IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-0661, p-ISSN: 2278-8727, Volume 21, Issue 3, Ser. 3 (May. – June. 2025), PP 29-35
www.iosrjournals.org

LRS Bianchi Type-V String Cosmological Model in Lyra's Manifold

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Abstract: *In this research work, a five-dimensional LRS Bianchi type-V string cosmological model with the electromagnetic field in the framework of Lyra's manifold is investigated. A determinate solution of the field equations is obtained with the help of the special law of variation for the Hubble parameter. Some physical and kinematical properties of the model are discussed.*
Key Words: *LRS Bianchi type-V; Lyra's manifold; String Cloud with Electromagnetic Field.*

Date of Submission: 14-06-2025 Date of Acceptance: 29-06-2025

I. INTRODUCTION

The LRS five-dimensional Bianchi Type-V spacetime describes a specific class of spacetime geometries with certain symmetries. It is an extension of the standard four-dimensional Bianchi Type V model, incorporating an additional spatial dimension. Which in general, describes an anisotropic universe, meaning that the expansion rates in different spatial directions are not necessarily the same. This model is particularly relevant for studying cosmological scenarios where rotational symmetry is locally preserved.

Lyra's manifold is an extension of the standard spacetime manifold. It incorporates an additional scalar field, called the Lyra field or the gauge function, which represents a coupling between the geometry of spacetime and the electromagnetic field. This extension allows for a varying gravitational coupling, which introduces modifications to Einstein's field equations.The displacement vector plays a significant role in the modified field equations of Lyra's field theory, influencing the dynamics of spacetime and matter.

The Einstein's modified field equation in normal gauge for Lyra's manifold obtained by Sen [1] is given by

$$R_{ij} - \frac{1}{2} g_{ij} R + \frac{3}{2}\phi_i \phi_j - \frac{3}{4}\phi_v \phi^v = -T_{ij} \tag{1}$$

Where, R_{ij} is Ricci tensor, g_{ij} is metric tensor, R is Ricci curvature, ϕ_i is displacement vector defined by $\phi_i = (\beta(t),0,0,0,0)$ Where, $\beta(t)$ is Gauge function, T_{ij} is Energy Momentum Tensor.

In the context of the LRS Bianchi Type-V string cosmological model, the presence of strings is considered. Strings are one-dimensional objects that play a significant role in string theory. They are fundamental entities that can interact with the gravitational and electromagnetic fields. Including strings in the cosmological model can have implications for the evolution and dynamics of the universe.

In recent years, numerous researchers have dedicated their efforts to studying a wide range of this topic. Mohanty et. al. [2,3,4] studied 'Five dimensional LRS Bianchi type-I string cosmological model in Saez and Ballester theory', 'Incompatibility of five dimensional LRS Bianchi type-V string and mesonic string cosmological models in general relativity' and 'Five-dimensional axially symmetric string cosmological model in Lyra manifold', Daimary et. al. [5,6] examined 'Five Dimensional Bianchi Type-I Anisotropic Cloud String Cosmological Model With Electromagnetic Field in Saez-Ballester Theory' and 'Five Dimensional Bianchi Type I Sting Cosmological Model in Electromagnetic Field', Samanta et. al. [7, 8] studied 'Five Dimensional Bulk Viscous String Cosmological Models in Saez and Ballester Theory of Gravitation', Khan et. al. [9] conducted a study on 'Symmetries of locally rotationally symmetric Bianchi type V spacetime', Baro et. al. [10] explored 'LRS Bianchi Type-I Cosmological Model with Strings in Lyra Geometry in Five Dimensional Space-time'., Bali et. al. [11, 12] studied 'Bianchi Type V Magnetized String Dust Universe with Variable Magnetic Permeability' and 'LRS Bianchi Type II Massive String Cosmological Models with Magnetic Field in Lyra's Geometry', Humad et. al. [13] analysed'LRS Bianchi Type –III Massive String Cosmological Model With Electromagnetic Field', Yadav et. al. [14] studied 'Some Bulk Viscous Magnetised LRS Bianchi Type I String

Cosmological Model in Lyra's Geometry', Priyokumar et. al. [15] studied 'Higher Dimensional LRS Bianchi Type-I String Cosmological Model with Bulk Viscosity in General Relativity'.

II. METRIC ANF FIELD EQUATION

LRS Five-dimensional Bianchi type-V Space-time is given as follows

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{2x}\left(dy^2 + dz^2\right) + C^2dm^2 \tag{2}$$

Here, A, B, C are the functions of time only.

In this research work, we have used String Cloud as source of gravitation with Electromagnetic Field.The Energy Momentum Tensor for String Cloud with Electromagnetic Field is given as

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j + E_{ij} \tag{3}$$

Where, $\rho,\ \lambda,\ \rho_p$ denotes rest energy density of the cloud strings with particles attached to them, tension density and particle density of the string. Also, u^i represents the five-velocity vector and x^i represents the unit space-like vector with the following condition

$$u^i u_i = -x^i x_i = -1 \text{ and } u^i x_i = 0 \tag{4}$$

There is relation between rest energy density and particle density given by,

$$p = \rho_p + \lambda \tag{5}$$

We consider the electromagnetic field E_{ij} as follows,

$$E_{ij} = \frac{1}{4\pi}\left(g^{\alpha\beta}F_{i\alpha}F_{i\beta} - \frac{1}{4}g_{ij}F_{\alpha\beta}F^{\alpha\beta}\right) \tag{6}$$

Where, $F_{\alpha\beta}$ denotes Electromagnetic field tensor.

When we compute the magnetic field along the x axis then we get that F_{01} is the only non-zero component of the electromagnetic field tensor F_{ij}. Hence, when we consider infinite electromagnetic conductivity, we get that $F_{02} = F_{03} = F_{12} = F_{13} = F_{14} = F_{23} = F_{24} = F_{34} = 0$

Hence from Equation (6), we get

$$E_0^0 = E_1^1 = E_2^2 = E_3^3 = E_4^4 = \frac{1}{16\pi A^2}{F_{01}}^2 \tag{7}$$

Hence from equations (3), (4) and (7), the components of the Energy Momentum Tensor are obtained as follows

$$T_0^0 = -\rho - \frac{1}{8\pi A^2}{F_{01}}^2,\ T_1^1 = -\lambda - \frac{1}{8\pi A^2}{F_{01}}^2,\ T_2^2 = T_3^3 = T_4^4 = \frac{1}{8\pi A^2}{F_{01}}^2 \tag{8}$$

Here we define the Average Scale Factor R spatial volume V, Expansion Scalar θ, Hubble Parameter H, the Deceleration Parameter q, the Shear Scalar σ^2 and the Anisotropy Parameter A_m for the metric (2) as follows,

Average Scale Factor $R = \left(AB^2C\right)^{\frac{1}{4}}$ (9)

Spatial Volume $V = R^4 = AB^2C$ (10)

Expansion Scalar $\theta = \frac{\dot{A}}{A} + \frac{2\dot{B}}{B} + \frac{\dot{C}}{C}$ (11)

Hubble parameter $H = \frac{1}{4}\left(\frac{\dot{A}}{A} + \frac{2\dot{B}}{B} + \frac{\dot{C}}{C}\right)$ (12)

Deceleration Parameter $q = -\frac{R\ddot{R}}{\dot{R}^2}$ (13)

Shear Scalar $\sigma^2 = \frac{3}{8}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right)^2$ (14)

Anisotropy Parameter $A_m = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i - H}{H}\right)^2 = \frac{3}{16H^2}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right)^2$ (15)

Therefore, the field equations of Lyra Geometryfor LRS five-dimensional Bianchi type-V space-time in the electromagnetic field are obtained as,

$$\frac{2\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}+\frac{2\dot{B}\dot{C}}{BC}+\frac{\dot{B}^2}{B^2}-\frac{3}{A^2}-\frac{3}{4}\beta^2=\rho+\frac{{F_{01}}^2}{8\pi A^2} \tag{16}$$

$$\frac{2\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{2\dot{B}\dot{C}}{BC}+\frac{\dot{B}^2}{B^2}-\frac{1}{A^2}+\frac{3}{4}\beta^2=\lambda+\frac{{F_{01}}^2}{8\pi A^2} \tag{17}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}-\frac{1}{A^2}+\frac{3}{4}\beta^2=-\frac{{F_{01}}^2}{8\pi A^2} \tag{18}$$

$$\frac{\ddot{A}}{A}+\frac{2\ddot{B}}{B}+\frac{2\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}-\frac{3}{A^2}+\frac{3}{4}\beta^2=-\frac{{F_{01}}^2}{8\pi A^2} \tag{19}$$

$$\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=0 \tag{20}$$

Here overhead dots denote the ordinary differentiation concerning time t.

Using the energy conservation equation $T^{j}_{i;j}=0$, the right-hand side of equation (1) leads to,

$$\frac{3}{2}\phi_i\left(\phi^j_{,j}+\phi^k\,\Gamma^j_{kj}\right)+\frac{3}{2}\phi^j\left(\phi_{i,j}-\phi_k\,\Gamma^k_{ij}\right)-\frac{3}{4}\phi_\nu\left(\phi^\nu_{,j}+\phi^k\,\Gamma^\nu_{kj}\right)-\frac{3}{4}\phi^\nu\left(\phi_{n,j}-\phi_k\,\Gamma^k_{\nu j}\right)=0 \tag{21}$$

Above equation (21) satisfies identically for $i=1,2,3,4$ and for $i=0$, we get,

$$\beta\dot{\beta}+\beta\left(\frac{\dot{A}}{A}+\frac{2\dot{B}}{B}+\frac{\dot{C}}{C}\right)=0 \tag{22}$$

III. COSMOLOGICAL SOLUTION

By use of equation (20), the field equations (16) to (19) can be written as,

$$\frac{3\dot{B}\dot{C}}{BC}+\frac{3\dot{B}^2}{B^2}-\frac{3}{B^2}-\frac{3}{4}\beta^2=\rho+\frac{{F_{01}}^2}{8\pi B^2} \tag{23}$$

$$\frac{2\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{2\dot{B}\dot{C}}{BC}+\frac{\dot{B}^2}{B^2}-\frac{1}{B^2}+\frac{3}{4}\beta^2=\lambda+\frac{{F_{01}}^2}{8\pi B^2} \tag{24}$$

$$\frac{2\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{2\dot{B}\dot{C}}{BC}+\frac{\dot{B}^2}{B^2}-\frac{1}{B^2}+\frac{3}{4}\beta^2=-\frac{{F_{01}}^2}{8\pi B^2} \tag{25}$$

$$\frac{3\ddot{B}}{B}+\frac{3\dot{B}^2}{B^2}-\frac{3}{B^2}+\frac{3}{4}\beta^2=-\frac{{F_{01}}^2}{8\pi B^2} \tag{26}$$

Also, equation (22) can be written as

$$\beta\dot{\beta}+\beta\left(\frac{3\dot{B}}{B}+\frac{\dot{C}}{C}\right)=0 \tag{27}$$

Since, the field equations (23) to (26) is system of four equations in six unknown parameters $B,\ C,\ \beta,\ \rho,\ \gamma$ and F_{01}. Hence to solve the above system of equations we assumed following conditions,

(i) Since the Shear Scalar σ is proportional to the Expansion Scalar θ (Collins et. al. [16]) so we may assume

$\sigma=k_1\theta$

Where k_1 is proportionality constant.

Hence from above relation between Shear Scalar and Expansion Scalar we get,

$$B=k_3C^N \tag{28}$$

Here, $k_3=k_2^{1/(1-2\sqrt{6}k_1)}$ and $N=\dfrac{1+\sqrt{\frac{8}{3}}k_1}{1-2\sqrt{6}k_1};\ k_1\neq\dfrac{1}{2\sqrt{6}}$

(ii) We also assume the special law of variation for Hubble parameter defined as (Berman [17])

$H=\dfrac{k_4}{R^m}$ where, k_4 and n are constants. (29)

Also, Hubble Parameter in terms of Average Scale factor is given by

$$H = \frac{\dot{R}}{R} \tag{30}$$

Now, from equation (29) and (30), we get,

$$R = (nk_4 t + k_5)^{\frac{1}{n}} \tag{31}$$

Comparing equations (9) and (31), we get

$$C = k_6 (nk_4 t + k_5)^{N_1} \tag{32}$$

Where, $k_6 = \left(\frac{1}{k_3}\right)^{\frac{3}{1+3N}}$ and $N_1 = \frac{4}{n(1+3N)}$; $N \neq -\frac{1}{3}$

From equation (28), we obtain,

$B = k_7 (nk_4 t + k_5)^{N_2}$ Where, $k_7 = k_3 k_6^{\ N}$ and $N_2 = NN_1$

Also,using equation (20), we have

$$A = B = k_7 (nk_4 t + k_5)^{N_2} \tag{33}$$

Using equations (32) and (33) the five-dimensional LRS Bianchi type-V space-time in equation (2) takes the form,

$$ds^2 = -dt^2 + k_7^{\ 2} (nk_4 t + k_5)^{2N_2} dx^2 + k_7^{\ 2} (nk_4 t + k_5)^{2N_2} e^{2x} \left(dy^2 + dz^2\right) + k_6^{\ 2} (nk_4 t + k_5)^{2N_1} dm^2 \tag{34}$$

Also, we calculated following terms with the help of metric coefficient and field equations

Gauge function-

$$\beta = \frac{k_9}{(nk_4 t + k_5)^{(3N+1)N_1}} \tag{35}$$

Electromagnetic Field Tensor-

$$F_{01} = -8\pi \left\{ k_{10} (nk_4 t + k_5)^{2(N_2 - 1)} + \frac{3}{4} k_7 k_9^{\ 2} (nk_4 t + k_5)^{N_3} - 3 \right\} \tag{36}$$

Where, $k_{10} = 3n^2 k_4^{\ 2} k_7^{\ 2} N_2 (2N_2 - 1)$ and $N_3 = 2N_2 - 2(1+3N)N_1$

Rest Energy Density $\rho = \frac{k_{11}}{(nk_4 t + k_5)^2} - \frac{6}{k_7^{\ 2} (nk_4 t + k_5)^{2N_2}}$ (37)

Where, $k_{11} = 3n^2 k_4^{\ 2} N_2 (N_1 + 3N_2 - 1)$

Tension Density- $\lambda = \frac{k_{12}}{(nk_4 t + k_5)^2} - \frac{4}{k_7^{\ 2} (nk_4 t + k_5)^{2N_2}} + \frac{3}{2k_9^{\ 2}} (nk_4 t + k_5)^{2N_1 (3N+1)}$ (38)

Where, $k_{12} = n^2 k_4^{\ 2} \left[5N_2 (N_2 - 1) + N_1 (N_1 - 1) + 3N_1 N_2 + 4N_2^{\ 2}\right]$

Particle Density-

$$\rho_p = \rho - \lambda = \frac{k_{13}}{(nk_4 t + k_5)^2} - \frac{2}{k_7^{\ 2} (nk_4 t + k_5)^{2N_2}} - \frac{3}{2k_9^{\ 2}} (nk_4 t + k_5)^{2N_1 (3N+1)} \tag{39}$$

Where, $k_{13} = k_{11} - k_{12}$

IV. PHYSICAL AND KINEMATICAL PROPERTIES

Spatial Volume- $V = (nk_4 t + k_5)^{\frac{4}{n}}$ (40)

Expansion Scaler- $\theta = \frac{4k_4 (N_1 + 3N_2)}{nk_4 t + k_5}$ (41)

Hubble Parameter- $H = \frac{4k_4}{nk_4 t + k_5}$ (42)

Deceleration Parameter- $q = -(1 - n)$ (43)

$q = n - 1$, When, $n \neq 0$

$q = -1$, When, $n = 0$

Shear Scalar- $\sigma^2 = \frac{3}{8}\left[\frac{nk_4(N_2 - N_1)}{nk_4 t + k_5}\right]^2$ (44)

Anisotropy Parameter- $A_m = \frac{3(N_1 - N_2)^2}{256}$ (45)

V. GRAPHICAL REPRESENTATION AND DISCUSION

Here we plotted some graphs of various Physical and Kinematical Parameters of the obtained Cosmological Model as follows

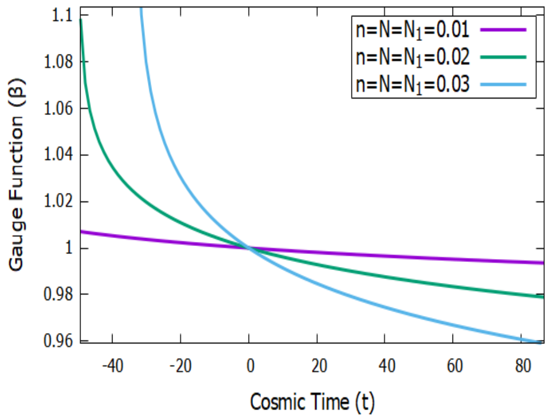

Figure 1: Plot of Gauge Function verses Cosmic time with $k_4 = k_5 = k_9 = 1$.

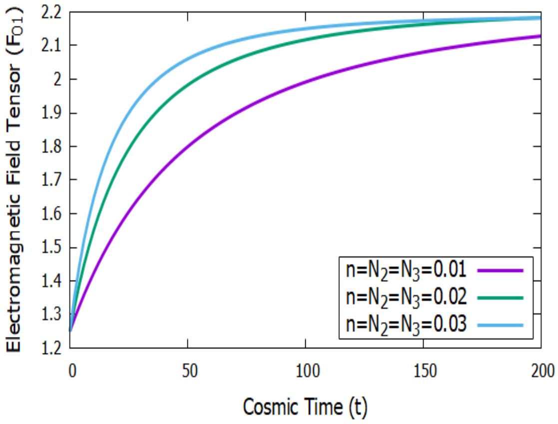

Figure 2: Plot of Electromagnetic Field Tensor verses Cosmic time with $k_4 = k_5 = k_7 = k_9 = k_{10} = 1$.

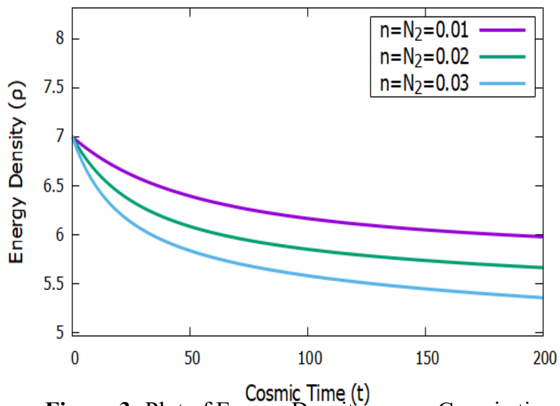

Figure 3: Plot of Energy Density verses Cosmic time with $k_4 = k_5 = k_9 = 1$.

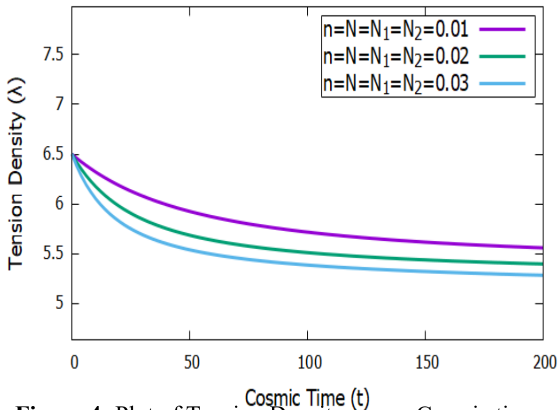

Figure 4: Plot of Tension Density verses Cosmic time with $k_4 = k_5 = k_9 = 1$.

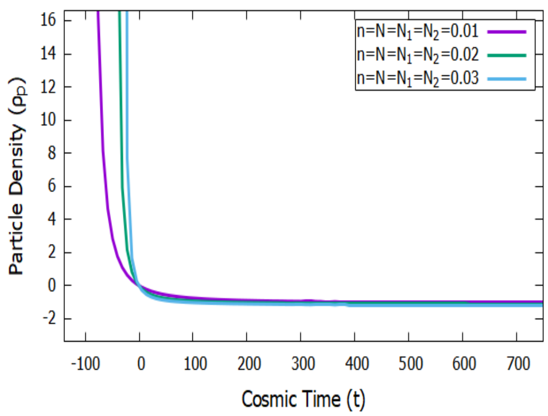

Figure 5: Plot of Particle Density verses Cosmic time with $k_4 = k_5 = k_9 = 1$.

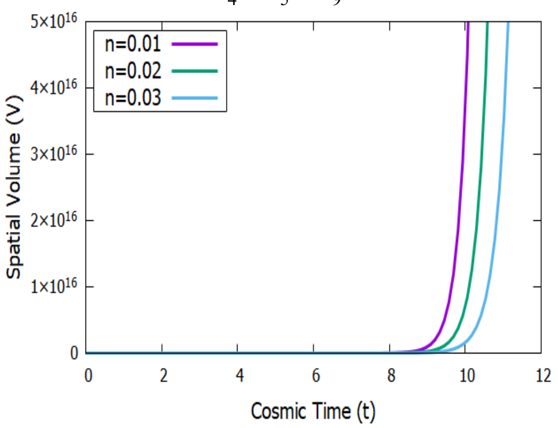

Figure 6: Plot of Spatial Volume verses Cosmic time with $k_4 = k_5 = k_9 = 1$.

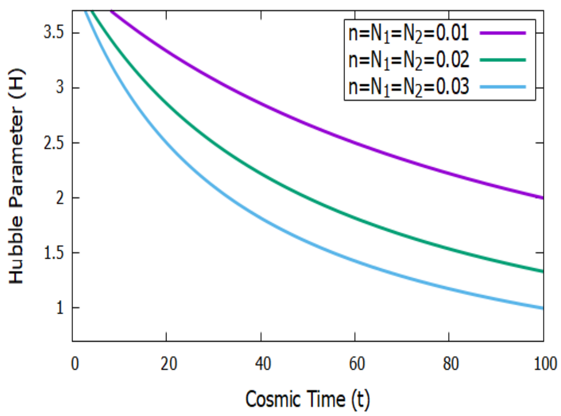

Figure 7: Plot of Hubble Parameter verses Cosmic time with $k_4 = k_5 = k_9 = 1$.

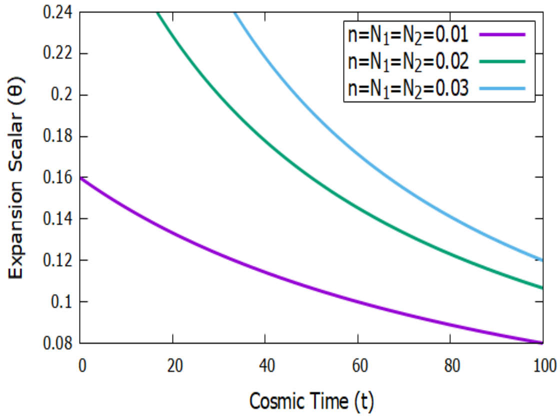

Figure 8: Plot of Expansion Scalar verses Cosmic time with $k_4 = k_5 = k_9 = 1$.

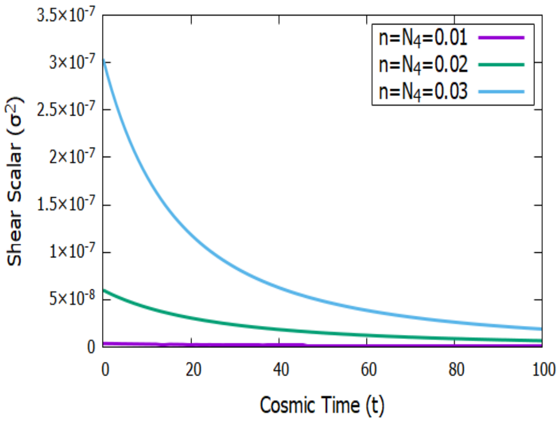

Figure 9: Plot of Shear Scalar verses Cosmic time with $k_4 = k_5 = k_9 = 1$.

Figure 1 depicts a graph illustrating the relationship between the gauge function and cosmic time. The plot reveals a gradual decrease in the gauge function as cosmic time increases. Notably, as time approaches infinity, the gauge function tends to zero.Hence, the model has singularity when the cosmic time is zero.

Figure 2 exhibits the correlation between the Electromagnetic Field Tensor and Cosmic Time. It is evident from the graph that the Electromagnetic Field Tensor does not vanish. Its significant involvement in the formation of strings during the early phases of the universe is apparent. During this period, both strings and the Electromagnetic field coexisted.

In Figures 3 and 4, the graph illustrates the variation of Energy Density and Tension Density with respect to Cosmic Time. Notably, both Energy Density and Tension Density exhibit comparable trends, but Energy Density diminishes more rapidly compared to Tension

Density. This observation aligns with our model, indicating a matter-dominated universe in the later stages, consistent with current observational data.

In figure 5, the graph depicts the evolution of Particle Density over Cosmic Time. Notably, at the time of the Big Bang $(t = 0)$, the Particle Density exhibits a significantly huge positive value. Over time, it gradually decreases but converges to a finite positive value as time approaches infinity $(t \to \infty)$. This observation suggests that particles continue to dominate the universe with the total number of particles remaining constant as time progresses.

In figure 6, the graph represents the relationship between Spatial Volume and Cosmic Time. The plot indicates that the volume initiates from zero at an initial time $(t = 0)$ and progressively expands as time advances. The Spatial Volume extends toward infinity $(V \to \infty)$ as time approaches to infinity $(t \to \infty)$.

In figures 7 and 8, the graphs portray the behaviour of the Hubble Parameter and Expansion Scalar in relation to Cosmic Time. Both the Hubble Parameter and Expansion Scalar are infinite at the initial time $(t = 0)$, decreases with increasing time and vanishes, when $t \to \infty$. This pattern suggests that the universe is undergoing expansion, and the expansion rate is accelerating, aligning with observational data.

In Figure 9, the graph illustrates the Shear Scalar plotted against Cosmic Time. As cosmic time increases, there is a decline in the Shear Scalar, indicating a shearing phenomenon within the model.

VI. Conclusion

In this research paper, we have obtained a cosmological model by solving field equations using Special law of variation of Hubble Parameter.

Along with Gauge function, Rest Energy density and Tension Density of String Cloud with Electromagnetic field are calculated. The universe seems to have singularity at $t = 0$. The correlation observed between the Electromagnetic Field Tensor and Cosmic Time implies the simultaneous existence of both strings and the Electromagnetic field during the early stages of the universe.

Examining the graphs depicting Energy Density and Tension Density concerning Cosmic Time reveals a dominance of matter in the later epochs of the universe. Particle Density, having a huge positive value at the time of the Big Bang, decreases gradually to a finite positive value, this suggests a continuous dominance of particles in the cosmos, with the total particle count remaining constant over time.

Various physical and kinematic properties of the derived model have been calculated and discussed. Equation (43) reveals that the Deceleration Parameter maintains a constant negative value, signifying that the cosmological model consistently undergoes inflation and expansion, since $t = 0$. The spatial volume graph aligns with this observation, illustrating a gradual expansion of the universe over cosmic time.

The Hubble Parameter and Expansion Scalar of the model both convey that the rate of the universe's expansion is accelerating. Additionally, the computation of the Shear Scalar indicates a shearing characteristic in the obtained cosmological model. Furthermore, the Anisotropy Parameter, as obtained in Equation (45), signifies that the universe maintains an anisotropic nature throughout its evolution.

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