}$$

$$f^*(u_{j,i}y_j)=\begin{cases} 0 \text{ if } i \text{ is odd and } 1 \le i \le p-1, 1 \le j \le 3 \\ 1 \text{ if } i \text{ is even and } 1 \le i \le p-1, 1 \le j \le 3, \end{cases}$$

$$f^*(u_{j,i}z_j)=\begin{cases} 1 \text{ if } i \text{ is odd and } 1 \le i \le p-1, 1 \le j \le 3 \\ 0 \text{ if } i \text{ is even and } 1 \le i \le p-1, 1 \le j \le 3, \end{cases}$$

$f^*(x_jy_j)=0$, $f^*(y_jz_j)=1$, $f^*(u_{1,1}u_{2,1})=1$, $f^*(u_{2,1}u_{3,1})=1$, $f^*(u_{3,1}u_{1,1})=0$.

we note that $e_f(0) = 3\left(\frac{3p-1}{2}\right)+1 = \frac{9p-1}{2}$ and $e_f(1) = 3\left(\frac{3p-1}{2}\right)+2 = \frac{9p+1}{2}$,

Thus $|e_f(0)-e_f(1)| \le 1$

Hence G is a SDC graph. □

Example 3.11:

A SDC labeling of the C (3. ($\overline{\Gamma(Z_{49})}$ + Γ(Z_6))) is given in figure 5 where n=3, p=7.

Let G= C (3. ($\overline{\Gamma(Z_{49})}$ + Γ(Z_6)))

V(G)= $\{u_{1,1}, u_{1,2}, \ldots, u_{1,6}, x_1, y_1, z_1, u_{2,1}, u_{2,2}, \ldots, u_{2,6}, x_2, y_2, z_2, u_{3,1}, u_{3,2}, \ldots, u_{3,6}, x_3, y_3, z_3\}$

E(G) = $\{u_{j,i}x_j, u_{j,i}y_j, u_{j,i}z_j, x_jy_j, y_jz_j, u_{1,1}u_{2,1}, u_{2,1}u_{3,1}, u_{3,1}u_{1,1}; 1 \le i \le 6 \text{ and } 1 \le j \le 3\}$

Therefore, | V (G) |=27 and | E (G) |= 63

Here $e_{f(0)}$ =31 and $e_{f(1)}$ =32

Hence $|e_f(0)-e_f(1)| \le 1$then C (3. ($\overline{\Gamma(Z_{49})}$ + Γ(Z_6))) is a SDC graph.

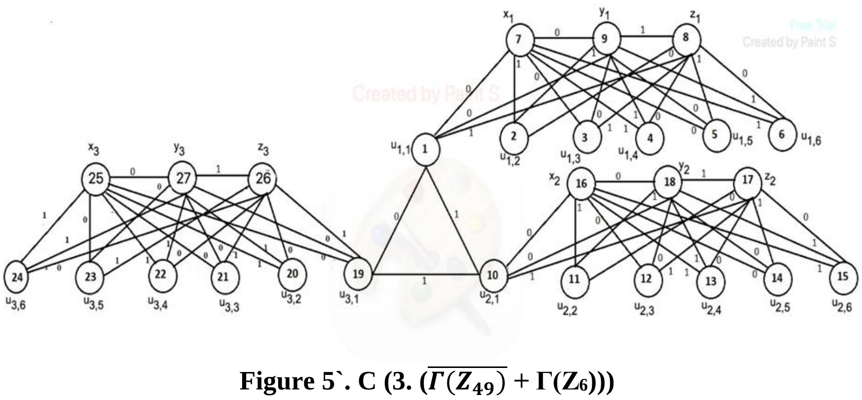

Figure 5`. C (3. ($\overline{\Gamma(Z_{49})}$ + Γ(Z_6)))

Theorem 3.12.

For any prime number p>2, cycle of join graph $\overline{\Gamma(Z_{p^2})}$ + Γ(Z_9) is a SDC graph when n ≥3.

Proof:

Let G = C (n. ($\overline{\Gamma(Z_{p^2})}$ + Γ (Z_9)))

The vertex set of the graph G is

V(G)= $\{u_{1,1}, u_{1,2}, \ldots, u_{1,\,p-1}, x_1, y_1, u_{2,1}, u_{2,2}, \ldots, u_{2,\,p-1}, x_2, y_2, \ldots, u_{n,1}, u_{n,2}, \ldots u_{n,\,p-1}, x_n, y_n,\}$

E(G)= $\{u_{j,\,i}x_j, u_{j,i}y_j, x_jy_j, u_{j,i}u_{j+1,i};\ 1 \le i \le p-1, 1 \le j \le n\}$

Therefore |V(G)| = n(p+1) and

|E(G)| = n (2p-1) + n = 2np

Define the vertex labeling.

f: V(G) → {1, 2, 3, …, n(p+1)} by

f ($u_{1,\,i}$) = i for $1 \le i \le p-1$,

f ($u_{2,\,i}$) =p+1+i for $1 \le i \le p-1$,

f ($u_{3,\,i}$) = 2p+2+i for $1 \le i \le p-1$,

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.

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f($u_{n,i}$)=(n-1)p+(n-1)+i for $1 \le i \le p-1$,

f(x_1) = p, f(x_2) = 2p+1, f(x_3) = 3p+2, …, f(x_n)= np+(n-1),

f(y_1) = p+1, f(y_2) =2p+2, f(y_3) =3p+3, …, f(y_n)= np+n.

Then the induced edge labeling is given by

$$f^*(u_{j,i}x_j)=\begin{cases} 0, & if\ i\ is\ odd\ and\ 1 \le i \le p-1, 1 \le j \le n \\ 1, & if\ i\ is\ even\ and\ 1 \le i \le p-1, 1 \le j \le n; \end{cases}$$

$$f^*(u_{j,i}y_j)=\begin{cases} 0, & if\ i\ is\ even\ and\ 1 \le i \le p-1, 1 \le j \le n \\ 1, & if\ i\ is\ odd\ and\ 1 \le i \le p-1, 1 \le j \le n; \end{cases}$$

$f^*(x_jy_j)=1$, $f^*(u_{j1}u_{j+1,\,1}) =0$, $f^*(u_{n1}u_{11}) = 0$.

we observe that $e_f(0)$ =n(p-1) +n=np and $e_f(1)$ =np Thus $|e_f(0) - e_f(1)| \le 1$

Hence G is a SDC graph. □

Example 3.13:

A SDC labeling of the C (4. ($\overline{\Gamma(Z_{25})}$ + Γ(Z_9))) is given in figure 6 where n=4, p=5.

Let G= C (4. ($\overline{\Gamma(Z_{25})}$ + Γ(Z_9))

V(G)= $\{u_{1,1}, u_{1,2}, \ldots, u_{1,\,4}, x_1, y_1, u_{2,1}, u_{2,2}, \ldots, u_{2,\,4}, x_2, y_2, \ldots, u_{4,1}, u_{4,2}, \ldots, u_{4,4}, x_4, y_4,\}$

E(G)= $\{u_{j,\,i}x_j, u_{j\,,i}y_j, x_jy_j, u_{j,1}u_{j+1,1}, u_{4,1}u_{1,\,1};\ 1 \le i \le 4, 1 \le j \le 4\}$

Therefore, | V (G)|=24 and | E (G) |= 40

Here $e_{f\,(0)}$ =20 and $e_{f\,(1)}$ =20

Hence $|e_f(0)-e_f(1)| \le 1$ then C (4. ($\overline{\Gamma(Z_{25})}$ + Γ(Z_9))) is a SDC graph.

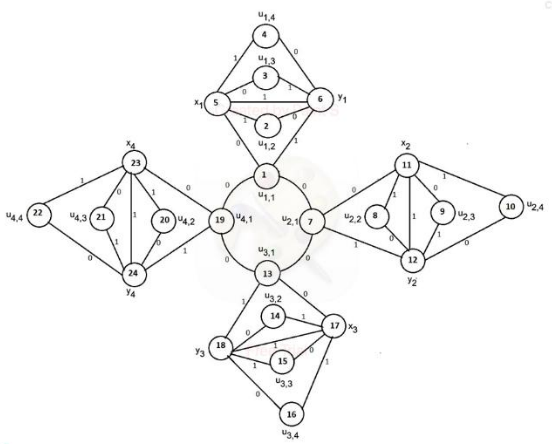

Figure 6. C (4. ($\overline{\Gamma(Z_{25})}$ + Γ(Z_9)))

Remark: In join graphs we used only paths in zero-divisor graphs, Z_4, Z_6, Z_9.

IV. Conclusion

The cycle of graphs are

- C (n. ($\Gamma(Z_{2p}) + \Gamma(Z_4)$)), n ≥ 3 and p > 2 is a SDC graph.
- C (n. ($\Gamma(Z_{2p}) + \Gamma(Z_6)$)), n ≥ 3 and p > 2 is a SDC graph.
- C (n. ($\Gamma(Z_{2p}) + \Gamma(Z_9)$)), n ≥ 3 and p > 2 is not a SDC graph.
- C (n. $\Gamma(Z_{p^2})$), n ≥ 3 and p > 2 is not a SDC graph.
- C (n. ($\overline{\Gamma(Z_{p^2})} + \Gamma(Z_4)$)), n = 3 and p > 2 is a SDC graph.
- C (n. ($\overline{\Gamma(Z_{p^2})} + \Gamma(Z_6)$)), n = 3 and p > 2 is a SDC graph.
- C (n. ($\overline{\Gamma(Z_{p^2})} + \Gamma(Z_9)$)), n ≥ 3 and p > 2 is a SDC graph.

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