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To find a non-split strong dominating set of an interval graph using an algorithm

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Abstract: *In graph theory, a connected component of an undirected graph is a sub graph in which any two vertices are connected to each other by paths. For a graph G, if the subgraph of G itself is a connected component then the graph is called connected, else the graph G is called disconnected and each connected component sub graph is called it's components. A dominating set D_{st} of graph G=(V,E) is a non-split strong dominating set if the induced sub graph < V-D_{st} > is connected. The non-split strong domination number of G is the minimum cardinality of a non-split strong dominating set . In this paper constructed a verification method algorithm for finding a non-split strong dominating set of an interval graph.*
Keywords: *Domination number, Interval graph, Strong dominating set, Strong domination number , split dominating set.*

I. Introduction

Let I = {I_1,I_2 ,….,I_n} be the given interval family. Each interval i in I is represented by $[a_i,b_i]$, for $i=1,2,.....,n$. Here a_i is called the left endpoint and b_i the right endpoint of the interval I_i .Without loss of generality we may assume that all end points of the intervals in I which are distinct between 1and 2n. The intervals are labelled in the increasing order of their right endpoints. Two intervals i and j are said to intersect each other, if they have non-empty intersection. Interval graphs play important role in numerous applications, many of which are scheduling problems. A graph $G=(V,E)$ is called an interval graph if there is a one-to-one correspondence between V and I such that two vertices of G are joined by an edge in E if and only if their corresponding intervals in I intersect. That is, if
$i=[a_i,b_i]$ and $j=[a_j,b_j]$, then i and j intersect means either $a_j < b_i$ or $a_i < b_j$.
Let G be a graph, with vertex set V and edge set E .
The open neighbourhood set of a vertex $v \in V$ is $nbd(v)=\{u \in V / uv \in E\}$.
The closed neighbourhood set of a vertex $v \in V$ is $nbd[v]=nbd(v) \cup \{v\}$.

A vertex in a graph G dominates itself and its neighbors. A set $D \subseteq V$ is called dominating set if every vertex in $<V-D>$ is adjacent to some vertex in D. The domination studied in [1-2]. The domination number γ of G is the minimum cardinality of a dominating set. The domination number is well-studied parameter. We can see this from the bibliography [3] on domination. In [4] , Sampathkumar and Pushpa Latha have introduced the concept of strong domination in graphs. Strong domination has been studied [5-7]. Kulli. V. R. et all [8] introduced the concept of split and non-split domination[9] in graphs. Also Dr.A. Sudhakaraiah et all [10] discussed an algorithm for finding a strong dominating set of an interval graph using an algorithm . A dominating set D is called split dominating set if the induced subgraph $<V-D>$ is disconnected. The split domination number of γ_s of G is the minimum cardinality of a split dominating set. Let $G=(V,E)$ be a graph and $u,v \in V$.
Then u strongly dominates v if
(i) $uv \in E$
(ii) $\deg v \leq \deg u$.

A set $D_{st} \subseteq V$ is a strong dominating set of G if every vertex in $V-D_{st}$ is strongly dominated by at least one vertex in D_{st}. The strong domination number $\gamma_{st}(G)$ of G is the minimum cardinality of a strong dominating set. A dominating set $D_{st} \subseteq V$ of a graph G is a Non-split strong dominating set if the induced subgraph $<V-D>$ is connected.Define $NI(i)=j$, if $b_i < a_j$ and there do not exist an interval k such that $b_i < a_k < a_j$. If there is no such j , then define $NI(i)=null$. N_{sd} (i) is the set of all neighbors whose degree is greater than degree of i and also greater than i.If there is no such neighbor then defines $N_{sd}(i) = null$. M (S) is the largest highest degree vertex in the set S.

II. Algorithms.

2.1.To find a Strong dominating set (SDS) of an interval graph using an algorithm[9].

Input : Interval family $I = \{I_1, I_2, ..., I_n\}$.

Output : Strong dominating set of an interval graph of a given interval family.

Step 1 : $S_1 = nbd$ [1].

Step 2 : S = The set of vertices in S_1 which are adjacent to all other vertices in S_1.

Step 3 : D_{st} = The largest highest degree interval in S.

Step 4 : LI = The largest interval in D_{st}

Step 5 : If $N_{sd}(LI)$ exists

Step 5.1 : a = $M(N_{sd}(LI))$.

Step 5.2 : b = The largest highest degree interval in nbd $[a]$.

Step 5.3 : $D_{st} = D_{st} \cup \{b\}$ goto step 4.

end if

else

Step 6 : Find NI(LI)

Step 6.1: If NI(LI) null goto step 7 .

Step 6.2 : $S_2 = nbd[NI(LI)]$.

Step 6.3 : S_3 = The set of all neighbors of $NI(LI)$ which are greater than or equal to $NI(LI)$.

Step 6.4 : S_4 = The set of all vertices in S_3 which are adjacent to all vertices in S_3.

Step 6.5 : c = The largest highest degree interval in S_4.

Step 6.6 : $D_{st} = D_{st} \cup \{c\}$ goto step 4.

Step 7 : End.

2.2.To find a Non-split Strong dominating set (NSSDS) of an interval graph using an algorithm.

Input : Interval family I= $\{I_1,I_2,I_3,$-------------$I_n\}$.

Output : Whether a strong dominating set is a non split strong dominating set or not.

Step1 : S_1=nbd[1]

Step2 : S=The set of vertices in S_1 which are adjacent to all other vertices in S_1.

Step3 : D_{st}=The largest highest degree interval in S .

Step4 : LI=The largest interval in D_{st}

Step5 : If W_{sd} (LI) exists

Step 5.1 : a = $M(N_{sd}(LI))$

Step 5.2 : b=The largest highest degree interval in nbd[a]

Step 5.3 : $D_{st} = D_{st} \cup \{b\}$ go to step 4

End if

Else

Step 6 : Find NI(LI).

Step 6.1 : If NI(LI)=null go to step 7.

Step 6.2 : S_2=nbd[NI(LI)]

Step 6.3 : S_3 = The set of all neighbors of $NI(LI)$ which are greater than or equal to $NI(LI)$.

Step 6.4 : S_4 = The set of all vertices in S_3 which are adjacent to all vertices in S_3.

Step 6.5 : c = The largest highest degree interval in S_4.

Step 6.6 : $D_{st} = D_{st} \cup \{c\}$ goto step 4.

Step 7 : V={1, 2,3,------------,n}

Step 8 : $|D_{st}|$=k

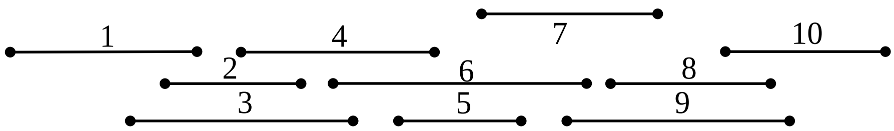
Step 9 : $S_N=\{V-D_{st}\}=\{S_1,S_2,S_3,$-----------$,S_k\}$, $k_1 \leq$ n-k

Step 10 : for (i = 1 to k_1-1)
{
For (j = i+1 to k_1)
{
If $(S_i,S_j)\in E$ of G then plot S_i to S_j
} }
The induced sub graph $G_1=V-D_{st}$ is obtained
Step11 : If $W(G_1)=1$
D_{st} is non split strong dominating set
Else
D_{st} is split strong dominating set
End.

III. Main Theorems

Theroem 1 : Let G be an interval graph corresponding to an interval family $I=\{I_1,I_2,I_3,------.I_n\}$. If i and j are any two intervals in I such that $i\in D_{st}$ is minimum strong dominating set of the given interval graph G, $j\neq 1$ and j is contained in i and if there is at least one interval to the left of j that intersects j and at least one interval $k\neq i$ to the right of j that intersects j then D_{st} is a non split strong domination.

Proof : Let G be an interval graph corresponding to an interval family $I = \{I_1,I_2,I_3,------I_n\}$. Let i and j be any two intervals in I such that $i\in D_{st}$,where D_{st} is a minimum strong dominating set of the given interval graph G, $j \neq 1$ and j is contained in i and suppose there is at least one interval to the left of j that intersects j and at least one interval $k \neq i$ to the right of j that intersects j.Then it is obviously we know that j is adjacent to k in the induced subgraph $<V-D_{st}>$.Then there will be a connection in $<V-D_{st}>$ to its left.



Interval family I

As follows an algorithm with illustration for neighbours as given interval family I. We construct an interval graph G from interval family I={1,2,3,--------10} as follows
nbd[1]={1,2,3}, nbd[2]={1,2,3,4}, nbd[3]={1,2,3,4,6},
nbd [4]={2,3,4,5,6}, nbd [5]={4,5,6,7}, nbd[6]={3,4,5,6,7,9},
nbd[7]={5,6,7,8,9}, nbd[8]={7,8,9,10}, nbd [9]={6,7,8,9,10},
nbd[10]={8,9,10}.
$N_{sd}(1)$={2,3}, $N_{sd}(2)$={3,4}, $N_{sd}(3)$={6}, $N_{sd}(4)$=6, $N_{sd}(5)$={6}, $N_{sd}(6)$=null, $N_{sd}(7)$=null, $N_{sd}(8)$={9}, $N_{sd}(9)$=null, $N_{sd}(10)$=null.
NI(1)=4, NI(2)=5, NI(3)=5, NI(4)=7, NI(5)=8, NI(6)=8, NI(7)=10, NI(8)=null, NI(9)=null, NI(10)=null.

Procedure for finding a non-split strong dominating set of an interval graph using an algorithm.

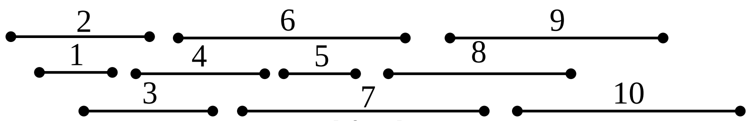
Step 1: S_1={1,2,3}.
Step 2: S={1,2,3}.
Step 3 : D_{st}={3}.
Step 4 : LI=3.
Step 5 : $N_{sd}(3)$={6}.
Step 5.1 : a=M($N_{sd}(3)$)=M({6})=6.
Step 5.2 : b=6.
Step 5.3 : $D_{st}=\{3\}\cup\{6\}=\{3,6\}$
Step 6 : LI=6.
Step 7 : NI(6)=8
Step7.1: S_2=nbd[8]={7,8,9,10}.
Step7.2: S_3={8,9,10}.
Step7.3: S_4={8,9,10}
Step7.4:c=9.
Step7.5 : $D_{st}= D_{st}\cup\{9\}=\{3,6\}\cup\{9\}=\{3,6,9\}$.
Step 8 : V={1,2,3,--------10}

Step 9 : $|D_{st}|=3$
Step10 : $S_N=\{1,2,3,4,5,6,8,10\}$
Step11 : for i=1, j=2, $(1,2)\in E$, plot 1 to 2
for i = 2 , j =3, $(2,3)\in E$, plot 2 to 3
for i = 3, j = 4, $(4,5)\in E$, plot 4 to 5
j = 5, $(4,6)\in E$, plot4 to 6
for i = 4, j = 5, $(5,6)\in E$, plot 5 to 6
j = 6, $(5,7)\in E$, plot 5 to 7
for i = 5, j=6, $(6,7)\in E$, plot 6 to7
for i = 6, j = 7, $(7,8)\in E$, plot 7 to 8
for i =7, j = 8, $(8,10)\in E$, plot 8 to 10
The induced sub graph $G_1=<V-D_{st}>$ is obtained.
Step12 : $W(G_1)=1$
Therefore D_{st} is the non split dominating set .
Step13: End .

Out put : {3,6,9} is a non split strong dominating set .

Theorem 2 : If i and j are two intervals in I such that $i\in D_{st}$ where D_{st} is a minimum dominating set of G, j=1 and j is contained in i and if there is one more interval other than i that intersects j then non-split strong domination occurs in G.

Proof : Let $I = \{I_1,I_2,I_3,I_4,\text{------},I_n\}$ be an interval family. Let j=1 be the interval contained in i where $i\in D_{st}$, where D_{st} is the minimum strong dominating set of G. Suppose k is an interval , $k\neq i$ and k intersect j. Since $i\in D_{st}$, the induced subgraph $<V-D_{st}>$ does not contain i. Further in $<V-D_{st}>$, the vertex j is adjacent to the vertex k and hence there will not be any disconnection in $<V-D_{st}>$. Therefore we get non split strong domination in G .In this connection as follows an algorithm .



Interval family I

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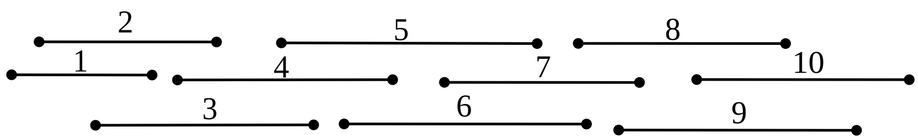
nbd [4]={2,3,4,6,7}, nbd [5]={5,6,7}, nbd[6]={3,4,5,6,7,8},
nbd[7]={4,5,6,7,8,9}, nbd[8]={6,7,8,9,10}, nbd [9]={ 7,8,9,10}, nbd[10]={8,9,10}.
$N_{sd}(1)=\{2,3\}$, $N_{sd}(2)=\{3,4\}$, $N_{sd}(3)=\{4\}$, $N_{sd}(4)=\{7\}$, $N_{sd}(5)=\{7\}$, $N_{sd}(6)=\{7\}$, $N_{sd}(7)=$ null, $N_{sd}(8)=$ null $N_{sd}(9)=$null, $N_{sd}(10)=$null.
NI(1)=4, NI(2)=5, NI(3)=5, NI(4)=5, NI(5)=8, NI(6)=9, NI(7)=10, NI(8)=null, NI(9)=null, NI(10)=null.

Procedure for finding a non-split strong dominating set of an interval graph using an algorithm.
Step 1 : $S_1=\{1,2,3\}$
Step 2 : S={1,2,3}
Step 3 : $D_{st}=3$
Step 4 : LI=3
Step 5 : $N_{sd}(3)=6$
Step 6 : a=6
Step 7 : b=7
Step 8 : $D_{st}=\{3\}\cup\{7\}=\{3,7\}$
Step 9 : LI=7
Step10 : NI(7)=10
Step10.1: $S_2=\{8,9,10\}$
Step10.2 : $S_3=\{10\}$
Step10.3 : $S_4=\{10\}$
Step10.4 : b=10
Step10.5 : $D_{st}=\{3,7,9\}$
Step11 : V={1,2,3,--------10}
Step12 : $|D_{st}|=3$
Step13 : $S_N=\{1,2,4,5,6,8,9\}$

Step14 : for i=1, j=2 ,(1,2)∈E , plot 1 to 2
for i = 2, j = 3, (2,4)∈E, plot 2 to 4
for i = 3, j = 4, (4,5)∈E, plot 4 to 5
for i = 4, j = 5, (5,6)∈E, plot 5 to 6
for i = 5, j =6, (6,8)∈E, plot 6 to 8
for i = 6,j =7, (8,9)∈E, plot 8 to 9
The induced subgraph G_1 = <V-D_{st}> is obtained .
Step15 : W(G_1)=1.
Therefore D_{st} is the non split strong dominating set.
Step16: End

Output : {3,7,10} is a non split strong dominating set .
Theorem 3 : Let D_{st} be a strong dominating set which is obtained by algorithm SDS. If i, j, k are three consecutive intervals such that i < j< k and j∈D_{st}, i intersects j, j intersect k and i interest k then non split strong domination occurs in G .
Proof : Suppose I = {I_1,I_2,I_3,------I_n} be an interval family . Let i, j, k be three consecutive intervals satisfying the hypothesis. Now i and k intersect implies that i and k are adjacent induced sub graph < V\D_{st}> an algorithm as follows .



Interval family I

As follows an algorithm with illustration for neighbours as given interval family I. We construct an interval graph G from interval family I={1,2,3,--------10} as follows
nbd[1]={1,2,3}, nbd[2]={1,2,3,4}, nbd[3]={1,2,3,4,5},
nbd [4]={2,3,4,5,6}, nbd [5]={3,4,5,6,7}, nbd[6]={4,5,6,7,8},
nbd[7]={5,6,7,8,9}, nbd[8]={6,7,8,9}, nbd [9]={7,8,9,10}, nbd[10]={9,10}.
N_{sd}(1)={2,3}, N_{sd}(2)={3,4}, N_{sd}(3)=null, N_{sd}(4)=null, N_{sd}(5)=null, N_{sd}(6)=null, N_{sd}(7)=null, N_{sd}(8)=null, N_{sd}(9)=null, N_{sd}(10)=null.
NI(1) = 4, NI(2) = 5, NI(3) = 6, NI(4) = 7, NI(5) = 8, NI(6) = 9, NI(7)=10, NI(8)=10, NI(9) = null, NI(10) = null.

Procedure for finding a non-split strong dominating set of an interval graph using an algorithm.
Step 1 : S_1={1,2,3}
Step 2 : S={1,2,3}
Step 3 : D_{st}=3
Step 4 : LI=3
Step 5 : NI(3)=6
Step 6 : Nbd[6]={4,5,6,7,8}
Step 6.1: S_3 = {6,7,8}
Step 6.2 : S_3 = {6,7,8}
Step 6.3 : S_4 = {6,7,8}
Step 6.4 : c=8
Step 6.5 : D_{st} ={3,8}
Step 7 : LI=8
Step 8 : NI(8)=null
Step 9 : V={1,2,3,--------10}
Step10 : |D_{st}|=2
Step11: S_N={1,2,4,5,6, 9,10}
Step12 : for i=1, j=2 ,(1,2)∈E, plot 1 to 2
for i=2, j=3, (2,4)∈E , plot 2 to 4
for i=3, j=4, (4,5)∈E, plot 4 to 5
j=5, (4,6)∈E, plot 4 to 6
for i=4, j=5, (5,6)∈E, plot 5 to 6
j=6, (5,7)∈E, plot 5 to 7

for i=5, j=6, (6,7)$\in$E, plot 6 to 7

for i=6,j=7, (7,9)$\in$E, plot 7 to 9

The induced sub graph G_1 is obtained .

Step13:W(G_1)=1

Therefore D_{st} is the non split strong dominating set.

Step14: End

Output: {3,8} is a non split strong dominating set .

IV. Conclusions

We studied the non-split strong domination in interval graphs. In this paper we discussed a verification method algorithm for finding a non-split strong dominating set of an interval graph.

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