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Jordan Higher (σ,τ)-Centralizer on Prime Ring

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Abstract: *Let R be a ring and σ,τ be an endomorphisms of R, in this paper we will present and study the concepts of higher (σ,τ)-centralizer, Jordan higher(σ,τ)-centralizer and Jordan triple higher (σ,τ)-centralizer and their generalization on the ring. The main results are prove that every Jordan higher (σ,τ)-centralizer of prime ring R is higher (σ,τ)-centralizer of R and we prove let R be a 2-torsion free ring,σ and τ are commutative endomorphism then every Jordan higher (σ,τ)-centralizer is Jordan triple higher (σ,τ)-centralizer.*

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I. Introcation

Throughout this paper, R is a ring and R is called prime if $aRb=(0)$ implies $a=0$ or $b=0$ and R is semiprime if $aRa=(0)$ implies $a=0$. A mapping $d:R\to R$ is called derivation if $d(xy)=d(x)y+xd(y)$ holds for all $x,y\epsilon R$. A left(right) centralizer of R is an additive mapping $T:R\to R$ which satisfies $T(xy)=T(x)y$ $(T(xy)=xT(y)$ for all $x,y\epsilon R$.

In [6] B.Zalar worked on centralizer of semiprime rings and defined that let R be a semiprimering. A left (right) centralizer of R is an additive mapping $T:R\to R$ satisfying $T(xy)=T(x)y$ $\big(T(xy)=xT(y)\big)$ for all $x,y\in R$. If T is a left and a right centralizer then T is a centralizer. An additive mapping $T:R\to R$ is called a Jordan centralizer if T satisfies $T(xy+yx)=T(x)y+yT(x)=T(y)x+xT(y)$ for all $x,y\in R$. A left (right) Jordan centralizer of R is an additive mapping $T:R\to R$ such that $T(x^2)=T(x)x$ $(T(x^2)=xT(x))$ for all $x\in R$.

Joso Vakman [3,4,5] developed some Remarks able results using centralizers on prime and semiprime rings.
E.Albas [1] developed some remarks able results using τ-centralizer of semiprime rings.

W.Cortes and G.Haetinger [2] defined a left(resp.right) Jordan σ-centralizer and proved any left(resp.right) Jordan σ-centralizer of 2-torsion free ring is a left(resp.right) σ-centralizer.
In this paper, we present the concept of higher (σ,τ)-centralizer, Jordan higher(σ,τ)-centralizer and Jordan triple higher (σ,τ)-centralizer and We have also prove that every Jordan higher (σ,τ)-centralizer of prime ring is higher (σ,τ)-centralizer Also prove that every Jordan higher (σ,τ)-centralizer of 2-torsion free ring is Jordan triple higher (σ,τ)-centralizer.

II. Higher(σ,τ)-Centralizer

Now we will the definition of higher (σ,τ)-centralizer, Jordan higher(σ,τ)-centralizer and Jordan triple higher (σ,τ)-centralizer on the ring R and other concepts which be used in our work.
We start work with the following definition:-

Definition (2.1):

let R be a ring, σ and τ are endomorphism of R and $T=(t_i)_{i\in N}$ be a family of additive mappings of R then T is said to be higher (σ, τ)- centralizer of R if

$$t_n(xy+zx)=\sum_{i=1}^{n} t_i(x)\,\sigma^i(y)+\tau^i(z)t_i(x)$$

for all $x,y,z\epsilon R$ and $n\epsilon N$
the following is an example of higher (σ, τ)-centralizer .

Example (2.2):

Let $R=\{\begin{pmatrix} x & o\\ o & x\end{pmatrix}: x\,\epsilon\, I\}$ be a ring, σ and τ are endomorphism of R such that

$$\sigma^i\begin{pmatrix} x & o\\ o & x\end{pmatrix}=\begin{pmatrix} \frac{x}{i} & o\\ o & o\end{pmatrix}\ and\ \tau^i\begin{pmatrix} x & o\\ o & x\end{pmatrix}=\begin{pmatrix} \frac{x}{i} & o\\ o & o\end{pmatrix}$$

we use the usual addition and multiplication on matrices of R ,and let
$t_n : R \to R$ be additive mapping defined by

$t_n \begin{pmatrix} x & o \\ o & x \end{pmatrix} = \begin{pmatrix} nx & 0 \\ 0 & 0 \end{pmatrix}$ if $\begin{pmatrix} x & 0 \\ 0 & x \end{pmatrix} \in R$ and $n \geq 1$.
it is clear that $T = (t_i)_{i\in N}$ is higher (σ,τ)-centralizer

Definition (2.3):

Let R be a ring, σ and τ are endomorphism of R and $T = (t_i)_{i\in N}$ be a family of additive mappings of R then T is said to be Jordan higher (σ,τ)-centralizer of R if for every $x, y \in R$ and $n \in N$

$$t_n(xy + yx) = \sum_{i=1}^{n} t_i(x)\sigma^i(y) + \tau^i(y)t_i(x)$$

It is clear that every higher (σ,τ)-centralizer of a ring R is Jordan higher (σ,τ)-centralizer of R, but the converse is not true in general as shown by the following example .

Example (2.4):

Let R be a ring , $t = (t_i)_{i\in N}$ be a higher (σ,τ)-centralizer and σ,τ are endomorphism of R define $R_1 = \{(x,x): x \in R\}$, σ_1 and τ_1 are endomorphism of R_1 defined by $\sigma_1^i(x,x) = (\sigma_i(x),\sigma_i(x))$ and $\tau_1^i(x,x) = (\tau_i(x),\tau_i(x))$ such that $x_1x_2 \neq x_2x_1$ but $x_1x_3 = x_3x_1$ for all $x_1x_2x_3 \in R$. Let the operation of addition and multiplication on R defined by
$(x_1,x_1) + (x_2,x_2) = (x_1 + x_2, x_1 + x_2)$
$(x_1,x_1)(x_2,x_2) = (x_1x_2, x_1x_2)$, for every $x_1, x_2 \in R$. let $t_n: R \to R$ be a higher (σ,τ)-centralizer mapping, and let $T_n: R_1 \to R_1$ be a higher (σ,τ)-centralizer mapping defined as $T_n(x,x) = (t_n(x), t_n(x))$ for $n \in N$. then T_n is a Jordan higher (σ_1,τ_1)-centralizer which is not higher (σ_1,τ_1)-centralizer of R_1.

Definition (2.5):

let R be a ring, σ and τ be endomorphism of R and $T = (t_i)_{i\in N}$ be a family of additive mappings of R then T is said to be a Jordan triple higher (σ,τ)-centralizer of R if for every x , $y \in R$ and $n \in N$

$$t_n(xyx) = \sum_{i=1}^{n} t_i(x)\,\sigma^i(y)\tau_i(x)$$

Now, we give some properties of higher (σ,τ)-centralizer of R.

lemma (2.6):

Let R be a ring, σ and τ are endomorphism of R and $T = (t_i)_{i\in N}$ be higher (σ,τ)-centralizer of R then for all $x, y, z, m, w \in R$ and N , the following statements holds

$$(i)\ t_n(xy + zw) = \sum_{i=1}^{n} t_i(x)\sigma^i(y) + \tau^i(z)t_i(w)$$

(ii) In particular $\sigma(R)$ is commutative

$$t_n(xyx + xyx) = \sum_{i=1}^{n} t_i(x)\sigma^i(y)\tau^i(x) + \tau^i(x)\sigma^i(y)t_i(x)$$

(iii) In particular $t_i(z)\sigma^i(y)\tau^i(x) = \tau^i(z)\sigma^i(y)t_i(x)$

$$t_n(xyz + zyx) = \sum_{i=1}^{n} t_i(x)\sigma^i(y)\tau^i(z) + \tau^i(z)\sigma^i(y)t_i(x)$$

(iv) In particular $\sigma(R)$ is commutative

$$t_n(xym + zwx) = \sum_{i=1}^{n} t_i(x)\sigma^i(y)\tau^i(m) + \tau^i(z)\sigma^i(w)t_i(x)$$

Proof:

(i) Replace $x + w$ for x and $y + m$ for y and $z + m$ for z in definition (2.1) we get
$t_n((x + w)(y + m) + (z + m)(x + w))$

$$= \sum_{i=1}^{n} t_i(x + w)\sigma^i(y + m) + \tau^i(z + m)t_i(x + w)$$

$$= \sum_{i=1}^{n} t_i(x)\sigma^i(y) + t_i(x)\sigma^i(m) + t_i(w)\sigma^i(y) + t_i(w)\sigma^i(m) + \tau^i(z)t_i(x) + \tau^i(z)t_i(w) + \tau^i(m)t_i(x) + \tau^i(m)t_i(w)$$

...(1)

On the other hand

$t_n((x+w)(y+m)+(z+m)(x+w))$
$= t_n(xy+xm+wy+wm+zx+zw+mx+mw)$
$= t_n(xy+zx)+t_n(wy+mw)+t_n(wm+mx)+t_n(xy+zw)$
$= \sum_{i=1}^{n} t_i(x)\sigma^i(y)+\tau^i(z)t_i(x)+t_i(w)\sigma^i(y)+\tau^i(m)t_i(x)+$
$t_i(w)\sigma^i(m)+\tau^i(m)t_i(x)+t_n(xy+zw)$

...(2)

Comparing (1) and (2) we get

$$t_n(xy+zw)=\sum_{i=1}^{n} t_i(x)\sigma^i(y)+\tau^i(z)t_i(w)$$

(ii) Replace $xy+yx$ for y in definition (2.3) we get
$t_n(x(xy+yx)+(xy+yx)x)$
$= t_n(xxy+xyx+xyx+yxx)$
$= t_n(xxy+yxx)+t_n(xyx+xyx)$
$= t_n\big(x(xy)+y(xx)\big)+t_n(xyx+xyx)$
$= \sum_{i=1}^{n} t_i(x)\sigma^i(xy)+\tau^i(y)t_i(xx)+t_n(xyx+xyx)$
$= \sum_{i=1}^{n} t_i(x)\sigma^i(x)\sigma^i(y)+\tau^i(y)\sigma^i(x)t_i(x)+t_n(xyx+xyx)$

...(1)

On the other hand
$t_n(x(xy+yx)+(xy+yx)x)$
$= t_n(xxy+xyx+xyx+yxx)$
$= t_n(xyx+yxx)+t_n(xxy+xyx)$
$= t_n\big(x(yx)+y(xx)\big)+t_n((xx)y+x(yx))$
$= \sum_{i=1}^{n} t_i(x)\sigma^i(yx)+\tau^i(y)t_i(xx)+\sum_{i=1}^{n} t_i(xx)\sigma^i(y)+\tau^i(x)t_i(yx)$
$= \sum_{i=1}^{n} t_i(x)\sigma^i(y)\sigma^i(x)+\tau^i(y)\sigma^i(x)t_i(x)+\sum_{i=1}^{n} t_i(x)\tau^i(x)\sigma^i(y)+$
$\tau^i(x)\sigma^i(y)t_i(x)$

...(2)

Comparing (1) and (2) we get

$$t_n(xyx+xyx)=\sum_{i=1}^{n} t_i(x)\sigma^i(y)\tau^i(x)+\tau^i(x)\sigma^i(y)t_i(x)$$

(iii) Replace $x+z$ for x in definition (2.5) we get

$$t_n\big((x+z)y(x+z)\big)=\sum_{i=1}^{n} t_i(x+z)\sigma^i(y)t_i(x+z)$$
$$=\sum_{i=1}^{n} t_i(x)\sigma^i(y)\tau^i(x)+t_i(x)\sigma^i(y)\tau^i(z)+$$
$$t_i(z)\sigma^i(y)\tau^i(x)+t_i(z)\sigma^i(y)\tau^i(z)$$

...(1)

On the other hand

$$t_n\big((x+z)y(x+z)\big)=t_n(xyx+xyz+zyx+zyz)$$
$$=t_n(xyx)+t_n(zyz)+t_n(xyz+zyx)$$
$$=\sum_{i=1}^{n} t_i(x)\sigma^i(y)\tau^i(x)+t_i(z)\sigma^i(y)\tau^i(z)+$$
$$=t_n(xyz+zyx)$$

. …(2)

Comparing (1) and (2) and since $t_i(z)\sigma^i(y)\tau^i(x) = \tau^i(z)\sigma^i(y)t_i(x)$ we get

$$t_n(xyz + zyx) = \sum_{i=1}^{n} t_i(x)\sigma^i(y)\tau^i(x) + \tau^i(z)\sigma^i(y)t_i(z)$$

(iv) Replace $ym + my$ for y and $zw + wz$ for z in definition (2.1) we get
$t_n(x(ym + my) + (zw + wz)x)$
$= t_n(xym + xmy + zwx + wzx)$
$= t_n(xym + wzx) + t_n(xmy + zwx)$
$= t_n\big(x(ym) + w(zx)\big) + t_n((xm)y + z(wx))$
$= \sum_{i=1}^{n} t_i(x)\sigma^i(ym) + \tau^i(w)t_i(zx) + \sum_{i=1}^{n} t_i(xm)\sigma^i(y) + \tau^i(z)t_i(wx)$
$= \sum_{i=1}^{n} t_i(x)\sigma^i(y)\sigma^i(m) + \tau^i(w)\sigma^i(z)t_i(x) + \sum_{i=1}^{n} t_i(x)\tau^i(m)\sigma^i(y) +$
$\tau^i(z)\sigma^i(w)t_i(x)$ ...(1)

On the other hand
$t_n(x(ym + my) + (zw + wz)x)$
$= t_n(xym + xmy + zwx + wzx)$
$= t_n(xmy + wzx) + t_n(xym + zwx)$
$= t_n\big(x(my) + w(zx)\big) + t_n(xym + zwx)$
$= \sum_{i=1}^{n} t_i(x)\sigma^i(my) + \tau^i(w)t_i(zx) + t_n(xym + zwx)$
$= \sum_{i=1}^{n} t_i(x)\sigma^i(m)\sigma^i(y) + \tau^i(w)\sigma^i(z)t_i(x) + t_n(xym + zwx)$ ...(2)

Comparing (1) and (2) we get

$$t_n(xym + zwx) = \sum_{i=1}^{n} t_i(x)\sigma^i(y)\tau^i(m) + \tau^i(z)\sigma^i(w)t_i(x)$$

Remark (2.7):

let R be a ring, σ and τ are endomorphism from R into R, and let $T = (t_i)_{i\in N}$ be a Jordan higher (σ,τ)-centralizer of R, we define $\delta_n: R \times R \to R$ by

$$\delta_n(\mathrm{x},\mathrm{y}) = t_n(xy + yx) - \sum_{i=1}^{n} t_i(x)\sigma^i(y) + \tau^i(y)t_i(x)$$

for every $x, y \epsilon R$ and $n \epsilon N$
Now, we introduce in the following lemma the properties of $\delta_n(x,y)$

Lemma (2.8):

let R a be ring $T = (t_i)_{i\in N}$ be a Jordan higher (σ,τ)-centralizer of R , then for all $x, y, z\epsilon R$, $n\epsilon N$
(i) $\delta_n(x + y, z) = \delta_n(x,z) + \delta_n(y,z)$
(ii) $\delta_n(x, y + z) = \delta_n(x,y) + \delta_n(x,z)$

Proof:

(i) $\delta_n(x + y, z) = t_n((x + y)z + z(x + y) - \sum_{i=1}^{n} t_i(x + y)\sigma^i(z) +$
$\tau^i(z)t_i(x + z)$
$= t_n(xz + yz + zx + zy) - \sum_{i=1}^{n} t_i(x)\sigma^i(z) + t_i(y)\sigma^i(z)$
$+\tau^i(z)t_i(x) + \tau^i(z)t_i(y)$

$$= t_n(xz+zx) - \sum_{i=1}^{n} t_i(x)\sigma^i(z) + \tau^i(z)t_i(x) +$$
$$t_n(yz+zy) - \sum_{i=1}^{n} t_i(y)\sigma^i(z) + \tau^i(z)t_i(y)$$
$$= \delta_n(x,z) + \delta_n(y,z)$$

$$(ii)\ \delta_n(x,y+z) = t_n(x(y+z)+(y+z)x) - \sum_{i=1}^{n} t_i(x)\sigma^i(y+z)$$
$$+\tau^i(y+z)t_i(x)$$
$$= t_n(xy+xz+yx+zx) - \sum_{i=1}^{n} t_i(x)\sigma^i(y) + t_i(x)\sigma^i(z)$$
$$+\tau^i(y)t_i(x) + \tau^i(z)t_i(x)$$
$$= t_n(xy+yx) - \sum_{i=1}^{n} t_i(x)\sigma^i(y) + \tau^i(y)t_i(x) +$$
$$t_n(xz+zx) - \sum_{i=1}^{n} t_i(x)\sigma^i(z) + \tau^i(z)t_i(x)$$
$$= \delta_n(x,y) + \delta_n(x,z)$$

Remark (2.9):

Note that $T = (t_i)_{i\in N}$ is higher (σ,τ)-centralizer of a ring R if and only if $\delta_n(x,y) = 0$,for all $x, y \in R$ and $n \in N$.

3) The Main Results

In this section, we introduce our main results. We have prove that every Jordan higher (σ,τ)-centralizer of prime ring is higher (σ,τ)-centralizer of R and we prove that Jordan higher (σ,τ)-centralizer of 2-torsion free ring R is Jordan triple higher (σ,τ)-centralizer.

Theorem (3.1):

Let R be a prime ring, $\sigma\ and\ \tau$ are commutating endomorphism from R into R and $T = (t_i)_{i\in N}$ be a Jordan higher (σ,τ)-centralizer from R into R, then $\delta_n(x,y) = 0$, for all $x, y \in R$ and $n \in N$.

Proof:

Replace $2yx$ for z in lemma (2.6)(iii) we get

$$t_n(xy(2yx) + (2yx)yx)$$
$$= t_n(xyyx + xyyx + yxyx + yxyx)$$
$$= t_n(xyyx + xyxy) + t_n(xyyx + xyxy)$$
$$= t_n\big((xy)yx + (yx)yx\big) + t_n((xy)(yx) + (yx)(yx))$$
$$= \sum_{i=1}^{n} t_i(xy)\sigma^i(y)\tau^i(x) + \tau^i(yx)\sigma^i(y)t_i(x) + \sum_{i=1}^{n} t_i(xy)\sigma^i(yx)$$
$$+\tau^i(yx)t_i(yx)$$
$$= \sum_{i=1}^{n} t_i(x)\sigma^i(y)\sigma^i(y)\tau^i(x) + \tau^i(y)\tau^i(x)\sigma^i(y)t_i(x) +$$
$$\sum_{i=1}^{n} t_i(x)\tau^i(y)\sigma^i(y)\sigma^i(x) + \tau^i(y)\tau^i(x)\sigma^i(y)t_i(x)$$
$$= \left(\sum_{i=1}^{n} t_i(x)\sigma^i(y) + \tau^i(y)t_i(x)\right).\sigma^n(y)\tau^n(x) +$$
$$\sum_{i=1}^{n} t_i(x)\tau^i(y)\sigma^i(y)\sigma^i(x) + \tau^i(y)\tau^i(x)\sigma^i(y)t_i(x)$$
$$= t_n(xy+yx).\sigma^n(y)\tau^n(x) + \sum_{i=1}^{n} t_i(x)\tau^i(y)\sigma^i(y)\sigma^i(x) +$$
$$\tau^i(y)\tau^i(x)\sigma^i(y)t_i(x)$$

...(1)

On the other hand

$t_n(xy(2yx)+(2yx)yx)$
$=t_n(xyyx+xyyx+yxyx+yxyx)$
$=t_n(xyyx+xyxy)+t_n(xyyx+xyxy)$
$=t_n(x(yy)x+(yx)yx)+t_n\big((xy)(yx)+(yx)(yx)\big)$

$$=\sum_{i=1}^{n}t_i(x)\sigma^i(yy)\tau^i(x)+\tau^i(yx)\sigma^i(y)t_i(x)+\sum_{i=1}^{n}t_i(xy)\sigma^i(yx)+\tau^i(yx)t_i(yx)$$

$$=\sum_{i=1}^{n}t_i(x)\sigma^i(y)\sigma^i(y)\tau^i(x)+\tau^i(y)\tau^i(x)\sigma^i(y)t_i(x)+\sum_{i=1}^{n}t_i(x)\tau^i(y)\sigma^i(y)\sigma^i(x)+\tau^i(y)\tau^i(x)\sigma^i(y)t_i(x)$$

...(2)

Comparing (1) and (2) we get

$$t_n(xy+yx).\sigma^n(y)\tau^n(x)=\left(\sum_{i=1}^{n}t_i(x)\sigma^i(y)+\tau^i(y)t_i(x)\right).\sigma^n(y)\tau^n(x)$$

$$\rightarrow\left(t_n(xy+yx)-\sum_{i=1}^{n}t_i(x)\sigma^i(y)+\tau^i(y)t_i(x)\right).\sigma^n(y)\tau^n(x)=0$$

$$\rightarrow\delta_n(x,y).\sigma^n(y)\tau^n(x)=0$$

And since R is prime ring then

$$\delta_n(x,y)=0$$

Corollary (3.2):

Every Jordan higher (σ,τ)-centralizer of prime ring R is higher (σ,τ)-centralizer of R .

Proof :

By theorem (3.1) and Remark (2.9) We obtain the required result .

Proposition (3.3):

Let R be 2-torsion free ring , σ and τ are commutating endomorphism of R then every Jordan higher (σ,τ)-centralizer is a Jordan triple higher (σ,τ)-centralizer.

Proof :

Let $(t_i)_{i\in N}$ be a higher (σ,τ)-centralizer of a ring

Replace $(xy+yx)$ for y in definition (2.3.3) ,we get

$t_n(x(xy+yx)+(xy+yx)x)$
$=t_n(xxy+xyx+xyx+yxx)$
$=t_n(xxy+yxx)+t_n(xyx+xyx)$
$=t_n\big(x(xy)+y(xx)\big)+t_n(2xyx)$

$$=\sum_{i=1}^{n}t_i(x)\sigma^i(xy)+\tau^i(y)t_i(xx)+2t_n(xyx)$$

$$=\sum_{i=1}^{n}t_i(x)\sigma^i(x)\sigma^i(y)+\tau^i(y)\sigma^i(x)t_i(x)+2t_n(xyx)$$

...(1)

On the other hand

$t_n(x(xy+yx)+(xy+yx)x)$
$=t_n(xxy+xyx+xyx+yxx)$
$=t_n(xyx+yxx)+t_n(xxy+xyx)$
$=t_n\big(x(yx)+y(xx)\big)+t_n((xx)y+x(yx))$

$$=\sum_{i=1}^{n}t_i(x)\sigma^i(yx)+\tau^i(y)t_i(xx)+\sum_{i=1}^{n}t_i(xx)\sigma^i(y)+\tau^i(x)t_i(yx)$$

$$= \sum_{i=1}^{n} t_i(x)\sigma^i(y)\sigma^i(x) + \tau^i(y)\sigma^i(x)t_i(x) + \sum_{i=1}^{n} t_i(x)\tau^i(x)\sigma^i(y) + \tau^i(x)\sigma^i(y)t_i(x) \quad ...(2)$$

Comparing (1) and (2) we get

$$2t_n(xyx) = 2\sum_{i=1}^{n} t_i(x)\sigma^i(y)\tau^i(x)$$

Since R is 2-torsion free then we get

$$t_n(xyx) = \sum_{i=1}^{n} t_i(x)\sigma^i(y)\tau^i(x)$$

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