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A Revised TOPSIS with Weighted Euclidean Distances

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Abstract
Multi-Criteria Decision Making (MCDM) techniques have attracted significant attention from researchers and professionals across diverse industries due to their ability to effectively evaluate, assess, and rank alternatives. However, choosing the most suitable MCDM method for a specific problem can be challenging, especially when several methods appear equally applicable. Among these methods, the Technique for Order of Performance by Similarity to Ideal Solution (TOPSIS) is widely used, leading researchers to propose various improved versions. This research focuses on enhancing the traditional TOPSIS method by integrating criteria weights along with performance ratings and alternative weights using Euclidean distances. This integration enables decision-makers to prioritize criteria based on their relative importance, ensuring that the distance calculation accurately reflects these priorities. Through this enhancement, the proposed approach aims to streamline and strengthen decision-making processes, addressing the complexities associated with selecting the optimal MCDM method for a given problem. In this study, both the traditional TOPSIS method and a revised version are applied to car selection using hypothetical data. The study evaluates the best automotive car based on specific criteria.
Keywords: *TOPSIS, Revised TOPSIS, Euclidean distance, Weighted Euclidean distance, Simulation comparison.*

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I. Introduction

Selecting an appropriate Multi-Criteria Decision Making (MCDM) method for a given MCDM problem is always a challenging task [50]. The need for comparative comparison for methods during selection has been highlighted in studies [11]. In the realm of MCDM, the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) is widely respected, utilized, and embraced as an MCDM approach because of its straightforwardness and core principle that the optimal solution is the one that is nearest to the positive ideal solution and farthest from the negative ideal solution [52]. A modified version of TOPSIS called Modified TOPSIS was created incorporating an entropy-based method for determining objective weights, under the premise that subjective weighting may not always be practical. This variant applies criteria weights in a distinct manner compared to TOPSIS when addressing MCDM problems [14]. TOPSIS has been widely employed in practical MCDM scenarios owing to its robust mathematical basis, simplicity, and straightforward applicability [51]. TOPSIS has served as a catalyst for numerous new methods and comparative analyses derived from it [53], establishing itself as a cornerstone among MCDM approaches [31]. TOPSIS has been extensively utilized in various domains such as decision-making for purchases and selecting outsource providers [24, 46], manufacturing decision processes [1, 42], analysis of financial performance [16], assessment of service quality [37], educational selection processes [39], technology selection tasks [27], material selection procedures [13], product selection scenarios [2], strategy evaluations [48], and critical mission planning [47]. The modified TOPSIS variant has been employed in comparative studies related to estimating criteria weights [17] and in developing objective composite indices [33]. This methodology has also found applications in resource management [36], software selection [7], environmental assessment [55], sustainability evaluations [54, 38], material selection processes [23], machine selection procedures [40], technology assessments [35], and in the development of various methods [56, 49]. The revised TOPSIS method represents a significant advancement by integrating not only criteria weights but also performance ratings and alternative weights through the calculation of Euclidean distances. This integration is designed with the specific aim of improving the accuracy and relevance of decision-making processes. Consequently, it becomes imperative to undertake a comprehensive evaluation and comparative analysis of these two methods to substantiate their suitability and delineate their respective applications in practical decision-making scenarios. This comparative analysis will shed light on the strengths, limitations, and unique contributions of each method, thereby providing valuable insights for decision-makers and researchers alike. In following sections, the TOPSIS model is first presented, followed by the revised TOPSIS model. Case study-based comparisons are then presented followed by mathematical analysis.

II. The General MCDM Problem and TOPSIS

The general MCDM problem is designed to assess and rank alternatives denoted as A_i (i = 1, . . . m) based on specific criteria C_j (j = 1, 2,…, n) [29]. The set of alternatives (A_i) represents the available options for the decision maker seeking prioritization. Criteria (C_j) constitute the factors influencing the decision maker's ranking of alternatives, and their respective weights W_j (j = 1, 2,…,n)indicate their relative importance. The criteria weights can be represented as a vector [10]:

$$W = [w_1, w_2, . . . ,w_n]^T \tag{1}$$

The decision maker's preferences for each alternative (A_i) with respect to each criterion (C_j) are termed as performance ratings x_{ij} (i = 1, 2, …, m; j = 1, 2,…, n). These ratings are organized into a decision matrix (X) [6] as follows:

$$X = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{pmatrix}. \tag{2}$$

The MCDM problem, is denoted as (X, W), where X is the decision matrix and W is the weight vector [9]. Various MCDM models can be applied to solve the given MCDM problem. Typically, these methods involve a normalization procedure to standardize performance ratings (x_{ij}) into a comparable measurement unit and a score aggregation technique [20]. Normalization transforms performance ratings to ensure uniformity, while a weighted score for each alternative (A_i) is calculated by aggregating weights with performance ratings. The final ranking of alternatives is determined based on the overall score, while integrating both criteria importance and performance evaluations [9].

2.1. The TOPSIS model

Developed in 1981 by Hwang and Yoon, the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) is a widely used method in MCDM that operates on the premise that each criterion demonstrates a consistent tendency of either increasing or decreasing utility [18]. This assumption allows for a clear definition of positive and negative ideal solutions, forming the basis for assessing the relative proximity of alternatives to these ideals. In the realm of MCDM, the TOPSIS stands out as a highly respected, widely utilized, and embraced approach [44]. Its popularity stems from its straightforwardness and adherence to a core principle: the optimal solution is the one that is closest to the positive ideal solution and farthest from the negative ideal solution [43].

TOPSIS operates on the premise that in decision-making involving multiple criteria, the criteria need to be evaluated simultaneously. These criteria can range from cost and efficiency to quality and sustainability, depending on the context of the decision [15]. TOPSIS facilitates this complex decision-making process by providing a systematic framework to rank and select the best alternatives from a set of options [28]. One of the key strengths of TOPSIS lies in its ability to handle both quantitative and qualitative data, making it applicable across various domains and industries. It transforms raw data into normalized decision matrices, where each criterion's importance is weighted according to its significance in the decision-making process [30]. This normalization process ensures a fair and unbiased evaluation of alternatives. The heart of TOPSIS lies in its calculation of the "closeness" of each alternative to the ideal solutions [53]. The positive ideal solution represents the best possible value for each criterion, while the negative ideal solution represents the worst possible value [32].

TOPSIS calculates the distance of each alternative to these ideal solutions using mathematical methods such as Euclidean distance or other similarity measures [25], By comparing the distances of alternatives to the positive and negative ideal solutions, TOPSIS generates a ranking that identifies the most desirable alternative, based on the one closest to the positive ideal solution and farthest from the negative ideal solution [26]. This ranking aids decision-makers in selecting the most suitable course of action or investment among competing alternatives. Furthermore, TOPSIS offers flexibility in sensitivity analysis, allowing decision-makers to assess the impact of changes in criteria weights or data inputs on the final rankings [4]. This feature enhances decision robustness and helps stakeholders understand the trade-offs involved in different decision scenarios [8].

Let a set of alternatives as A_i (i = 1, . . .m), and a set of criteria C_j (j = 1, 2,…, n), be given, and let W_j (j= 1, 2,…, n) be a set of criteria weights, where $W_j > 0$ and $\sum_{j=1}^{n} w_j = 1$. Let $X = [\![x_{ij}]\!]_{m \times n}$ represent the decision matrix in where x_{ij} is the performance of the ith alternative with respect to the jth criterion [41]. The TOPSIS method is as follows:

Step 1: *Formation of a Decision Matrix*

The performance ratings x_{ij} (i = 1, 2, , m; j = 1, 2, …, n) of the preferences for each alternative A_i with respect to each criterion C_j [31] are organized into a decision matrix|:

$$X = [\![x_{ij}]\!]_{m \times n} \tag{1}$$

Step 2: *Construction of Normalized Decision Matrix*

Vector Normalisation is applied to obtain normalised performance ratings from (1). In this procedure, each performance rating x_{ij} in X is divided by its norm. The normalized ratings r_{ij} (i = 1, 2, . . ., m; j = 1, 2, . . ., n) are obtained as follows:

$$R = (r_{ij}),$$

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{n} x_{ij}^2}}, \; (i = 1, 2, . . .,m, j = 1, 2, . . . , n) \tag{2}$$

This conversion method facilitates comparison among criteria more effortlessly using dimensionless units. Nonetheless, it faces difficulties in enabling direct comparison because of varying scale lengths [43].

Step 3*: Construction of Weighted Normalized Decision Matrix*

The weighted normalised performance ratings u_{ij} (i = 1, 2,…, m; j= 1, 2,…,n) is calculated as follows. These weighted ratings are combined to form the weighted-normalised decision matrix U.

$$u_{ij} = r_{ij} \times w_j , \; i = 1, 2, . . .,m, j = 1, 2, . . . , n \tag{3}$$

Step 4*: Determination of Positive Ideal and Negative Ideal Solution values*

Determine the Positive ideal solution (PIS) values (A^+) and the Negative ideal solution (NIS) values (A^-). The PIS is the best performance of all alternatives based on each criterion, and the NIS is the worst performance of all alternatives based on each criteria. The criteria set involves two subset, beneficial criteria set, and non-beneficial criteria set [57].

Positive ideal solution (PIS) values: A^+ = {best u_{ij} },

$$A^+ = \{u_1^+, u_2^+, ..., u_n^+\} . \tag{4(a)}$$

Negative ideal solution (NIS) values:

$$A^- = \{\text{worst } u_{ij} \},$$

$$A^- = \{u_1^-, u_2^-, . ..,u_n^-\} . \tag{4(b)}$$

where, (i) u_j^+ = max u_{ij} , if j is beneficial criteria; u_j^+ = min u_{ij} , if j is non-beneficial criteria.

(ii) u_j^- = min u_{ij} , if j is beneficial criteria; u_j^- = max u_{ij} , if j is non-beneficial criteria.

Step 5: *Obtain the Separation Values*

The measure of separation involves determining the distance of every alternative rating from both the PIS and NIS, utilizing the principles of Euclidean distance theory (Huang *et al*., 2018). The following outlines the steps for calculating PIS (S_i^+) and NIS (S_i^-) separations separately.

$$S_i^+ = \sqrt{\sum_{j=1}^{n} (u_{ij} - u_j^+)^2} \qquad (5a)$$

$$S_i^- = \sqrt{\sum_{j=1}^{n} (u_{ij} - u_j^-)^2} \qquad (5b)$$

Step 6: *Calculate the overall Preference Score*

The overall preference score φ_i for each alternative A_i is calculated as

$$\varphi_i = \frac{S_i^-}{S_i^+ + S_i^-}, \ i = 1, \ . \ . \ . \ , m; \qquad (6)$$

where $0 \leq \varphi_i \leq 1$. The larger the index value (φ_i), the better the performance of the alternative.

Step 7: *Ranking Alternatives by the Preference Scores*

A set of alternatives can now be ranked in descending order according to the value of φ_i.

2.2. The Revised TOPSIS model

In the conventional TOPSIS, only the decision matrix is impacted by weights w_j. Suppose that the Euclidean distance computation is altered by an extra weight α.

Modified separation measures:

$$E_i^+ = \sqrt{\sum \alpha_j (v_{ij} - v_j^+)^2}, \qquad E_i^- = \sqrt{\sum \alpha_j (v_{ij} - v_j^-)^2}.$$

The relative significance of the separation measure itself for criteria *j* is captured by α_j in this case. When certain criteria have a greater influence on the closeness measures than others, assigning weights α_j to the Euclidean distance alters the influence of the separation measures on its final ranking, which may result in a better discriminating among alternatives.

Let the discrimination capability $D(C_i)$ of an alternative *i* be the ability of C_i to distinguish between close alternatives. Using conventional TOPSIS:

$D(C_i) = |C_i - C_k|$ for alternatives *i* and *k*.

Using modified TOPSIS

$D'(C_i) = |C_i' - C_k'|$, where C_i' incorporates the additional weight α_j.

The modified TOPSIS increases the sensitivity of the closeness measure by scaling the Euclidean distance through α_j.

i. Assume $\alpha_j = 1$ for all *j* initially (reduces to TOPSIS)

ii. Adjust α_j based on the variability in the Euclidean distance calculation, we achieve finer distinctions among alternatives:

$D'(C_i) \succ D(C_i)$ [greater separation between similar rankings]

Let R_i be the rank of alternative *i* under TOPSIS and R_i' be its rank under modified TOPSIS. For a well chosen α_j, the modified TOPSIS should yield

R_i' = true optimal rank of *i*, whereas TOPSIS may misclassify *i* due to inadequate discrimination among alternatives.

By introducing α_j, the flexibility of the model increases, allowing it to a better align rankings with decision-maker preferences and real-world conditions. This improved adaptability is mathematically expressed as

$$\text{Revised } RC = \frac{\sum \alpha_j E_i^-}{\sum \alpha_j E_i^+ + \sum \alpha_j E_i^-},$$

which accommodates more complex adjustments.

Through the assignment of weights to both the Euclidean distance (α_j) and the decision matrix (w_j), the modified TOPSIS provides superior ranks by improving alternative discrimination and alignment with decision-making priorities. The improved sensitivity and adaptability of the separation measures provide the mathematical foundation.

Step 1: *Formation of a Decision Matrix*

A decision matrix *X* is formed from the performance ratings x_{ij} ($i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$) of the DM's preferences for each alternative A_i with respect to each criterion

C_j . These ratings are organized into a decision matrix.

$$X = [\![x_{ij}]\!]_{m \times n} \qquad (1)$$

Step 2: *Construction of Normalized Decision Matrix*

Vector Normalisation is applied to obtain normalised performance ratings from (1). In this procedure, each performance rating x_{ij} in X is divided by its norm. The normalized ratings r_{ij} (i = 1, 2, . . ., m; j = 1, 2, . . ., n) are obtained as follows:

$$R = (r_{ij}),$$

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{n} x_{ij}^2}}, \; (i = 1, 2, \ldots, m, \; j = 1, 2, \ldots, n.) \qquad (2)$$

Step 3: *Construction of Weighted Normalized Decision Matrix*

The weighted normalised performance ratings u_{ij} (i = 1, 2, …, m; j = 1, 2,…,n) is calculated as follows. These weighted ratings are combined to form the weighted-normalised decision matrix U.

$$u_{ij} = r_{ij} \times w_j, \; (i = 1, 2, \ldots, m, \; j = 1, 2, \ldots, n) \qquad (3)$$

Step 4: *Determination of Positive Ideal and Negative Ideal Solution values*

Determine the Positive ideal solution (PIS) values (A^+) and the Negative ideal solution (NIS) values (A^-). The PIS is the best performance of all alternatives based on each criterion, and the NIS is the worst performance of all alternatives based on each criteria. The criteria set involves two subset, beneficial criteria set, and non-beneficial criteria set.

Positive ideal solution (PIS) values: $A^+ = \{\text{best}\, u_{ij}\}$,

$$A^+ = \{u_1^+, u_2^+, \ldots, u_n^+\}. \qquad 4(a)$$

Negative ideal solution (NIS) values:

$$A^- = \{\text{worst}\, u_{ij}\},$$

$$A^- = \{u_1^-, u_2^-, \ldots, u_n^-\}. \qquad 4(b)$$

where, (i) u_j^+ = max u_{ij} , if j is beneficial criteria, u_j^+ = min u_{ij} , if j is non-beneficial criteria.

(ii) u_j^- = min u_{ij} , if j is beneficial criteria, u_j^- = max u_{ij} , if j is non-beneficial criteria.

Step 5: *Calculation of the separation measure from the positive ideal and the negative ideal solutions S_i^+ and S_i^- , respectively using the weighted Euclidean distance metric*. The main purpose of assigning weights to the Euclidean distance in MCDM is to precisely highlight the importance of each criterion within the decision-making process.

$$S_i^+ = \sqrt{\sum_{j=1}^{n} w_j (u_{ij} - u_j^+)^2} \qquad (5a)$$

$$S_i^- = \sqrt{\sum_{j=1}^{n} w_j (u_{ij} - u_j^-)^2}, \qquad (5b)$$

where i = 1, 2,…, m; j = 1, 2,…,n.

Step 6. *Calculate the relative closeness to the ideal solution* (C_i). *The relative closeness of the i*th *alternative A_i with respect to the* PIS *can be expressed as*

$$C_i = \frac{S_i^-}{S_i^+ + S_i^-}, \; i = 1, \ldots, m \qquad (6)$$

where $0 \le C_i \le 1$. The larger the index value (C_i), the better the performance of the alternative.

Step 7. *Rank the preference order*

A set of alternatives can now be preference ranked according to the descending order of the value of C_i.

III. Comparisons of TOPSIS and the Revised TOPSIS

In this study, a detailed comparison is conducted between the traditional TOPSIS method and the revised iteration of TOPSIS. The comparison is carried out under the consideration of two distinct weight configurations, each representing a unique set of criteria. This approach allows for a comprehensive evaluation and understanding of how these methods perform under varying weight distributions, shedding light on their strengths, weaknesses, and potential applicability in different decision-making scenarios. The comparative analysis aim to provide insights into the effectiveness and robustness of both methodologies, contributing to the advancement of decision-making techniques in various domains.

3.1. A comparative analysis based on a practical case study

Mr. John is in the market to purchase a car for his family. To ensure he makes the best choice, he enlists the expertise of a team comprising three decision-makers. The experts start by narrowing down the options to four top automotive car showrooms in the city, denoted as

A = {Civic, Corolla, Swift, Hyundai}. They also collectively determine four evaluation criteria, represented by C = {Style, Safety, Fuel Efficiency, Expenses}, which will be used to select the most suitable automotive car from the available options [57].

Solutions to the problem using equal and non-equal criteria weight settings

Case I: *Equal criteria weight settings*

Solution by TOPSIS model

Step 1: *Formation of the Decision Matrix*

The decision matrix is given in the following table.

Table 1: Decision Matrix $D = [\![x_{ij}]\!]_{m \times n}$

	Style	Safety	Fuel Efficiency	Expenses
Civic	7	9	9	8
Corolla	8	7	8	7
Swift	9	6	8	9
Hyundai	6	7	8	6
$\sqrt{\sum_{j=1}^{n} x_{ij}^2}$	15.17	14.66	16.52	15.17

Step 2: *Construction of Normalized Decision Matrix*

To normalize the decision matrix, divide each entry by $\sqrt{\sum_{j=1}^{n} x_{ij}^2}$

Table 2: Normalized Decision Matrix (*NDM*) $R = (r_{ij})$,

	Style	Safety	Fuel Efficiency	Expenses
Civic	0.46	0.61	0.54	0.53
Corolla	0.53	0.48	0.48	0.46
Swift	0.59	0.41	0.48	0.59
Hyundai	0.40	0.48	0.48	0.40

Step 3: *Computation of the weight matrix*

The weights assigned by the experts (decision makers) to the criteria are given by the matrix

$W = [\, w_1$ (Style) = 0.25, w_2 (Safety) = 0.25, w_3 (Fuel Efficiency) = 0.25, w_4 (Expenses) = 0.25$]^T$

Step 4: *Computation of Weighted Normalized Decision Matrix* (*WNDM*)

To get *WNDM*, multiplying each column of *NDM* in Table 3 by weights w_j, of weight vector computed in the step 3.

Table 3: Weighted Normalized Decision Matrix $U(u_{ij}) = r_{ij} \times w_j$

	Style (0.25)	Safety (0.25)	Fuel Efficiency (0.25)	Expenses (0.25)
Civic	0.115	0.153	0.135	0.133
Corolla	0.133	0.120	0.120	0.115
Swift	0.148	0.103	0.120	0.148
Hyundai	0.100	0.120	0.120	0.100

Step 5: *Calculation of Positive Ideal Solution* (PIS) *and Negative Ideal Solution* (NIS)

To find the PIS (A^+) and NIS (A^-).

Table 4: Positive Ideal Solution

	Benefit Criteria			Cost Criteria
	Style (0.25)	Safety (0.25)	Fuel Efficiency (0.25)	Expenses (0.25)
Civic	0.115	0.153	0.135	0.133
Corolla	0.133	0.120	0.120	0.115
Swift	0.148	0.103	0.120	0.148
Hyundai	0.100	0.120	0.120	0.100
A^+	0.148	0.153	0.135	0.100
A^-	0.100	0.103	0.120	0.148

Step 6: *Determination of the separation measures for each alternative*

Calculating separation from PIS (S_i^+)

Table 5a: Calculation of $S_i^+ = \sqrt{\sum_{j=1}^{n} (u_{ij} - u_j^+)^2}$

	Style	**Safety**	**Fuel Efficiency**	**Expenses**	$\sum_{j=1}^{n} (u_{ij} - u_j^+)^2$	S_i^+
Civic	$(0.115 - 0.148)^2$	$(0.153 - 0.153)^2$	$(0.135 - 0.135)^2$	$(0.133 - 0.100)^2$	0.002178	0.047
Corolla	$(0.133 - 0.148)^2$	$(0.120 - 0.153)^2$	$(0.120 - 0.135)^2$	$(0.115 - 0.100)^2$	0.001764	0.042
Swift	$(0.148 - 0.148)^2$	$(0.103 - 0.153)^2$	$(0.120 - 0.135)^2$	$(0.148 - 0.100)^2$	0.005029	0.071
Hyundai	$(0.100 - 0.148)^2$	$(0.120 - 0.153)^2$	$(0.120 - 0.135)^2$	$(0.100 - 0.100)^2$	0.003618	0.060

Calculating separation from NIS (S_i^-)

Table 5b: Calculation of $S_i^- = \sqrt{\sum_{j=1}^{n} (u_{ij} - u_j^-)^2}$

	Style	**Safety**	**Fuel Efficiency**	**Expenses**	$\sum_{j=1}^{n} (u_{ij} - u_j^-)^2$	S_i^-
Civic	$(0.115 - 0.100)^2$	$(0.153 - 0.103)^2$	$(0.135 - 0.120)^2$	$(0.133 - 0.148)^2$	0.003175	0.056
Corolla	$(0.133 - 0.100)^2$	$(0.120 - 0.103)^2$	$(0.120 - 0.120)^2$	$(0.115 - 0.148)^2$	0.002467	0.050
Swift	$(0.148 - 0.100)^2$	$(0.103 - 0.103)^2$	$(0.120 - 0.120)^2$	$(0.148 - 0.148)^2$	0.002304	0.048
Hyundai	$(0.100 - 0.100)^2$	$(0.120 - 0.103)^2$	$(0.120 - 0.120)^2$	$(0.100 - 0.148)^2$	0.002593	0.051

Step 6: *Calculation of the relative closeness to the ideal solution* (C_i).

The relative closeness to the ideal solution is calculated as follows:

$$C_i = \frac{S_i^-}{S_i^+ + S_i^-}.$$

Civic: $C_1 = \frac{0.056}{0.047 + 0.056} = 0.544$

Corolla: $C_2 = \frac{0.050}{0.042 + 0.050} = 0.543$

Swift: $C_3 = \frac{0.048}{0.071 + 0.048} = 0.403$

Hyundai: $C_4 = \frac{0.051}{0.060 + 0.051} = 0.459$

Step 7. *Rank the preference order*

A set of alternatives can now be preference ranked according to the descending order of the value of C_i. From *Step* 7, it is observed that Civic is the best automotive car since it has the highest C_i value of 0.544; whist Swift is the worst automotive car since it has the least C_i value of 0.403 based on the evaluation criteria. Hence $C_1 \succ C_2 \succ C_4 \succ C_3$.

Solution by revised TOPSIS model

Steps 1 *to* 5 *of the TOPSIS model solution are same under the revised TOPSIS*.

Step 6: *Determination of the separation measures for each alternative using weighted Euclidean distance metric.*

Calculating separation from PIS (S_i^+)

Table 5a: Calculation of $S_i^+ = \sqrt{\sum_{j=1}^{n} w_j (u_{ij} - u_j^+)^2}$

	Style (0.25)	**Safety** (0.25)	**Fuel Efficiency** (0.25)	**Expenses** (0.25)	$\sum_{j=1}^{n} w_j (u_{ij} - u_j^+)^2$	S_i^+
Civic	$(0.115 - 0.148)^2$	$(0.153 - 0.153)^2$	$(0.135 - 0.135)^2$	$(0.133 - 0.100)^2$	0.000545	0.023
Corolla	$(0.133 - 0.148)^2$	$(0.120 - 0.153)^2$	$(0.120 - 0.135)^2$	$(0.115 - 0.100)^2$	0.000409	0.020
Swift	$(0.148 - 0.148)^2$	$(0.103 - 0.153)^2$	$(0.120 - 0.135)^2$	$(0.148 - 0.100)^2$	0.001257	0.035
Hyundai	$(0.100 - 0.148)^2$	$(0.120 - 0.153)^2$	$(0.120 - 0.135)^2$	$(0.100 - 0.100)^2$	0.000905	0.030

Calculating separation from NIS (S_i^-)

Table 5b: Calculation of $S_i^- = \sqrt{\sum_{j=1}^{n} w_j (u_{ij} - u_j^-)^2}$

	Style (0.25)	**Safety** (0.25)	**Fuel Efficiency** (0.25)	**Expenses** (0.25)	$\sum_{j=1}^{n} w_j (u_{ij} - u_j^-)^2$	S_i^-
Civic	$(0.115 - 0.100)^2$	$(0.153 - 0.103)^2$	$(0.135 - 0.120)^2$	$(0.133 - 0.148)^2$	0.007938	0.028
Corolla	$(0.133 - 0.100)^2$	$(0.120 - 0.103)^2$	$(0.120 - 0.120)^2$	$(0.115 - 0.148)^2$	0.000617	0.023
Swift	$(0.148 - 0.100)^2$	$(0.103 - 0.103)^2$	$(0.120 - 0.120)^2$	$(0.148 - 0.148)^2$	0.000576	0.024
Hyundai	$(0.100 - 0.100)^2$	$(0.120 - 0.103)^2$	$(0.120 - 0.120)^2$	$(0.100 - 0.148)^2$	0.000648	0.025

Step 7: *Calculation of the relative closeness to the ideal solution* (C_i).

The relative closeness to the ideal solution is calculated as follows:

$$C_i = \frac{S_i^-}{S_i^+ + S_i^-}.$$

Civic: $C_1 = \frac{0.028}{0.023 + 0.028} = 0.549$ **Corolla:** $C_2 = \frac{0.023}{0.020 + 0.023} = 0.535$

Swift: $C_3 = \frac{0.024}{0.035 + 0.024} = 0.407$ **Hyundai:** $C_4 = \frac{0.025}{0.030 + 0.025} = 0.455$

Step 8. *Rank the preference order*

A set of alternatives can now be preference ranked according to the descending order of the value of C_i. From *Step* 7, it is observed that Civic is the best automotive car since it has the highest C_i value of 0.549, whist Swift is the worst automotive car since it has the least C_i value of 0.407 based on the evaluation criteria. Hence $C_1 \succ C_2 \succ C_4 \succ C_3$.

Case II: *Non-equal criteria weight settings*

Solution by TOPSIS model
Steps 1 *and* 2 *under TOPSIS model solution of equal weights are same under the revised TOPSIS of unequal weights*

Step 3: *Computation of the weight matrix*
The weights assigned by the experts (decision makers) to the criteria are given by the matrix
W = [w_1 (Style) = 0.1, w_2 (Safety) = 0.4, w_3 (Fuel Efficiency) = 0.3, w_4 (Expenses) = 0.2]^T
Step 4: *Computation of Weighted Normalized Decision Matrix* (*WNDM*)
To get *WNDM*, multiplying each column of *NDM* in Table 2 by weights w_j, of weight vector computed in the step 3.

Table 3: Weighted Normalized Decision Matrix $U(u_{ij}) = r_{ij} \times w_j$

	Style (0.1)	Safety (0.4)	Fuel Efficiency (0.3)	Expenses (0.2)
Civic	0.046	0.244	0.162	0.106
Corolla	0.053	0.192	0.144	0.092
Swift	0.059	0.164	0.144	0.118
Hyundai	0.040	0.192	0.144	0.080

Step 5: *Calculation of Positive Ideal Solution* (PIS) *and Negative Ideal Solution* (NIS)
To find the PIS (A^+).

Table 4: PIS (A^+) and NIS (A^-) values

	Benefit Criteria			Cost Criteria
	Style	Safety	Fuel Efficiency	Expenses
Civic	0.046	0.244	0.162	0.106
Corolla	0.053	0.192	0.144	0.092
Swift	0.059	0.164	0.144	0.118
Hyundai	0.040	0.192	0.144	0.080
A^+	0.059	0.244	0.162	0.080
A^-	0.040	0.164	0.144	0.118

Step 6: *Determination of the separation measures for each alternative*
Calculating separation from PIS (S_i^+)

Table 6a: Calculation of $S_i^+ = \sqrt{\sum_{j=1}^{n} (u_{ij} - u_j^+)^2}$

	Style	**Safety**	**Fuel Efficiency**	**Expenses**	$\sum_{j=1}^{n} (u_{ij} - u_j^+)^2$	S_i^+
Civic	$(0.046 - 0.059)^2$	$(0.244 - 0.244)^2$	$(0.162 - 0.162)^2$	$(0.106 - 0.080)^2$	0.000845	0.029
Corolla	$(0.053 - 0.059)^2$	$(0.192 - 0.244)^2$	$(0.144 - 0.162)^2$	$(0.092 - 0.080)^2$	0.003208	0.057
Swift	$(0.059 - 0.059)^2$	$(0.164 - 0.244)^2$	$(0.144 - 0.162)^2$	$(0.118 - 0.080)^2$	0.008168	0.090
Hyundai	$(0.040 - 0.059)^2$	$(0.192 - 0.244)^2$	$(0.144 - 0.162)^2$	$(0.080 - 0.080)^2$	0.003389	0.058

Calculating separation from NIS (S_i^-)

Table 6b: Calculation of $S_i^- = \sqrt{\sum_{j=1}^{n} (u_{ij} - u_j^-)^2}$

Style	**Safety**	**Fuel**	**Efficiency**	**Expenses**	$\sum_{j=1}^{n} (u_{ij} - u_j^-)^2$	S_i^-
Civic	$(0.046 - 0.040)^2$	$(0.244 - 0.164)^2$	$(0.162 - 0.144)^2$	$(0.106 - 0.118)^2$	0.006904	0.083
Corolla	$(0.053 - 0.040)^2$	$(0.192 - 0.164)^2$	$(0.144 - 0.144)^2$	$(0.092 - 0.118)^2$	0.001629	0.040
Swift	$(0.059 - 0.040)^2$	$(0.164 - 0.164)^2$	$(0.144 - 0.144)^2$	$(0.118 - 0.118)^2$	0.000361	0.019
Hyundai	$(0.040 - 0.040)^2$	$(0.192 - 0.164)^2$	$(0.144 - 0.144)^2$	$(0.080 - 0.118)^2$	0.002228	0.047

Step 7: *Calculation of the relative closeness to the ideal solution* (C_i).
The relative closeness to the ideal solution is calculated as follows:

$$C_i = \frac{S_i^-}{S_i^+ + S_i^-}.$$

Civic: $C_1 = \frac{0.083}{0.029 + 0.083} = 0.741$ **Corolla:** $C_2 = \frac{0.040}{0.057 + 0.040} = 0.412$

Swift: $C_3 = \frac{0.019}{0.090 + 0.019} = 0.174$ **Hyundai:** $C_4 = \frac{0.047}{0.058 + 0.047} = 0.448$

Step 8. *Rank the preference order*
A set of alternatives can now be preference ranked according to the descending order of the value of C_i .From *Step* 7, it is observed that Civic is the best automotive car since it has the highest C_i value of 0.741, whist Swift is the worst automotive car since it has the least C_i value of 0.174 based on the evaluation criteria. Hence $C_1 \succ C_4 \succ C_2 \succ C_3$.

Solution by revised TOPSIS model
Steps 1 *to* 5 *under TOPSIS model for unequal weights are same here.*

Step 6: *Determination of the separation measures for each alternative using weighted Euclidean distance metric.*
Calculating separation from PIS (S_i^+)

Table 6a: Calculation of $S_i^+ = \sqrt{\sum_{j=1}^{n} w_j (u_{ij} - u_j^+)^2}$

	Style (0.1)	**Safety** (0.4)	**Fuel Efficiency** (0.3)	**Expenses** (0.2)	$\sum_{j=1}^{n} w_j (u_{ij} - u_j^+)^2$	S_i^+
Civic	$(0.046 - 0.059)^2$	$(0.244 - 0.244)^2$	$(0.162 - 0.162)^2$	$(0.106 - 0.080)^2$	0.000152	0.012
Corolla	$(0.053 - 0.059)^2$	$(0.192 - 0.244)^2$	$(0.144 - 0.162)^2$	$(0.092 - 0.080)^2$	0.001211	0.035
Swift	$(0.059 - 0.059)^2$	$(0.164 - 0.244)^2$	$(0.144 - 0.162)^2$	$(0.118 - 0.080)^2$	0.002946	0.054
Hyundai	$(0.040 - 0.059)^2$	$(0.192 - 0.244)^2$	$(0.144 - 0.162)^2$	$(0.080 - 0.080)^2$	0.001215	0.035

Calculating separation from NIS (S_i^-)

Table 6b: Calculation of $S_i^- = \sqrt{\sum_{j=1}^{n} w_j (u_{ij} - u_j^-)^2}$

	Style (0.1)	**Safety** (0.4)	**Fuel Efficiency** (0.3)	**Expenses** (0.2)	$\sum_{j=1}^{n} w_j (u_{ij} - u_j^-)^2$	S_i^-
Civic	$(0.046 - 0.040)^2$	$(0.244 - 0.164)^2$	$(0.162 - 0.144)^2$	$(0.106 - 0.118)^2$	0.002690	0.052
Corolla	$(0.053 - 0.040)^2$	$(0.192 - 0.164)^2$	$(0.144 - 0.144)^2$	$(0.092 - 0.118)^2$	0.000466	0.022
Swift	$(0.059 - 0.040)^2$	$(0.164 - 0.164)^2$	$(0.144 - 0.144)^2$	$(0.118 - 0.118)^2$	0.000036	0.006
Hyundai	$(0.040 - 0.040)^2$	$(0.192 - 0.164)^2$	$(0.144 - 0.144)^2$	$(0.080 - 0.118)^2$	0.000602	0.025

Step 7: *Calculation of the relative closeness to the ideal solution* (C_i).
The relative closeness to the ideal solution is calculated as follows:

$$C_i = \frac{S_i^-}{S_i^+ + S_i^-}.$$

Civic: $C_1 = \frac{0.052}{0.012 + 0.052} = 0.8125$ **Corolla:** $C_2 = \frac{0.022}{0.035 + 0.022} = 0.3860$

Swift: $C_3 = \frac{0.006}{0.054+0.006} = 0.1000$ **Hyundai:** $C_4 = \frac{0.025}{0.035+0.025} = 0.4167$

Step 8. *Rank the preference order*

The set of alternatives can now be preference ranked according to the descending order of the value of C_i. From *Step* 7, it is observed that Civic is the best automotive car since it has the highest C_i value of 0.8125, whist Swift is the worst automotive car since it has the least C_i value of 0.1000 based on the evaluation criteria. Hence $C_1 \succ C_4 \succ C_2 \succ C_3$.

3.2. Comparison with equal weight settings

3.2.1. Simulation results

The combined evidence from simulations and mathematical scrutiny reinforces the conclusion that under the condition of equal criteria weights, both the TOPSIS and revised TOPSIS methods exhibit identical ranking outcomes. This reaffirms the robustness and reliability of these decision-making methodologies within well-defined parameter settings, enhancing confidence in their applicability in practical decision-making scenarios.

3.2.2. Mathematical demonstration

Expanding the TOPSIS equation $C_i = \frac{S_i^-}{S_i^- + S_i^+}$, where $S_i^+ = \sqrt{\sum_{j=1}^{n}(u_{ij}-u_j^+)^2}$ and $S_i^- = \sqrt{\sum_{j=1}^{n}(u_{ij}-u_j^-)^2}$, the TOPSIS equation becomes

$$C_i = \frac{\sqrt{\sum_{j=1}^{n}(u_{ij}-u_j^-)^2}}{\sqrt{\sum_{j=1}^{n}(u_{ij}-u_j^+)^2} + \sqrt{\sum_{j=1}^{n}(u_{ij}-u_j^-)^2}}. \quad (1a)$$

u_{ij} is the performance score of the weighted normalized decision matrix which is a product of the normalized matrix r_{ij} and criteria weight w_j .ie $u_{ij} = r_{ij} \times w_j$. Hence eqn, (1a) becomes

$$C_i = \frac{\sqrt{\sum_{j=1}^{n}(w_j r_{ij}-w_j r_j^-)^2}}{\sqrt{\sum_{j=1}^{n}(w_j r_{ij}-w_j r_j^+)^2} + \sqrt{\sum_{j=1}^{n}(w_j r_{ij}-w_j r_j^-)^2}}, \quad (2a)$$

Factorizing w_j, eqn. (2a) becomes

$$C_i = \frac{\sqrt{\sum_{j=1}^{n}w_j^2(r_{ij}-r_j^-)^2}}{\sqrt{\sum_{j=1}^{n}w_j^2(r_{ij}-r_j^+)^2} + \sqrt{\sum_{j=1}^{n}w_j^2(r_{ij}-r_j^-)^2}}, \quad (3a)$$

When using equal weights for criteria, applying w_j = w eqn, (3a) becomes

$$\Rightarrow \qquad C_i = \frac{\sqrt{\sum_{j=1}^{n} w^2 (r_{ij} - r_j^-)^2}}{\sqrt{\sum_{j=1}^{n} w^2 (r_{ij} - r_j^+)^2} + \sqrt{\sum_{j=1}^{n} w^2 (r_{ij} - r_j^-)^2}} \tag{4a}$$

But $\sum_{jj=1}^{n} w_{jj} = 1$. Hence eqn. (4a) becomes

$$C_i = \frac{\sqrt{\sum_{j=1}^{n} (r_{ij} - r_j^-)^2}}{\sqrt{\sum_{j=1}^{n} (r_{ij} - r_j^+)^2} + \sqrt{\sum_{j=1}^{n} (r_{ij} - r_j^-)^2}} . \tag{5a}$$

Next, expanding the revised TOPSIS equation $C_i = \frac{S_i^-}{S_i^- + S_i^+}$. Substituting

$S_i^+ = \sqrt{\sum_{j=1}^{n} w_j (u_{ij} - u_j^+)^2}$ and $S_i^- = \sqrt{\sum_{j=1}^{n} w_j (u_{ij} - u_j^-)^2}$, the revised TOPSIS equation becomes

$$C_i = \frac{\sqrt{\sum_{j=1}^{n} w_j (u_{ij} - u_j^-)^2}}{\sqrt{\sum_{j=1}^{n} w_j (u_{ij} - u_j^+)^2} + \sqrt{\sum_{j=1}^{n} w_j (u_{ij} - u_j^-)^2}} , \tag{1b}$$

u_{ij} is the performance score of the weighted normalized decision matrix which is a product of the normalized matrix r_{ij} and criteria weight w_j .ie $u_{ij} = r_{ij} \times w_j$. Hence eqn. (1b) becomes

$$C_i = \frac{\sqrt{\sum_{j=1}^{n} w_j (w_j r_{ij} - w_j r_j^-)^2}}{\sqrt{\sum_{j=1}^{n} w_j (w_j r_{ij} - w_j r_j^+)^2} + \sqrt{\sum_{j=1}^{n} w_j (w_j r_{ij} - w_j r_j^-)^2}} , \tag{2b}$$

$$\Rightarrow \qquad C_i = \frac{\sqrt{\sum_{j=1}^{n} w_j [w_j (r_{ij} - r_j^-)]^2}}{\sqrt{\sum_{j=1}^{n} w_j [w_j (r_{ij} - r_j^+)]^2} + \sqrt{\sum_{j=1}^{n} w_j [w_j (r_{ij} - r_j^-)]^2}} , \tag{3b}$$

Factorizing w_j , eqn. (3b) becomes

$$\Rightarrow \qquad C_i = \frac{\sqrt{\sum_{j=1}^{n} w_j \times w_j^2 (r_{ij} - r_j^-)^2}}{\sqrt{\sum_{j=1}^{n} w_j \times w_j^2 (r_{ij} - r_j^+)^2} + \sqrt{\sum_{j=1}^{n} w_j \times w_j^2 (r_{ij} - r_j^-)^2}} , \tag{4b}$$

$$\Rightarrow \qquad C_i = \frac{\sqrt{\sum_{j=1}^{n} w_j^3 (r_{ij} - r_j^-)^2}}{\sqrt{\sum_{j=1}^{n} w_j^3 (r_{ij} - r_j^+)^2} + \sqrt{\sum_{jj=1}^{n} w_j^3 (r_{ij} - r_j^-)^2}}, \tag{5b}$$

With the equal criteria weight settings, applying w_j = w eqn. (5b) becomes

$$C_i = \frac{\sqrt{\sum_{j=1}^{n} w^3 (r_{ij} - r_j^-)^2}}{\sqrt{\sum_{j=1}^{n} w^3 (r_{ij} - r_j^+)^2} + \sqrt{\sum_{j=1}^{n} w^3 (r_{ij} - r_j^-)^2}}, \tag{6b}$$

But $\sum_{jj=1}^{n} w_{jj} = 1$. Hence eqn. (5b) becomes

$$C_i = \frac{\sqrt{\sum_{j=1}^{n} (r_{ij} - r_j^-)^2}}{\sqrt{\sum_{j=1}^{n} (r_{ij} - r_j^+)^2} + \sqrt{\sum_{j=1}^{n} (r_{ij} - r_j^-)^2}}. \tag{7b}$$

When Equations (5a) and (7b) are compared, it becomes clear that the two models are essentially the same. This mathematical clarification provides evidence supporting the consistency in ranking results and emphasizes the significant structural similarities between the two models. The primary difference between TOPSIS and revised TOPSIS lies in how criteria weights are integrated during calculations. A careful examination of the extended TOPSIS equation (3a) and the extended revised TOPSIS equation (5b) reveals that the only distinction between the two methods is the use of w_j^2 in TOPSIS and w_j^3 in revised TOPSIS when calculating distances from the positive and negative ideal solutions.

3.3. Comparison with non-equal criteria weight settings

The study begins with a simulation and presents its findings, followed by a mathematical comparison between the TOPSIS and revised TOPSIS methods in scenarios where weights are not equal.

3.3.1. Simulation results

In this simulation analysis, the decision matrix from the car purchasing scenario outlined by [57] is utilized. The decision matrix is depicted in Table 1.

The simulation began by assigning equal weights (W = 0.25, 0.25, 0.25, 0.25) to the four criteria. Under this equal weight configuration, both the TOPSIS and revised TOPSIS methods were applied to solve the decision problem. The resulting rankings ($C_1 \succ C_2 \succ C_4 \succ C_3$) were consistent and served as the baseline rankings. Subsequently, unequal weights (W = 0.1, 0.4, 0.3, 0.2) were employed for the criteria, and the MCDM problem was addressed using both TOPSIS and revised TOPSIS models, yielding identical rankings ($C_1 \succ C_4 \succ C_2 \succ C_3$).

The simulation findings, along with earlier sections discussing equal weight configurations, emphasize that the disparity between TOPSIS and revised TOPSIS lies solely in their treatment of criteria weights during computations. A detailed examination of the extended TOPSIS equation (3a) and extended revised TOPSIS equation (5b) reveals that the key distinction between the two approaches lies in TOPSIS utilizing w_j^2 while revised TOPSIS uses w_j^3 when calculating distances from both positive and negative ideal solutions.

IV. Conclusions

This study extensively compared the well-known TOPSIS model with its proposed revised version through simulations and mathematical proofs. The findings supported the advantages of the revised TOPSIS model, particularly the inclusion of criteria weights and alternative weights using Euclidean distances. In contrast, the traditional TOPSIS model relies on non-weighted Euclidean distances, potentially overlooking the significance of weighted distances and addressing possible shortcomings of the model. The revised TOPSIS model represents a notable advancement in integrating criteria weights and performance ratings, distinguishing it from its conventional counterpart. While they exhibit structural similarities, this research's mathematical clarification ensures consistency in ranking results, highlighting significant parallels between the two models.

By incorporating criteria weights along with performance ratings and alternative weights using Euclidean distances, the revised TOPSIS model aims to enhance accuracy and relevance in decision-making processes. This strategic modification is expected to preserve and potentially strengthen the TOPSIS model's strengths, offering a more nuanced and precise evaluation framework. Such adaptation proves especially beneficial in scenarios where the relative importance of criteria and alternatives significantly influences decision outcomes.

REFERENCES

[1]. Agrawal, V., V. Kohli, S. Gupta, Computer aided robot selection: the 'multiple attribute decisionmaking'approach, Int. J. Prod. Res. **29**(8) (1991) 1629–1644.

[2]. Akgül, E., Bahtiyari, M. I., KizilkayaAydoğan, E., and Benli, H. (2022). Use of TOPSIS method for designing different textile products in coloration via natural source "madder". Journal of Natural Fibers, **19**(14), 8993-9008.

[3]. Arabameri, A., Rezaei, K., Cerda, A., Lombardo, L., and Rodrigo-Comino, J. (2019). GIS-based groundwater potential mapping in Shahroud plain, Iran. A comparison among statistical (bivariate and multivariate), data mining and MCDM approaches. Science of the total environment, 658, 160-177.

[4]. Athawale, V. M., and Chakraborty, S. (2010, January). A TOPSIS method-based approach to machine tool selection. In Proceedings of the 2010 International Conference on Industrial Engineering and Operations Management Dhaka, Bangladesh (Vol. 2, pp. 83-94).

[5]. Aydogan, E. K. Performance measurement model for Turkish aviation firms using the rough-AHP and TOPSIS methods under fuzzy environment, Expert Syst. Appl. **38** (4) (2011) 3992–3998.

[6]. Belton, V., and Stewart, T. (2002). Multiple criteria decision analysis: an integrated approach. Springer Science & Business Media.

[7]. Byun, H., K. Lee, A decision support system for the selection of a rapid prototyping Process using the modified TOPSIS method, Int. J. Adv. Manuf. Technol. **26**(11–12) (2005) 1338–1347.

[8]. Çelikbilek, Y., and Tüysüz, F. (2020). An in-depth review of theory of the TOPSIS method: An experimental analysis. Journal of Management Analytics, **7**(2), 281-300.

[9]. Chakraborty, S. (2022). TOPSIS and Modified TOPSIS: A comparative analysis. Decision Analytics Journal, **2**, 100021.

[10]. Chakraborty, S., and Mandal, A. (2018, September). A novel TOPSIS based consensus technique for multi-attribute group decision making. In 2018 18th International Symposium on Communications and Information Technologies (ISCIT) (pp. 322-326). IEEE.

[11]. Chakraborty, S., C.-H. Yeh, A simulation based comparative study of normalization procedures in multi-attribute decision making. in: Proceedings of the 6th Conference on 6th WSEAS Int. Conf. on Artificial Intelligence, Knowledge Engineering and Data Bases, 2007.

[12]. Chang, Y. H., C.-H. Yeh, Evaluating airline competitiveness using multi-attribute decision making, Omega **29** (5) (2001) 405–415.

[13]. Chede, S. J., B.R. Adavadkar, A.S. Patil, H.K. Chhatriwala, M.P. Keswani, Materialselection for design of powered hand truck using TOPSIS, Int. J. Ind. Syst. Eng. **39**(2) (2021) 236–246.

[14]. Deng, H., Yeh, C. H., and Willis, R. J. (2000). Inter-company comparison using Modified TOPSIS with objective weights. Computers and Operations Research, **27**(10), 963-973.

[15]. Eraslan, S. (2015). A decision making method via TOPSIS on soft sets. Journal of New results in Science, **4**(8), 57-70.

[16]. Feng, C. M., R.-T. Wang. Considering the financial ratios on the performance evaluation of highway bus industry, Trans. Rev. **21**(4) (2001) 449–467.

[17]. García, F., F. Guijarro, I. Moya, A goal programming approach to estimating performance weights for ranking firms, Comput. Oper. Res. **37**(9) (2010), 1597–1609.

[18]. Ginting, G., Fadlina, M., Siahaan, A. P. U., and Rahim, R. (2017). Technical approach of TOPSIS in decision making. Int. J. Recent Trends Eng. Res, **3**(8), 58-64.

[19]. Greenacre, M. (2008). Chapter 4 Measures of distance between samples: Euclidean. Department of Statistics, Stanford University.

[20]. Henig, M. I., and Buchanan, J. T. (1996). Solving MCDM problems: Process concepts. Journal of Multi-Criteria Decision Analysis, **5**(1), 3-21.

[21]. Huang, Y., and Jiang, W. (2018). Extension of TOPSIS Method and its Application in Investment. Arabian Journal for Science and Engineering, **43**, 693-705.

[22]. Hwang, C. I., K. Yoon, Multiple Attribute Decision Making Methods and Applications, Springer, Berlin Heidelberg, 1981.

[23]. Jahan, A., Mustapha, F., Sapuan, S. M., Ismail, M. Y., and Bahraminasab, M. (2012). A framework for weighting of criteria in ranking stage of material selection process. The International Journal of Advanced Manufacturing Technology, **58**, 411-420.

[24]. Kahraman, C., Engin, O., Kabak, Ö., and Kaya, İ. (2009). Information systems outsourcing decisions using a group decision-making approach. Engineering Applications of artificial intelligence, **22**(6), 832-841.

[25]. Khan, A., and Maity, K. P. (2016). Application of MCDM-based TOPSIS method for the optimization of multi quality characteristics of modern manufacturing processes. International Journal of Engineering Research in Africa, **23**, 33-51.

[26]. Khan, A., and Maity, K. (2017). Application of MCDM-based TOPSIS method for the selection of optimal process parameter in turning of pure titanium. Benchmarking: An International Journal, **24**(7), 2009-2021.

[27]. Khan, M. S., S.I.A. Shah, A. Javed, N.M. Qadri, N. Hussain, Drone selection using multi-criteria decision-making methods, in: 2021 International Bhurban Conference on Applied Sciences and Technologies, IBCAST, IEEE, 2021, pp. 256–270.

[28]. Kirubakaran, B., and Ilangkumaran, M. (2016). Selection of optimum maintenance strategy based on FAHP integrated with GRA-TOPSIS. Annals of Operations Research, **245**, 285-313.

[29]. Kornyshova, E., and Salinesi, C. (2007, April). MCDM techniques selection approaches: state of the art. In 2007 ieee symposium on computational intelligence in multi-criteria decision-making (pp. 22-29). IEEE.

[30]. Krishna Kumar, K., Karthikeyan, A., and Elango, M. (2022). Selection of a best humanoid robot using "topsis" for rescue operation. In Advances in Mechanical and Materials Technology: Select Proceedings of EMSME 2020 (pp. 943-953). Springer Singapore.

[31]. Kuo, T. (2017). A modified TOPSIS with a different ranking index. European journal of operational research, **260**(1), 152-160.

[32]. Lamrini, L., Abounaima, M. C., and TalibiAlaoui, M. (2023). New distributed-topsis approach for multi-criteria decision-making problems in a big data context. Journal of Big Data, **10**(1), 97.

[33]. Liern, V., B. Pérez-Gladish, Building composite indicators with unweighted-TOPSIS, IEEE Trans. Eng. Manage. (2021).

[34]. Lin, C. T., C.-B. Chen, Y.-C. Ting, An ERP model for supplier selection in electronics industry, Expert Syst. Appl. **38** (3) (2011) 1760–1765.
[35]. Lundgren, C., E. TuranogluBekar, M. Bärring, J. Stahre, A. Skoogh, B. Johansson, R. Hedman, Determining the impact of 5G-technology on manufacturing performance using a modified TOPSIS method, Int. J. Comput. Integr. Manuf. (2021), 1–22.
[36]. Meshram, S. G., Alvandi, E., Meshram, C., Kahya, E., and Fadhil Al-Quraishi, A. M. (2020). Application of SAW and TOPSIS in prioritizing watersheds. Water Resources Management, **34**, 715-732.
[37]. Mukherjee, A., and Nath, P. (2005). An empirical assessment of comparative approaches to service quality measurement. Journal of services marketing, **19**(3), 174-184.
[38]. Narayanan, A., M. Jenamani, B. Mahanty, Determinants of sustainability and prosperityin Indian cities, Habitat Int. **118** (2021) 102456.
[39]. Nanayakkara, C., Yeoh, W., Lee, A., and Moayedikia, A. (2020). Deciding discipline, course and university through TOPSIS. Studies in Higher Education, **45**(12), 2497-2512.
[40]. Özcan, S., A.K. Çelik, A comparison of TOPSIS, grey relational analysis and COPRAS methods for machine selection problem in the food industry of Turkey, Int. J. Prod. Manag. Eng. **9** (2) (2021) 81–92.
[41]. Palczewski, K., and Sałabun, W. (2019). The fuzzy TOPSIS applications in the last decade. Procedia computer science, **159**, 2294-2303.
[42]. Parkan, C., M.-L. Wu, Decision-making and performance measurement models with applications to robot selection, Comput. Ind. Eng. **36**(3) (1999) 503–523.
[43]. Pavić, Z., and Novoselac, V. (2013). Notes on TOPSIS method. International Journal of Research in Engineering and Science, **1**(2), 5-12.
[44]. Sarkar, A. (2013). A TOPSIS method to evaluate the technologies. International Journal of Quality and Reliability Management, **31**(1), 2-13.
[45]. Shannon, C. E. (1948). A mathematical theory of communication. The Bell system technical journal, **27**(3), 379-423.
[46]. Shyur, H. J., H.-S. Shih, A hybrid MCDM model for strategic vendor selection, Math. Comput. Model. **44**(7–8) (2006) 749–761.
[47]. Tavana, M., and Hatami-Marbini, A. (2011). A group AHP-TOPSIS framework for human spaceflight mission planning at NASA. Expert Systems with Applications, **38**(11), 13588-13603.
[48]. Ture, H., S. Dogan, D. Kocak, Assessing euro strategy using multi-criteria decision making methods: VIKOR and TOPSIS, Soc. Ind. Res. **142**(2) (2020) 645–665.
[49]. Wang, C., G.G. Yen, F. Zou, A novel predictive method based on key points for dynamic multi-objective optimization, Expert Syst. Appl. (2021) 116127.
[50]. Yeh, C.H.. A problem-based selection of multi-attribute decision-making methods, Int. Trans. Oper. Res. **9**(2) (2002) 169–181.
[51]. Yeh, C. H. The selection of multiattribute decision making methods for scholarship student selection, Int. J. Sel. Ass. **11**(4) (2003) 289–296.
[52]. Yoon, K. P., C.-L. Hwang, Multiple Attribute Decision Making: An Introduction, Vol. 104, Sage publications, 1995.
[53]. Zavadskas, E. K., Mardani, A., Turskis, Z., Jusoh, A., and Nor, K. M. (2016). Development of TOPSIS method to solve complicated decision-making problems – An overview on developments from 2000 to 2015. International journal of information technology and decision making, **15**(03), 645-682.
[54]. Zhang, L., Xu, Y., Yeh, C. H., Liu, Y., and Zhou, D. (2016). City sustainability evaluation using multi-criteria decision making with objective weights of interdependent criteria. Journal of Cleaner Production, **131**, 491-499.
[55]. Zhou, P., B. Ang, K. Poh, Comparing aggregating methods for constructing the composite environmental index: An objective measure, Ecol. Econom. **59**(3), (2006) 305–311.
Zhou, M., Liu, X. B., Yang, J. B., Chen, Y. W., and Wu, J. (2019). Evidential reasoning approach with multiple kinds of attributes and entropy-based weight asignment. Knowledge-Based Systems, **163**, 358-375.
[56]. Zulqarnain, R. M., Saeed, M., Ahmad, N., Dayan, F., and Ahmad, B. (2020). Application of TOPSIS method for decision making. Int. J. Sci. Res. in Mathematical and Statistical Sciences, **7**(2).