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Difference Cordial Labeling On Zero Divisor Graphs

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Abstract:
Assume that G is a (p, q) graph. Let V(G) to {1,2,...,p} be mapped by f. Assign the label |f (u)− f(v) | to each edge uv. It's referred to as differential cordial labeling. When f is 1 −1 and $\left|e_f(0) - e_f(1)\right| \le 1$*, the number of edges labeled with 1 and those not labeled with 1 are indicated by* $e_f(1)$ *and* $e_f(0)$ *, respectively. A difference cordial graph is one that has a difference cordial labeling.Thus , by using the above definitions here we investigate that zero divisor graphs* $\Gamma(Z_{np})$, $\Gamma(Z_{p^n})$, $\Gamma(Z_{pq})$, *for difference cordial labeling.*

Keywords And Phrases: *Cordial labeling, Difference cordial labeling , Zero divisors, Zero divisor graphs*

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I. Introduction

Let the graph G = (V,E) be (p,q). Only simple, undirected graphs have been examined in this article. The order of G is the number of its vertices, while the size of G is the number of its edges. Numerous scientific and technological fields, including astronomy, radar, circuit design, and database administration, use labeled graphs [9]. Graceful labeling, which was first presented by Rosa [5] in the year 1967, is considered to be the founding principle of graph labeling. Chait [3] first proposed the cordial labeling of graphs in 1980. Numerous publications have examined the cordiality behavior of various graphs [4,5,6,7]. The concept of differential cordial labeling was suggested by R. Ponraj [10]. R. Ponraj [11] A remark on difference cordial graphs and the study of Signed Product Cordial Labeling by Jayapal Baskar Babujee and Shobana Loganathan [8]. Zero divisor graphs were examined by D. Bharathi and D. Eswara Rao [12]. Sk. Sajana and D. Bharathi [13] investigated intersection graphs of zero divisors. The results of Difference Cordial labeling on Zero divisor graphs $\Gamma(Z_{np})$, $\Gamma(Z_{p^n})$, and $\Gamma(Z_{pq})$ were expanded in this paper.

II. Preliminaries

Definition 2.1 Cordial Labeling

Each vertex in a graph is given a cordial label of either 0 or 1, so that the number of vertices with the labels 0 and 1 differs by no more than 1, and the same is true for the number of edges with the labels 0 and 1 based on their endpoint labels.

Definition 2.2 Difference Cordial Labeling

Assume that G is a (p, q) graph. Let V(G) to {1,2,…,p} be mapped by f. Assign the label |f (u)− f(v) | to each edge uv . It's referred to as differential cordial labeling. When f is 1 −1 and $\left|e_f(0) - e_f(1)\right| \le 1$, the number of edges labeled with 1 and those not labeled with 1 are indicated by $e_f(1)$ and $e_f(0)$, respectively. A difference cordial graph is a graph that has a difference cordial labeling.

Definition 2.3 Divisor graph

A divisor graph is one in which two vertices are connected by an edge if one number divides the other, where vertices represent elements of a set of positive integers.

Definition 2.4 zero Divisors

Let R be a ring, If b ∈ R,b ≠0 such that a. b = 0 or b. a = 0, then an element a ∈ R,a ≠0 is referred to as a zero divisor.

Definition 2.5 Zero divisor graph $\Gamma(R)$

Assuming R is a commutative ring with unity, the zero divisor graph Γ(R) corresponds to the zero divisors of R. Each vertex represents a non-zero divisor in R. An edge exists between two vertices a and b if and only if a·b=0. In other words, two vertices are adjacent when the product of the respective elements is zero in the ring.

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III. Main Section

Theorem 3.1 [10] Every path graph is a difference cordial graph
Theorem 3.2 [10] K_n represents a difference graph of cordial if n ≤ 4
Theorem 3.3 [10] $K_{1,n}$ represents a difference graph of cordial if n ≤ 4
Theorem 3.4 [10] $K_{2,n}$ represents a difference graph of cordial if n ≤ 4
Theorem 3.5 The Zero divisor graph $\Gamma(Z_p)$ is not a difference cordial graph for prime numbers P>2.
Proof: Since Z_p has no zero divisors other than zero for prime numbers p>2
Theorem 3.6 The graph of the zero divisors $\Gamma(Z_{2p})$ is a difference cordial graph for prime numbers P ≤ 5.
Proof: If p is a prime number, then the vertex set of $\Gamma(Z_{2p})$ is
V(G) = $\{2,4,6 \dots\dots\dots 2(p-1), p\}$
Let V = $\{v_1, v_2, \dots\dots..v_p\}$
From this , we have that $E(\Gamma(Z_{2p}))$ = $\{v_i v_p, 1 \le i \le p-1\}$
Here , The graph of the zero divisors $\Gamma(Z_{2p}) \cong K_{1,p-1}$ then the graph is not difference cordial According to the theorem 3.3, for p-1≤ 4.
Therefore, the graph of the zero divisors $\Gamma(Z_{2p})$ is a difference cordial graph for prime numbers P ≤ 5
Example 3.7 The graph of the zero divisors $\Gamma(Z_{10})$, is a difference cordial graph.
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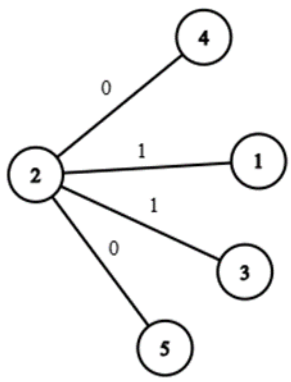
The vertex set of $\Gamma(Z_{2p})$ is V $(\Gamma(Z_{10}))$ = {2,4,6,10,5} =$\{v_1, v_2, v_3, v_4, v_5\}$
Here the zero divisor graph $\Gamma(Z_{10}) \cong K_{1,4}$
Let $v_1 = 1, v_2 = 3, v_3 = 4, v_4 = 5, and\ v_5 = 2$ then difference cordial Labelling of $\Gamma(Z_{10})$ is given Figure (i)



D.C.L of $(\Gamma(Z_{10}))$ **Figure (i)**

Theorem 3.8 The difference cordial graph $\Gamma(Z_{3p})$ is a zero divisor graph for primes P ≤ 5.
Proof: In the event that p is greater than or equal to 2, the vertex set of $\Gamma(Z_{3p})$ is
V $(\Gamma(Z_{3p}))$ = $\{3,6 \dots\dots\dots 3(p-1), p, 2p\}$
i.e V = $\{x_1, x_2, \dots\dots..x_{p-1}, y_1, y_2\}$
From this, we have that $E(\Gamma(Z_{3p}))$ = $\{y_1x_i, y_2x_i, 1 \le i \le p-1\}$
i) For p = 2 , $\Gamma(Z_{3p}) = \Gamma(Z_6)$, Hence $\Gamma(Z_{3p})$ is a difference cordial graph by theorem 3.6
ii)For p = 3 $\Gamma(Z_{3p}) = \Gamma(Z_9)$ and V $(\Gamma(Z_9))$ = {3,6} by theorem 3.1 it is a difference cordial graph
iii) For p = 5 $\Gamma(Z_{3p}) = \Gamma(Z_{15})$ and V $(\Gamma(Z_{15}))$ = {5,10,3,6,9,12}

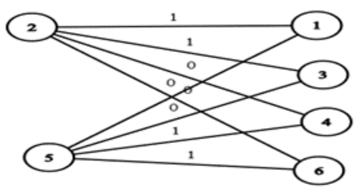
i.e. V = $\{y_1, y_2, x_1, x_2, x_3, x_4\}$

Here y_1, y_2 have four edges each without loss of generality say $f(y_1) = r$, $f(y_2) = s$
To obtain the edge label 1, the sole condition is that $f(x_i) = r-1$ and $f(x_j) = r + 1$ for certain indices i and j.

Similary $f(x_i) = s-1$ $f(x_j) = s+1$ for certain indices i and j.

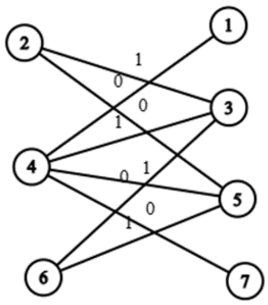
Then $e_f(1) \leq 4$, Hence $|e_f(0) - e_f(1)| \geq 4 - 4 = 0 \leq 1$
Hence , for p = 5 $\Gamma(Z_{3p}) = \Gamma(Z_{15})$ is a difference cordial graph.
We now presume that p is greater than 7, and we assume that f is a difference cordial
For p =7 the vertex set of $\Gamma(Z_{21})$ is
V = {3,6,9,12,15,18,7,14} i.e. $V = \{x_1, x_2, x_3, x_4, x_5, x_6, y_1, y_2\}$
Here y_1,y_2 have six edges each without loss of generolity say $f(y_1) = r$, $f(y_2) = s$.
For some i and j, the only possible outcome is that $f(x_i) = r-1$ and $f(x_j) = r + 1$; this is the only way to obtain the edge label 1.

Similary $f(x_i) = s-1$, $f(x_j) = s+1$ for certain indices i and j .
.
This suggests $e_f(1) \leq 4$, Hence $|e_f(0) - e_f(1)| \geq 8 - 4 = 4 \geq 1$
Which is contradiction.
Therefore for prime number $P \leq 5$, the graph of the zero divisor $\Gamma(Z_{3p})$ is difference cordial graph
Example 3.9 A difference cordial graph is represented by the zero divisor graph $\Gamma(Z_{15})$
V $(\Gamma(Z_{15}))$ = {5,10,3,6,9,12} , the graph of $\Gamma(Z_{15})$ is given in figure (ii)



D.C.L of $(\Gamma(Z_{15}))$ **Figure (ii)**

Theorem 3.10 For prime integer q<5, the graph of the zero divisors $\Gamma(Z_{4q})$ is difference cordial graph.
Proof: In the event that q is greater than or equal to 2, the vertex set of $\Gamma(Z_{4q})$ can be parition into Vertex set $V_1 = \{q, 2q, 3q\} = \{l_1, l_2, l_3\}$ and Vertex set $V_2 = \{2,4 \ldots\ldots\ldots 2(q-1), 2(q+1) \ldots., 2(2q-1)\} = \{m_1, m_2, \ldots\ldots.. m_{q-1,} m_{q+1,\ldots\ldots} m_{2q-1}\}$
i) For p = 2 , $\Gamma(Z_{4q}) = \Gamma(Z_8)$,
Vertex set of $\Gamma(Z_8)$ is V = {2,4,6} then $\Gamma(Z_8) \cong \Gamma(Z_6)$
Hence $\Gamma(Z_{4p})$ is a difference cordial graph by theorem 3.6
ii) For p = 3 $\Gamma(Z_{4p}) = \Gamma(Z_{12})$ The vertex set of $\Gamma(Z_{12})$ is V = {3,6,9,2,4,8,10} i.e $V = \{l_1, l_2, l_3, m_1, m_2, m_3, m_4\}$ Say $l_1 = 2$, $l_2 = 4$, $l_3 = 6$, $m_1 = 1$, $m_2 = 3$, $m_3 = 5$, $m_4 = 7$ the graph of $\Gamma(Z_{12})$ is given in figure(iii)

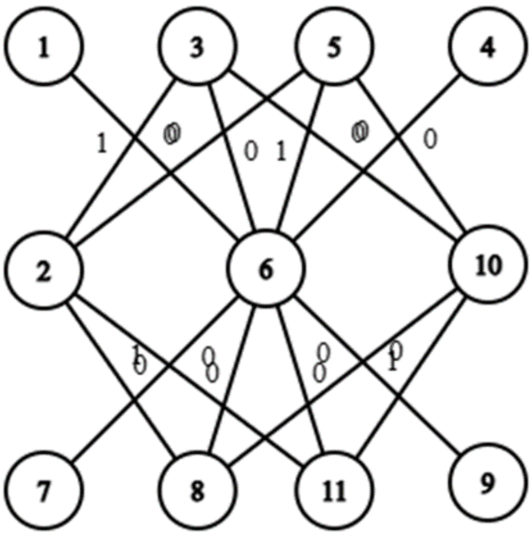


D.C.L of $(\Gamma(Z_{12}))$ **Figure (iii)**

Here $e_f(1) = 4$ and $e_f(0) = 4$, Hence $|e_f(0) - e_f(1)| = 0 \leq 1$
Hence $\Gamma(Z_{4p})$ is a difference cordial graph
iii) For p = 5 $\Gamma(Z_{4p}) = \Gamma(Z_{20})$
The vertex set $V(\Gamma(Z_{20})) = \{5,10,15,2,4,6,8,12,14,16,18\}$
i.e $V = \{w_1, w_2, w_3, z_1, z_2, z_3, z_4, z_5, z_6, z_7, z_8\}$
Here w_1, w_2, w_3 have 4 edges, 8 edges, 4 egdes respectively , without loss of generality
Say $f(w_1) = r$, $f(w_2) = s$, $f(w_3) = t$
For some i and j, the only possible outcome is that $f(z_i) = r-1$ and $f(z_j) = r + 1$; this is the only way to obtain the edge label 1.

Similary $f(z_i) = s-1$, $f(z_j) = s+1$ for some i,j ; $f(z_i) = t-1$, $f(z_j) = t+1$ for certain i and j
This indicates $e_f(1) \leq 6$, Hence $|e_f(0) - e_f(1)| \geq 10 - 6 = 4 \geq 1$
Which is contradiction.
Consequently, for the prime number q < 5, the graph of the zero divisors $\Gamma(Z_{4q})$ is classified as a difference cordial graph.

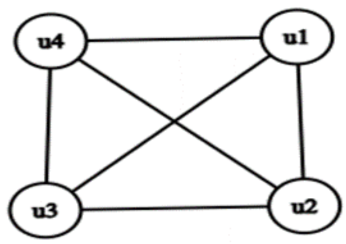
Example 3.11 the graph of the zero divisors $\Gamma(Z_{20})$ is not difference cordial graph .
$V = \{5,10,15,2,4,6,8,12,14,16,18\}$
i.e $V = \{w_1, w_2, w_3, z_1, z_2, z_3, z_4, z_5, z_6, z_7, z_8\}$
Say $w_1 = 2$, $w_2 = 6$, $w_3 = 10$, $z_1 = 1$, $z_2 = 3$, $z_3 = 4$, $z_4 = 5$
$z_5 = 7$, $z_6 = 8$, $z_7 = 9$, $z_8 = 11$
the graph of $\Gamma(Z_{20})$ is given in figure(iv)



D.C.L of $(\Gamma(Z_{20}))$ **Figure (iv)**

Here $e_f(1) = 6$ and $e_f(0) = 10$, Hence $|e_f(0) - e_f(1)| = 4 \geq 1$
Accordingly, the zero divisor graph $\Gamma(Z_{20})$ does not constitute a difference cordial graph.

Theorem 3.12 For prime integer P≤5 the graph of the zero divisors $\Gamma(Z_{5p})$ is classified as a difference cordial graph.
Proof: If p is a prime number, then the set of all possible vertices in $\Gamma(Z_{5p})$ equals
$V = \{p, 2p, 3p, 4p, 5, 10, \dots \dots 5(P-1)\}$ i.e $V = \{u_1, u_2, u_3, u_4, v_1, v_2, \dots, v_{p-1}\}$
i) For p = 2 , $\Gamma(Z_{5p}) = \Gamma(Z_{10})$, Hence $\Gamma(Z_{10})$ is a difference cordial graph by theorem 3.6
ii) For p = 3 $\Gamma(Z_{5p}) = \Gamma(Z_{15})$ Hence $\Gamma(Z_{15})$ is a difference cordial graph by theorem 3.8
iii) For p = 5 $\Gamma(Z_{5p}) = \Gamma(Z_{25})$ the vertex set of $\Gamma(Z_{25})$ is $V = \{5,10,15,20\}$
Here $\Gamma(Z_{25}) \cong K_4$, Therefore, theorem 3.2 indicates that $\Gamma(Z_{25})$ is a difference cordial graph.
Figure (v) shows the graph of $\Gamma(Z_{25})$



Zero divisor graph of $\Gamma(Z_{25})$ **Figure (v)**

iv) For p = 7 the vertex set of $\Gamma(Z_{35})$ is V = {7,14,21,28,5,10,15,20,25,30}
i.e V $= \{a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4, b_5, b_6\}$

Here a_1, a_2, a_3, a_4 have 6 edges each respectively , without loss of generality
Say $f(a_1) = r$, $f(a_2) = s$, $f(a_3) = t$, $f(a_4) = w$
For the edge label 1, the sole possibility is that $f(b_i) = r-1$ and $f(b_j) = r + 1$ for certain i,j.

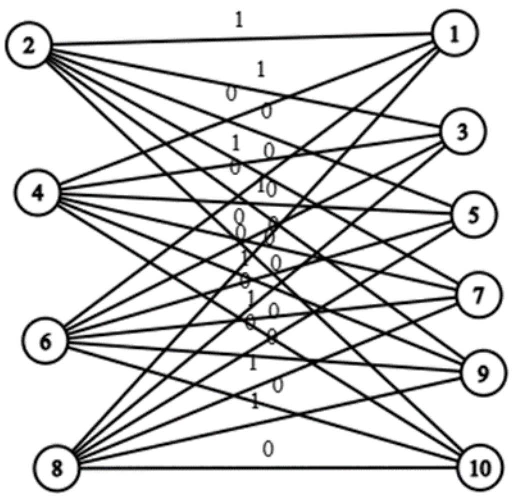
Similary $f(b_i) = s-1$, $f(b_j) = s +1$ for some i,j ; $f(b_i) = t-1$, $f(b_j) = t+1$ for certain i,j;

$f(b_i) = w-1$, $f(b_j) = w+1$ for certain i,j
This implies $e_f(1) \leq 8$, Hence $\left|e_f(0) - e_f(1)\right| \geq 16 - 8 \geq 1$
Which is contradiction.
As a result, the graph of the zero divisors $\Gamma(Z_{5p})$ is difference cordial graph for prime numbers P that are less than or equal to 5.

Example 3.13 There is no difference cordial graph with zero divisor of $\Gamma(Z_{35})$.
the vertex set of $\Gamma(Z_{35})$ is V = {7,14,21,28,5,10,15,20,25,30}

V $= \{a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4, b_5, b_6\}$

Say $a_1 = 2$, $a_2 = 4$, $a_3 = 6$, $a_4 = 8$, $b_1 = 1$, $b_2 = 3$, $b_3 = 5$, $b_4 = 7$, $b_5 = 9$, $b_6 = 10$
the graph of $\Gamma(Z_{35})$ is given in figure(vi)



D.C.L of ($\Gamma(Z_{35})$) **Figure (vi)**

Here $e_f(1) = 8$ and $e_f(0) = 16$, Hence $\left|e_f(0) - e_f(1)\right| = 8 \geq 1$

There is no difference cordial graph with zero divisor of $\Gamma(Z_{35})$.

Theorem 3.14 For the prime number P=2, the graph of the zero divisors $\Gamma(Z_{6p})$ is classified as difference cordial graph.

Proof: If we assume that p is a prime number, then the vertex set of the set $\Gamma(Z_{6p})$

V = $\{x \in Z_{6p}/\gcd(x, 6p) \neq 1\}$

For p= 2 , $\Gamma(Z_{6p})$ = $\Gamma(Z_{12})$ it difference cordial graph by theorem 3.10

For p = 3 , $\Gamma(Z_{6p}) = \Gamma(Z_{18})$

The vertex set of $\Gamma(Z_{18})$ is V= {2,3,4,6,8,9,10,12,14,15,16} = {3,9,15,2,4,6,8,10,12,14,16} i.e V = $\{r_1, r_2, r_3, s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8\}$

Here r_2 has 8 edges, without loss of generality

Say f(r_2) = x , The only way to get the edge label 1 is

that f(s_i) = x-1 , f(s_j) = x + 1 for some indices i and j

This implies $e_f(1) \leq 5$, (since $r_1, r_3, s_3, s_6,$ may have maximum edge label 1 is three)

Hence $|e_f(0) - e_f(1)| \geq 7 - 5 \geq 1$

Which is contradiction.

Consequently, the Zero divisor graph $\Gamma(Z_{6p})$ is not a difference cordial graph for prime integer p = 3.

Hence, For the prime number P=2, the graph of the zero divisors $\Gamma(Z_{6p})$ is classified as adifference cordial graph.

Example 3.15 There is no difference cordial graph found in the zero divisor graph $\Gamma(Z_{18})$.

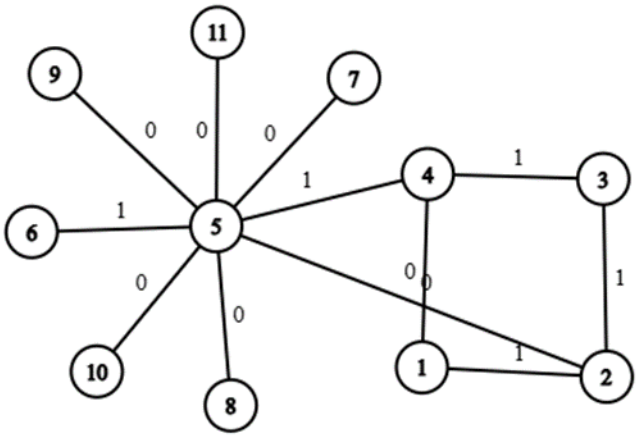
The vertex set of $\Gamma(Z_{18})$ is V= {3,9,15,2,4,6,8,10,12,14,16}

i.e V = $\{k_1, k_2, k_3, m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8\}$ to maximize edge label 1

consider k_1 = 1 , k_2= 6 , k_3 = 3 , m_1= 6 , m_2 = 7, m_3 = 2, m_4 = 8,

m_5 = 9 , m_6 =4, m_7 = 10, m_8 = 11

the graph of $\Gamma(Z_{18})$ is given in figure(vii)



D.C.L of ($\Gamma(Z_{18})$) **Figure (vii)**

Here $e_f(1)$ =5 and $e_f(0)$ =7 , Hence $|e_f(0) - e_f(1)| = 2 \geq 1$

Hence , There is no difference cordial graph found in the zero divisor graph $\Gamma(Z_{18})$.

Corollary 3.16 : For n > 6 and P > 5, the zero divisor graph $\Gamma(Z_{np})$ does not show a difference cordial.

Proof: Since

i) For n=1, $\Gamma(Z_{np}) = \Gamma(Z_p)$ according to theorem 3.5, the cordial graph can't be difference cordial

ii) For n=2 , $\Gamma(Z_{np}) = \Gamma(Z_{2p})$ is not difference cordial graph for p>5 by theorem 3.6

iii) For n=3 , $\Gamma(Z_{np}) = \Gamma(Z_{3p})$ is not difference cordial graph for p>5 by theorem 3.8

iv) For n=4 , $\Gamma(Z_{np}) = \Gamma(Z_{4p})$ is not difference cordial graph for p>3 by theorem 3.10

v) For n=5 , $\Gamma(Z_{np}) = \Gamma(Z_{5p})$ is not difference cordial graph for p>5 by theorem 3.12

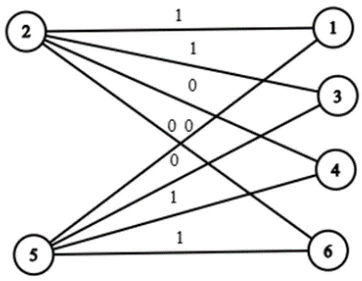
vi) For n=6 , $\Gamma(Z_{np}) = \Gamma(Z_{6p})$ is not difference cordial graph for p>3 by theorem 3.14

Therefore, we can deduce that the graph of the zero divisors $\Gamma(Z_{np})$ does not exhibit the characteristics of a difference cordial graph for n > 6 and P > 5.

Theorem 3.17 the graph of the zero divisors $\Gamma(Z_{pq})$ is classified as difference cordial graph when 2≤p,q≤5.
Proof: let p,q are prime numbers So, the vertex set of $\Gamma(Z_{pq})$ is
$V_1 = \{p, 2p, 3p \dots \dots .(q-1)p\}$ and $V_2 = \{q, 2q, 3q \dots \dots \dots (p-1)q\}$
i.e $V_1 = \{v_1, v_2, \dots \dots .., v_p\}$ and $V_2 = \{u_1, u_2, \dots \dots .., u_q\}$
$E(\Gamma(Z_{pq})) = \{u_i v_j : u_i \in V_2, v_j \in V_1 \text{ for } 1 \le i \le p-1, 1 \le j \le q-1\}$
i) For q = 2 , $\Gamma(Z_{pq}) = \Gamma(Z_{2p})$, Hence $\Gamma(Z_{2p})$ is a difference cordial graph for $p \le 5$ by theorem 3.6
ii) For q = 3 $\Gamma(Z_{pq}) = \Gamma(Z_{3p})$ Hence $\Gamma(Z_{3p})$ is a difference cordial graph for $p \le 5$ by theorem 3.8
iii) For q = 5 $\Gamma(Z_{pq}) = \Gamma(Z_{5p})$ Hence $\Gamma(Z_{5p})$ is a difference cordial graph for $p \le 5$ by theorem 3.12
iv) For q = 7 $\Gamma(Z_{pq}) = \Gamma(Z_{7p})$ is not difference cordial for any prime number p by theorem 3.6, 3.8,3.12

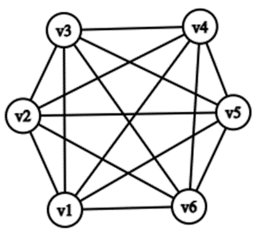
Consequently, the graph of the zero divisors $\Gamma(Z_{pq})$ is classified as a difference cordial network if 2≤p,q≤5.

Example 3.18
The graph of the zero divisors $\Gamma(Z_{pq})$ is classified as a difference cordial graph when p = 3 and q = 5.
For p = 3 & q =5 $\Gamma(Z_{pq}) = \Gamma(Z_{15})$ and $V(\Gamma(Z_{15})) = \{5,10,3,6,9,12\}$
i.e $V = \{w_1, w_2, z_1, z_2, z_3, z_4\}$
Say $w_1 = 2$, $w_2 = 5$, $z_1 = 1$, $z_2 = 3$, $z_3 = 4$, $z_4 = 6$
the graph of $\Gamma(Z_{15})$ is given in figure(viii)



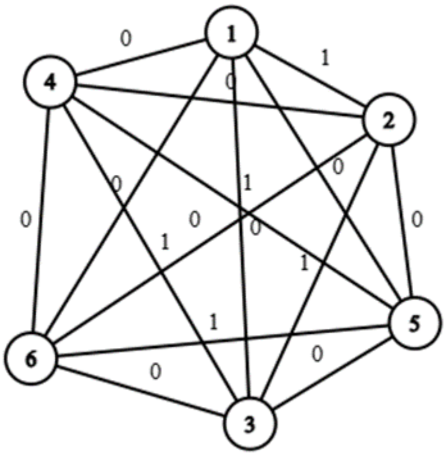
D.C.L of $(\Gamma(Z_{15}))$ **Figure (viii)**

Here $e_f(1) = 4$ and $e_f(0) = 4$, Hence $|e_f(0) - e_f(1)| = 0 \le 1$
Hence it is difference cordial graph for p = 3 ,q=5
Theorem 3.19 The Zero divisor graph $\Gamma(Z_{p^2})$ repregents a difference cordial graph for $p \le 5$.
Proof: Let p be a prime number; then the vertex set of $\Gamma(Z_{p^2})$ is
$V = \{p, 2p, 3p \dots \dots .(p-1)p\}$ i.e. $V = \{v_1, v_2, \dots \dots .., v_{p-1}\}$
i) For p = 2 , $\Gamma(Z_{p^2}) = \Gamma(Z_4)$, Therefore, by theorem 3.6, it is a difference cordial graph.
ii) For p = 3 $\Gamma(Z_{p^2}) = \Gamma(Z_9)$ Consequently, it is a difference cordial graph, as per theorem 3.8.
iii) For p = 5 $\Gamma(Z_{p^2}) = \Gamma(Z_{25})$ Therefore, it is a difference cordial graph as stated in Theorem 3.12.
iv) For p = 7 $\Gamma(Z_{p^2}) = \Gamma(Z_{49})$ the vetex set of $\Gamma(Z_{49})$ is
$V = \{7,14,21,28,35,42\}$ i.e. $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$
Here $\Gamma(Z_{49}) \cong K_6$, Based to theorem 3.2, $\Gamma(Z_{49})$ is not a difference cordial graph.
The graph of $\Gamma(Z_{49})$ is given figure (ix)



Zero divisor graph of $\Gamma(Z_{49})$ **Figure (ix)**

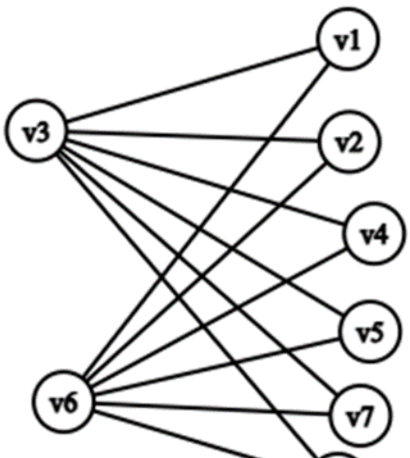
Example 3.20 There is no difference cordial graph in the zero divisor graph $\Gamma(Z_{49})$.
the vetex set of $\Gamma(Z_{49})$ is V = {7,14,21,28,35,42} i.e. V = $\{d_1, d_2, d_3, d_4, d_5, d_6\}$
Say $d_1 = 1$, $d_2 = 2$, $d_3 = 3$, $d_4 = 4$, $d_5 = 5$, $d_6 = 6$
the graph of $\Gamma(Z_{49})$ is given in figure(x)



D.C.L of ($\Gamma(Z_{49})$) **Figure (x)**

Here $e_f(1) = 5$ and $e_f(0) = 10$, Hence $|e_f(0) - e_f(1)| = 5 \geq 1$
The zero divisor graph $\Gamma(Z_{49})$ cannot be considered a difference cordial graph.

Theorem 3.21 the graph of the zero divisors $\Gamma(Z_{p^3})$ is difference cordial graph only when p=2.
Proof: Let p is prime number then the vertex set of $\Gamma(Z_{p^3})$ is
V $=\{p, 2p, 3p \dots\dots (p^2-1)p\}$ i.e. V = $\{v_1, v_2, \dots\dots\dots, v_{p^2-1}\}$
i) For p = 2 , $\Gamma(Z_{p^3}) = \Gamma(Z_8)$, Hence it is a difference cordial graph by theorem 3.10
ii) For p = 3 $\Gamma(Z_{p^3}) = \Gamma(Z_{27})$
The vertex set is V ={3,6,9,12,15,18,21,24}
i.e.V = $\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$
Here $\Gamma(Z_{27}) \cong K_{2,6}$, hence $\Gamma(Z_{p^3})$ is not difference cordial graph for p=3 by theorem 3.4.
The graph of the $\Gamma(Z_{27})$ is given in figure (xi)

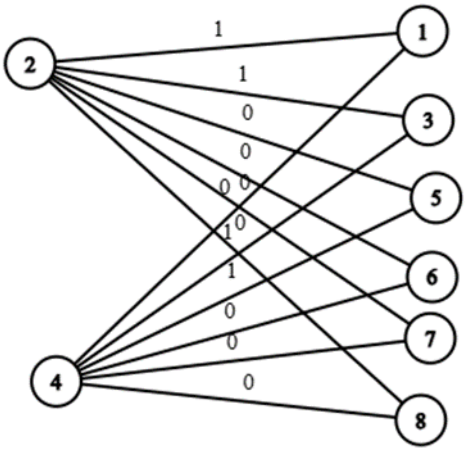


Zero divisor graph of $\Gamma(Z_{27})$ **Figure (xi)**

Therefore, for p>2, the Zero divisor graph $\Gamma(Z_{p^3})$ is not difference cordial graph.

Example 3.22 A difference cordial graph does not exist for the zero divisor graph $\Gamma(Z_{27})$.

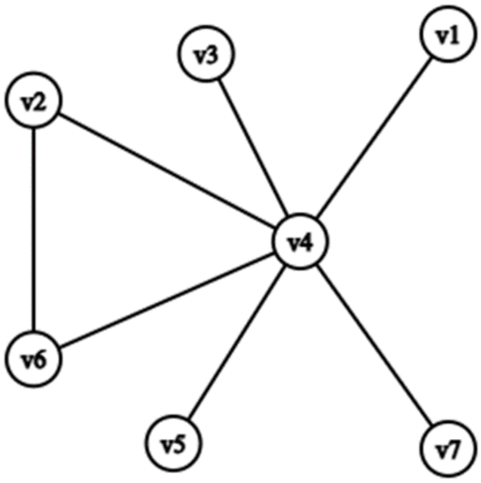
The vertex set is V ={3,6,9,12,15,18,21,24} i.e.V = $\{c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8\}$
Say $c_1 = 1$, $c_2 = 3$, $c_3 = 2$, $c_4 = 5$, $c_5 = 6$, $c_6 = 4$, $c_7 = 7$, $c_8 = 8$
the graph of $\Gamma(Z_{27})$ is given in figure(xii)



D.C.L of $(\Gamma(Z_{27}))$ **Figure (xii)**

Here $e_f(1) = 4$ and $e_f(0) = 8$, Hence $|e_f(0) - e_f(1)| = 4 \geq 1$
Therefore, the zero divisor graph $\Gamma(Z_{27})$ is not a difference cordial graph.

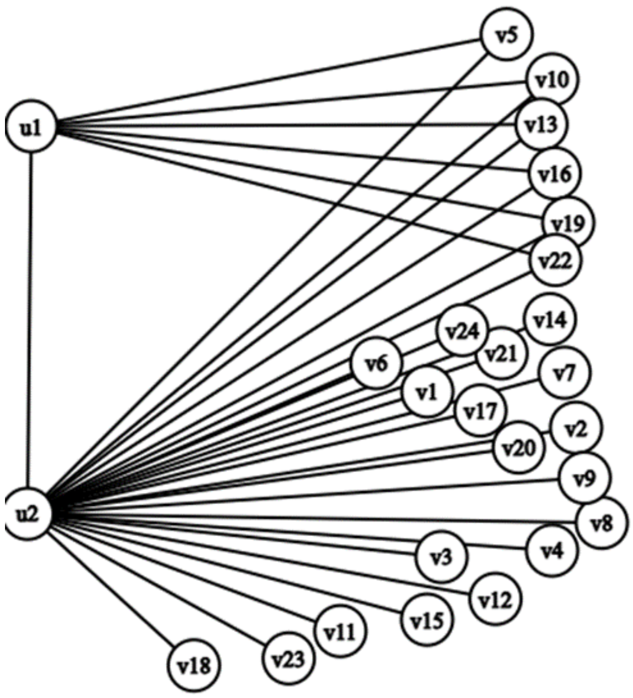
Theorem 3.23 The graph of the zero divisors $\Gamma(Z_{p^4})$ is a difference cordial graph for every prime number p = 2.
Proof: Let p is prime number then the vertex set of $\Gamma(Z_{p^4})$ is
Vertex set $=\{p, 2p, 3p \dots \dots (p^4 - p), p^2, 2p^2, \dots \dots \dots (p^4 - p^2), p^3, 2p^3, \dots (p^4 - p^3)\}$
For p=2 $\Gamma(Z_{p^4}) = \Gamma(Z_{16})$, the vertex set V = {2,4,6,8,10,12,14}
i.e V = $\{l_1, l_2, l_3, l_4, l_5, l_6, l_7\}$
The graph of $\Gamma(Z_{16})$ is shown in figure (xiii)



Zero divisor graph $\Gamma(Z_{16})$ **Figure (xiii)**

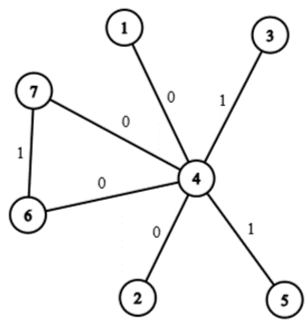
Here l_4 has 6 edges, without loss of generality
Say $f(l_4) = x$, After that, the only way to get the edge mark 1 is
that $f(l_i) = x-1$, $f(l_j) = x + 1$ for some of the i, j

This implies $e_f(1) \leq 3$, (since v_2 , v_6 may have edge label 1)
Hence $\left|e_f(0) - e_f(1)\right| \geq 7 - 3 - 4 = 0 \leq 1$
For prime number P=2, the Zero divisor graph $\Gamma(Z_{p^4})$ is difference cordial.
For p=3 $\Gamma(Z_{p^4})$ = $\Gamma(Z_{81})$, the vertex set V = {9,27,3,6,12, 15,18,21,24,30,33,36,39,42,45,48,51,54,57,60,63,66,69,72,75,78}
i.e V = $\{u_1, u_2, v_1, v_2, v_3, \dots\dots\dots.., v_{23}, v_{24}\}$ where u_1= 9 & u_2 = 27
The graph of $\Gamma(Z_{81})$ is given in figure (xiv)



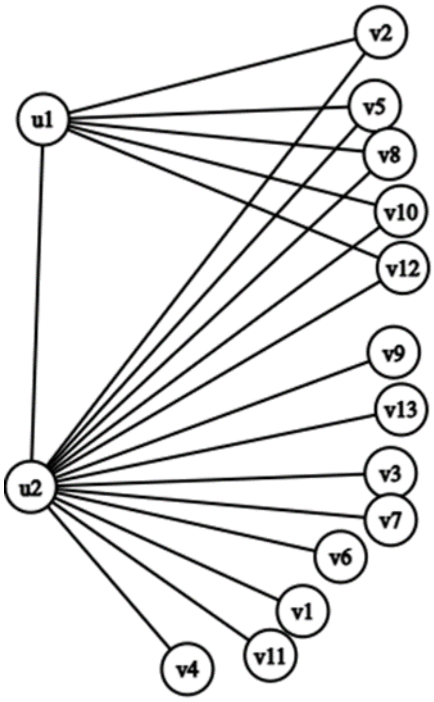
Zero divisor graph $\Gamma(Z_{81})$ **figure (xiv)**

Here u_1, u_2 have 7, 25 edges each respectively , without loss of generality
Say $f(u_1)$ = r , $f(u_2)$ = s
To obtain the edge label 1, the only possibility is that $f(v_i)$ = r-1 and $f(v_j)$ = r + 1 for some i , j.
Similary $f(v_i)$ = s-1 , $f(v_j)$ = s +1 for some integers i and j
In this case $e_f(1) \leq 4$, Hence $\left|e_f(0) - e_f(1)\right| \geq 31 - 4 - 4 \geq 1$
Which is contradiction.
Therefore, for prime integer P=3, the Zero divisor graph $\Gamma(Z_{p^4})$ is not difference cordial graph.
There fore, the graph of the zero divisors $\Gamma(Z_{p^4})$ is difference cordial graph for any prime number p>2
Example 3.24 Zero divisor graph $\Gamma(Z_{16})$ is difference cordial graph
the vertex set V = {2,4,6,8,10,12,14} i.e V = $\{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$
Say v_1 = 1 , v_2 = 7, v_3 = 3, v_4 = 4 ,v_5 = 5 , v_6 =4, v_7 = 2
the graph of $\Gamma(Z_{16})$ is given in figure(xv)



D.C.L of ($\Gamma(Z_{16})$) **Figure (xv)**

Here $e_f(1) = 3$ and $e_f(0) = 8$, Hence $|e_f(0) - e_f(1)| = 4 \geq 1$
Hence , Zero divisor graph $\Gamma(Z_{27})$ is difference cordial graph
Theorem 3.25 The Zero divisor graph $\Gamma(Z_{p^5})$ is difference cordial graph for any prime number p≥2
Proof: Let p is prime number then the vertex set of $\Gamma(Z_{p^5})$ is
V $=\{p, 2p, 3p \dots \dots (p^4 - 1)p\}$
For P =2 , $\Gamma(Z_{p^5}) = \Gamma(Z_{32})$, the vertex set is
V = {8,16,2,4,6,10,12,14,18,20,22,24,26,28,30}
i.e V = $\{u_1, u_2, v_1, v_2, v_3, \dots \dots \dots, v_{12}, v_{13}\}$ where $u_1 = 4$ & $u_2 = 16$
The graph of $\Gamma(Z_{32})$ is given in figure (xvi)



Zero divisor graph of $\Gamma(Z_{32})$ in figure (xvi)

Here u_1, u_2 have 6, 14 edges each respectively , without loss of generality
Say $f(u_1) = r$, $f(u_2) = s$
The edge label 1 can only be obtained by setting $f(v_i) = r-1$ and $f(v_j) = r + 1$ for some i,j.

Similary $f(v_i) = s-1$, $f(v_j) = s+1$ for some i and j
That means $e_f(1) \leq 4$, Hence $|e_f(0) - e_f(1)| \geq 19 - 4 - 4 \geq 1$
Which is contradiction.
Hence, for prime number P= 2 , The zero divisor graph $\Gamma(Z_{p^5})$ does not exhibit the properties of a difference cordial graph.
There fore, $\Gamma(Z_{p^5})$ is a difference cordial graph for all prime numbers p that are greater than or equal to 2.
Corollary 3.26 For n ≥5 and any prime number p ≥ 2, the graph of the zero divisors $\Gamma(Z_{p^n})$ is not a difference cordial graph.

IV. Conclusion:

In closing, we investigate that zero divisor graphs

i) For $n \leq 10$ $\Gamma(Z_{np})$ is for $\begin{cases} n = 2,3,5 \text{ and } p \leq 5 \text{ differrence cordial} \\ n = 4 \text{ and } p = 2, 3 \text{ differrence cordial} \\ n = 6, 8, 10 \text{ and } p = 2 \text{ difference cordial} \end{cases}$

ii) For $n \leq 10$ $\Gamma(Z_{np})$ is for n = 1,7,9 it does not difference cordial

iii) For $n \leq 4$ $\Gamma(Z_{p^n})$ is for $\begin{cases} n = 2 \text{ and } p \leq 5 \text{ difference cordial} \\ n = 3,4 \text{ and } p = 2 \text{ difference cordial} \end{cases}$

iv) $\Gamma(Z_{pq})$ is difference cordial for $2 \leq p, q \leq 5$

A similar investigation can be carried out for some other graphs.

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