On The Polar Curves Functions Treatment By Different Software Tools: A Didactic Approach

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Abstract:

This article presents an analysis of various software tools for the processing and visualization of functions in polar coordinates. It highlights specialized mathematical programs and computational environments that allow for graphing polar curves, analyzing their behavior, and performing dynamic simulations. Among the main analytical advantages are the ease of graphically representing complex functions, the ability to perform precise area and length calculations, and the option of immediately exploring transformations and parameter variations. In the educational field, the use of these tools contributes to improving university students' conceptual understanding, as it facilitates the geometric interpretation of functions, promotes visual and interactive learning, and supports the development of skills for relating theory to practice. In conclusion, the software constitutes an essential resource for both teaching and research, offering a dynamic environment that simplifies mathematical analysis and strengthens academic training at the higher education level.

Keywords: polar functions, analytic geometry, two-dimensional elements, academic performance, computational tools

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I. INTRODUCTION

The concept of polar coordinates, which is the foundation for polar functions, goes back to the work of Greek mathematicians. Hipparchus (2nd century BCE) and later Ptolemy (2nd century CE) studied angles and chords in circles, which foreshadowed the idea of relating points to a radius and an angle. The formal use of polar coordinates began much later. In the 17th century, with the rise of analytic geometry, mathematicians such as René Descartes (Cartesian geometry) and Bonaventura Cavalieri started to explore new ways to describe curves. It was Grégoire de Saint-Vincent and Blaise Pascal who began using radial distances in some problems, but the first clear use of polar coordinates is often credited to Isaac Newton and Jacob Bernoulli in the late 1600s [1-5].

One of the earliest famous polar functions was the spiral of Archimedes, studied by Archimedes around 225 BCE. Later, in the 17th century, Jacob Bernoulli investigated the logarithmic spiral, calling it the "miraculous spiral" because of its self-similarity. These examples showed that polar equations could represent curves more naturally than Cartesian coordinates in many cases. During the 18th century, polar coordinates became standard in calculus and mechanics. Mathematicians like Euler and Laplace used them in physics and astronomy to describe planetary motion, since circular and spiral paths are easier to model in polar form. By the 19th century, polar functions were a regular part of mathematical analysis, especially in complex numbers, where some expressions linked algebra, geometry, and trigonometry elegantly. [6-10]

Today, polar functions are essential in mathematics, physics, and engineering. They are used to model oscillations, electromagnetic fields, control systems, and wave patterns. With the aid of modern software, plotting and analyzing polar functions has become accessible, making them an important didactic tool in universities.

Polar functions often describe curves like spirals, roses, or lemniscates that are not easy to imagine by hand. Without software, students struggle to connect the symbolic expression with its geometric representation. To draw polar graphs by hand, students must calculate many points for different values of θ , convert them into Cartesian coordinates, and then carefully sketch the curve. This process is time-consuming and error-prone.

Determining intersections, areas, or lengths of curves in polar coordinates requires integral calculus and trigonometric manipulations. Without visualization, the meaning of these results can remain abstract. When

the focus is only on tedious calculations, students may feel frustrated and disconnected from the practical beauty of polar functions. [11-14]

Some of the advantages of using software tools are the generation of accurate polar plots in seconds, allowing students to see instantly how changes in the equation affect the shape of the curve. By interacting with dynamic graphs, students develop stronger intuition about symmetry, periodicity, and the behavior of polar functions. Instead of spending hours on manual plotting, students can dedicate time to analyzing properties, comparing functions, and solving real applications. Software minimizes mistakes in calculations and drawing, helping students focus on learning concepts rather than struggling with arithmetic. Interactive and visually appealing representations make learning more enjoyable and closer to professional practice in science and engineering [15-20].

II. GRAPHS OF THE POLAR EQUATIONS

The software MATLAB, GeoGebra, and Winplot were used to present the polar functions. Let us consider the specific polar functions as shown in equations (1-6), correspondent to the cardioid, 4-petal rose, 3-petal rose, lemniscate, circumference, and Archimedean spiral respectively. Table 1 shows the values of $r_i(\theta)$ in the interval $[0,2\pi]$ in increments of $\pi/18$ thou the real increment was $\pi/180$.

Table 1: Different values for $r_i(\theta)$ in the interval $[0, 2\pi]$

				· ` ′			
n	θ	$r_1(\theta)$	$r_2(\theta)$	$r_3(\theta)$	$r_4(\theta)$	$r_5(\theta)$	$r_6(\theta)$
1	0	4	0	2	0	0	0
2	$\pi/18$	3.47905547	1.02606043	1.73205081	Complex	0.86824089	0.17453293
3	π/9	2.97393957	1.92836283	1	Complex	1.71010072	0.34906585
4	π/6	2.5	2.59807621	0	Complex	2.5	0.52359878
5	$2\pi/9$	2.07163717	2.95442326	-1	Complex	3.21393805	0.6981317
6	$5\pi/18$	1.70186667	2.95442326	-1.7320508	Complex	3.83022222	0.87266463
7	$\pi/3$	1.40192379	2.59807621	-2	Complex	4.33012702	1.04719755
8	$7\pi/18$	1.18092214	1.92836283	-1.7320508	Complex	4.6984631	1.22173048
9	$4\pi/9$	1.04557674	1.02606043	-1	Complex	4.92403877	1.3962634
10	$\pi/2$	1	0	0	0	5	1.57079633
11	$5\pi/9$	1.04557674	-1.0260604	1	1.75447465	4.92403877	1.74532925
12	$11\pi/18$	1.18092214	-1.9283628	1.73205081	2.40522109	4.6984631	1.91986218
13	$2\pi/3$	1.40192379	-2.5980762	2	2.79181458	4.33012702	2.0943951
14	$13\pi/18$	1.70186667	-2.9544232	1.73205081	2.97712441	3.83022222	2.26892803
15	$7\pi/9$	2.07163717	-2.9544232	1	2.97712441	3.21393805	2.44346095
16	$5\pi/6$	2.5	-2.5980762	0	2.79181458	2.5	2.61799388
17	8π/9	2.97393957	-1.9283628	-1	2.40522109	1.71010072	2.7925268
18	$17\pi/18$	3.47905547	-1.0260604	-1.7320508	1.75447465	0.86824089	2.96705973
19	π	4	0	-2	0	0	3.14159265
19 20	$\frac{\pi}{19\pi/18}$	4 4.52094453	0 1.02606043	-2 -1.7320508	0 Complex	0 -0.8682408	3.14159265 3.31612558
					9	,	
20	$19\pi/18$	4.52094453	1.02606043	-1.7320508	Complex	-0.8682408	3.31612558
20 21	$19\pi/18$ $10\pi/9$	4.52094453 5.02606043	1.02606043 1.92836283	-1.7320508 -1	Complex Complex	-0.8682408 -1.7101007	3.31612558 3.4906585
20 21 22	$19\pi/18$ $10\pi/9$ $7\pi/6$	4.52094453 5.02606043 5.5	1.02606043 1.92836283 2.59807621	-1.7320508 -1	Complex Complex Complex	-0.8682408 -1.7101007 -2.5	3.31612558 3.4906585 3.66519143
20 21 22 23	$19\pi/18$ $10\pi/9$ $7\pi/6$ $11\pi/9$	4.52094453 5.02606043 5.5 5.92836283	1.02606043 1.92836283 2.59807621 2.95442326	-1.7320508 -1 0 1	Complex Complex Complex Complex	-0.8682408 -1.7101007 -2.5 -3.2139380	3.31612558 3.4906585 3.66519143 3.83972435
20 21 22 23 24	$19\pi/18$ $10\pi/9$ $7\pi/6$ $11\pi/9$ $23\pi/18$	4.52094453 5.02606043 5.5 5.92836283 6.29813333	1.02606043 1.92836283 2.59807621 2.95442326 2.95442326	-1.7320508 -1 0 1 1.73205081	Complex Complex Complex Complex Complex	-0.8682408 -1.7101007 -2.5 -3.2139380 -3.8302222	3.31612558 3.4906585 3.66519143 3.83972435 4.01425728
20 21 22 23 24 25	$19\pi/18$ $10\pi/9$ $7\pi/6$ $11\pi/9$ $23\pi/18$ $4\pi/3$	4.52094453 5.02606043 5.5 5.92836283 6.29813333 6.59807621	1.02606043 1.92836283 2.59807621 2.95442326 2.95442326 2.59807621	-1.7320508 -1 0 1 1.73205081 2	Complex Complex Complex Complex Complex Complex	-0.8682408 -1.7101007 -2.5 -3.2139380 -3.8302222 -4.3301270	3.31612558 3.4906585 3.66519143 3.83972435 4.01425728 4.1887902
20 21 22 23 24 25 26	$19\pi/18$ $10\pi/9$ $7\pi/6$ $11\pi/9$ $23\pi/18$ $4\pi/3$ $25\pi/18$	4.52094453 5.02606043 5.5 5.92836283 6.29813333 6.59807621 6.81907786	1.02606043 1.92836283 2.59807621 2.95442326 2.95442326 2.59807621 1.92836283	-1.7320508 -1 0 1 1.73205081 2	Complex Complex Complex Complex Complex Complex Complex Complex	-0.8682408 -1.7101007 -2.5 -3.2139380 -3.8302222 -4.3301270 -4.6984631	3.31612558 3.4906585 3.66519143 3.83972435 4.01425728 4.1887902 4.36332313
20 21 22 23 24 25 26 27	$19\pi/18$ $10\pi/9$ $7\pi/6$ $11\pi/9$ $23\pi/18$ $4\pi/3$ $25\pi/18$ $13\pi/9$	4.52094453 5.02606043 5.5 5.92836283 6.29813333 6.59807621 6.81907786 6.95442326	1.02606043 1.92836283 2.59807621 2.95442326 2.95442326 2.59807621 1.92836283 1.02606043	-1.7320508 -1 0 1 1.73205081 2 1.73205081 1	Complex Complex Complex Complex Complex Complex Complex Complex Complex	-0.8682408 -1.7101007 -2.5 -3.2139380 -3.8302222 -4.3301270 -4.6984631 -4.9240387	3.31612558 3.4906585 3.66519143 3.83972435 4.01425728 4.1887902 4.36332313 4.53785606
20 21 22 23 24 25 26 27 28	$ \begin{array}{c} 19\pi/18 \\ 10\pi/9 \\ 7\pi/6 \\ 11\pi/9 \\ 23\pi/18 \\ 4\pi/3 \\ 25\pi/18 \\ 13\pi/9 \\ 3\pi/2 \\ \end{array} $	4.52094453 5.02606043 5.5 5.92836283 6.29813333 6.59807621 6.81907786 6.95442326 7	1.02606043 1.92836283 2.59807621 2.95442326 2.95442326 2.59807621 1.92836283 1.02606043	-1.7320508 -1 0 1 1.73205081 2 1.73205081 1 0	Complex	-0.8682408 -1.7101007 -2.5 -3.2139380 -3.8302222 -4.3301270 -4.6984631 -4.9240387 -5	3.31612558 3.4906585 3.66519143 3.83972435 4.01425728 4.1887902 4.36332313 4.53785606 4.71238898
20 21 22 23 24 25 26 27 28 29	$ \begin{array}{c} 19\pi/18 \\ 10\pi/9 \\ 7\pi/6 \\ 11\pi/9 \\ 23\pi/18 \\ 4\pi/3 \\ 25\pi/18 \\ 13\pi/9 \\ 3\pi/2 \\ 14\pi/9 \end{array} $	4.52094453 5.02606043 5.5 5.92836283 6.29813333 6.59807621 6.81907786 6.95442326 7 6.95442326	1.02606043 1.92836283 2.59807621 2.95442326 2.95442326 2.59807621 1.92836283 1.02606043 0 -1.0260604	-1.7320508 -1 0 1 1.73205081 2 1.73205081 1 0 -1	Complex 1.75447465	-0.8682408 -1.7101007 -2.5 -3.2139380 -3.8302222 -4.3301270 -4.6984631 -4.9240387 -5 -4.9240387	3.31612558 3.4906585 3.66519143 3.83972435 4.01425728 4.1887902 4.36332313 4.53785606 4.71238898 4.88692191
20 21 22 23 24 25 26 27 28 29 30	$ \begin{array}{c} 19\pi/18 \\ 10\pi/9 \\ 7\pi/6 \\ 11\pi/9 \\ 23\pi/18 \\ 4\pi/3 \\ 25\pi/18 \\ 13\pi/9 \\ 3\pi/2 \\ 14\pi/9 \\ 29\pi/18 \\ \end{array} $	4.52094453 5.02606043 5.5 5.92836283 6.29813333 6.59807621 6.81907786 6.95442326 7 6.95442326 6.81907786	1.02606043 1.92836283 2.59807621 2.95442326 2.95442326 2.59807621 1.92836283 1.02606043 0 -1.0260604 -1.9283628	-1.7320508 -1 0 1 1.73205081 2 1.73205081 1 0 -1 -1.7320508	Complex 20 1.75447465 2.40522109	-0.8682408 -1.7101007 -2.5 -3.2139380 -3.8302222 -4.3301270 -4.6984631 -4.9240387 -5 -4.9240387 -4.6984631	3.31612558 3.4906585 3.66519143 3.83972435 4.01425728 4.1887902 4.36332313 4.53785606 4.71238898 4.88692191 5.06145483
20 21 22 23 24 25 26 27 28 29 30 31	$ \begin{array}{c} 19\pi/18 \\ 10\pi/9 \\ 7\pi/6 \\ 11\pi/9 \\ 23\pi/18 \\ 4\pi/3 \\ 25\pi/18 \\ 13\pi/9 \\ 3\pi/2 \\ 14\pi/9 \\ 29\pi/18 \\ 5\pi/3 \\ \end{array} $	4.52094453 5.02606043 5.5 5.92836283 6.29813333 6.59807621 6.81907786 6.95442326 7 6.95442326 6.81907786 6.59807621	1.02606043 1.92836283 2.59807621 2.95442326 2.95442326 2.59807621 1.92836283 1.02606043 0 -1.0260604 -1.9283628 -2.5980762	-1.7320508 -1 0 1 1.73205081 2 1.73205081 1 0 -1 -1.7320508 -2	Complex 2.40522109 2.79181458	-0.8682408 -1.7101007 -2.5 -3.2139380 -3.8302222 -4.3301270 -4.6984631 -4.9240387 -5 -4.9240387 -4.6984631 -4.3301270	3.31612558 3.4906585 3.66519143 3.83972435 4.01425728 4.1887902 4.36332313 4.53785606 4.71238898 4.88692191 5.06145483 5.23598776
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20 21 22 23 24 25 26 27 28 29 30 31 32 33	$ \begin{array}{c} 19\pi/18 \\ 10\pi/9 \\ 7\pi/6 \\ 11\pi/9 \\ 23\pi/18 \\ 4\pi/3 \\ 25\pi/18 \\ 13\pi/9 \\ 3\pi/2 \\ 14\pi/9 \\ 29\pi/18 \\ 5\pi/3 \\ 31\pi/18 \\ 16\pi/9 \\ \end{array} $	4.52094453 5.02606043 5.5 5.92836283 6.29813333 6.59807621 6.81907786 6.95442326 7 6.95442326 6.81907786 6.59807621 6.29813333 5.92836283	1.02606043 1.92836283 2.59807621 2.95442326 2.95442326 2.59807621 1.92836283 1.02606043 0 -1.0260604 -1.9283628 -2.5980762 -2.9544232 -2.9544232	-1.7320508 -1 0 1 1.73205081 2 1.73205081 0 -1 -1.7320508 -2 -1.7320508 -1	Complex 2.40522109 2.79181458 2.97712441 2.97712441	-0.8682408 -1.7101007 -2.5 -3.2139380 -3.8302222 -4.3301270 -4.6984631 -4.9240387 -5 -4.9240387 -4.6984631 -4.3301270 -3.8302222 -3.2139380	3.31612558 3.4906585 3.66519143 3.83972435 4.01425728 4.1887902 4.36332313 4.53785606 4.71238898 4.88692191 5.06145483 5.23598776 5.41052068 5.58505361
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$$r_1(\theta) = 4 - 3\sin(\theta) \tag{1}$$

$$r_2(\theta) = 3\sin(2\theta) \tag{2}$$

$$r_3(\theta) = 2\cos(3\theta) \tag{3}$$

$$r_4(\theta) = \sqrt{-9\sin(2\theta)} \tag{4}$$

$$r_5(\theta) = 5\sin(\theta) \tag{5}$$

$$r_6(\theta) = \theta \tag{6}$$

Figs. 2 and 4 show the graphs for the polar functions in MATLAB and Winplot respectively, while Figs. 5-10 show the code and graphs in GeoGebra. In MATLAB, it is necessary to create the vectors indicating an appropriate plotting step; also, especially for Eq. (4), the complex values for the function must be discriminated by code, while GeoGebra and Winplot do not present this issue. Fig. 1 contains the MATLAB code for the creation of polar function graphs. Fig.3 shows the six polar functions in Winplot as well as a box configuration for the 1st polar function.

```
1 -
       clc, clear all
 2 -
       t=0:pi/180:2*pi;
 3 —
       r1=4-3*sin(t); r2=3*sin(2*t); r3=2*cos(3*t);
       r4 = sqrt((-9*sin(2*t))); r4 = real(r4); r5 = 5*sin(t); r6 = t;
 4 -
 5 -
       subplot(2,3,1),polarplot(t,r1,'r','Linewidth',3)
 6 -
       set(gca, 'Fontsize', 12), title('r 1(\theta)')
 7 -
       subplot(2,3,2),polarplot(t,r2,'y','Linewidth',3)
 8 -
       set(gca, 'Fontsize', 12), title('r 2(\theta)')
       subplot(2,3,3),polarplot(t,r3,'b','Linewidth',3)
 9 -
10 -
       set(gca, 'Fontsize', 12), title('r 3(\theta)')
       subplot(2,3,4),polarplot(t,r4,'m','Linewidth',3)
11 -
       set(gca, 'Fontsize', 12), title('r_4(\theta)')
12 -
       subplot(2,3,5),polarplot(t,r5,'k','Linewidth',3)
13 -
14 -
       set(gca, 'Fontsize', 12), title('r 5(\theta)')
15 -
       subplot(2,3,6),polarplot(t,r6,'c','Linewidth',3)
       set(gca, 'Fontsize', 12), title('r 6(\theta)')
16 -
           Figure 1. MATLAB code for the generation of the polar function graphs
```

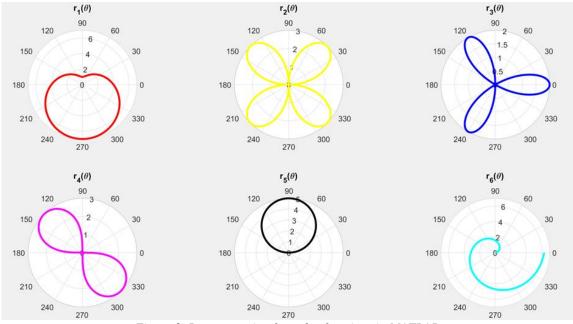


Figure 2. Representation for polar functions in MATLAB

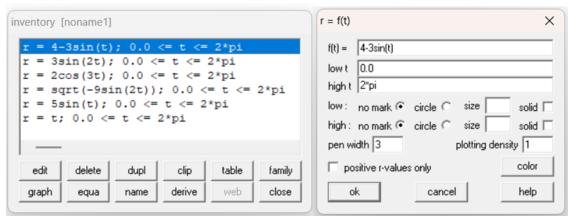


Figure 3. Equation configuration windows in Winplot

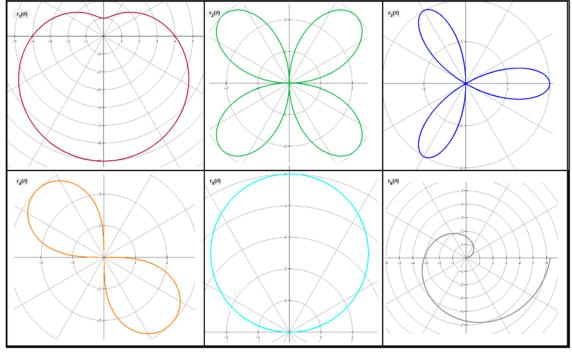


Figure 4. Representation for polar functions in Winplot

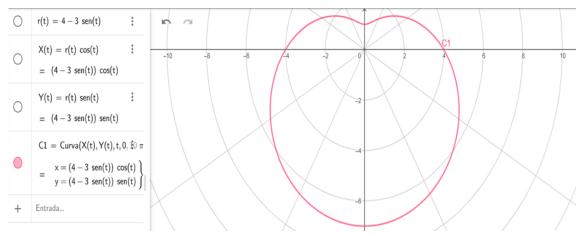


Figure 5. Representation for polar function cardioid in GeoGebra

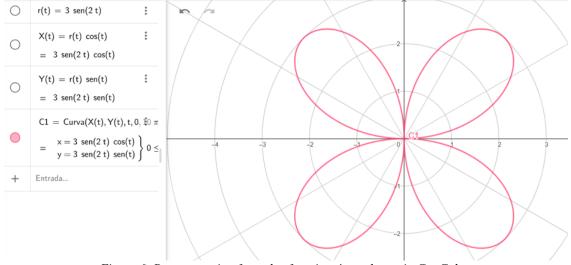


Figure 6. Representation for polar function 4-petal rose in GeoGebra

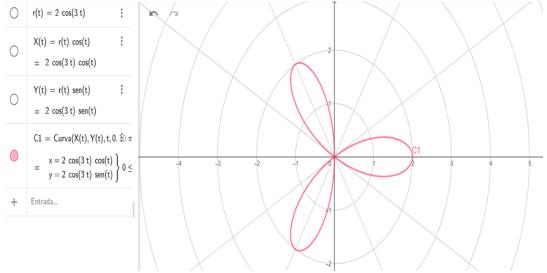


Figure 7. Representation for polar function 3-petal rose in GeoGebra

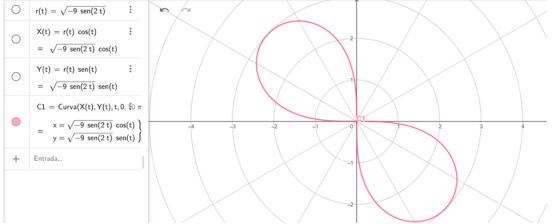


Figure 8. Representation for polar function lemniscate in GeoGebra

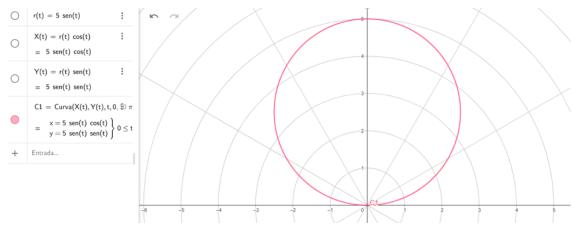


Figure 9. Representation for polar function circumference in GeoGebra

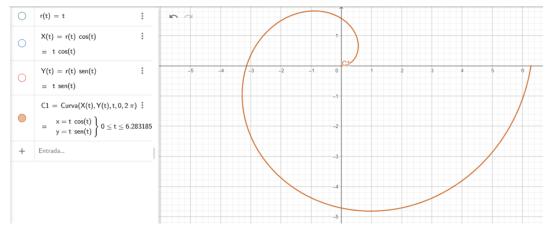


Figure 10. Representation for polar function Archimedean spiral in GeoGebra

III. DISCUSSION

Each of these tools (MATLAB, Winplot, and GeoGebra) has its strengths for students learning and creating polar functions (like cardioids, rose curves, spirals, lemniscates, etc.). For instance, MATLAB is powerful and professional; it is widely used in engineering and science, so students learn a tool relevant for research and industry, and students can link polar function exploration to applications in physics, robotics, signal processing, antenna design, etc. On the other hand, the commands must be known for programming the graphs generation code; besides, in some cases, additional commands have to be used for eliminating complex results, as in Eq. (4).

On the other hand, Winplot is simple to install and use, making it very accessible for students, who can input polar equations directly, without programming knowledge. It offers fast visualization, is also good for quickly generating graphs and experimenting with different equations. Its simplicity makes it excellent for beginners who need to understand the shape of functions without worrying about coding or advanced settings.

Finally, GeoGebra is Interactive and dynamic, sliders let students vary parameters in real time, instantly seeing how curves change; it runs on browsers and mobile devices, lowering barriers for students. GeoGebra also emphasizes exploration and intuition through geometry and interactivity, perfect for high school and early university levels, besides many prebuilt activities and shared examples exist for polar functions. It also connects algebra, geometry, and calculus in one environment, reinforcing learning links.

IV. CONCLUSION

This work provided various software tools to generate graphs for polar functions. Different mathematical software tools for the treatment and graphical representation of polar functions demonstrated their significant value in both analytical and didactic contexts. These platforms not only simplified the visualization of complex curves but also enhanced accuracy in calculations related to areas, lengths, and intersections. For our students, the use of software reduced the cognitive load associated with manual plotting, allowing them to focus on developing conceptual understanding and problem-solving skills. Moreover, interactive visualizations strengthened the connection between abstract mathematical expressions and their geometric meaning, fostering deeper learning and engagement. Therefore, the integration of mathematical software into the study of polar

functions should be considered an essential pedagogical strategy in higher education, as it bridges the gap between theory and practice while preparing students for professional applications in science and engineering.

In summary, without software, polar functions can appear abstract, tedious, and difficult to visualize. With software, the students gained clarity, efficiency, and motivation, transforming the study of polar coordinates into a more meaningful and engaging experience.

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