IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN:2319-765X. Volume 9, Issue 4 (Jan. 2014), PP 62-70
www.iosrjournals.org

Effect of Michell function on the thickness of circular plate with internal heat generation

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Abstract: *The present paper deals with the determination of displacement and thermal transient stresses in a thick* $(M \neq 0)$ *circular plate with internal heat generation. External arbitrary heat supply is applied at the upper surface of a thick* $(M \neq 0)$ *circular plate, whereas the lower surface of a thick* $(M \neq 0)$ *circular plate is insulated and the heat is dissipated due to convection in surrounding through lateral surface. Here we compute the effect of Michell function on the thickness of circular plate with internal heat generation. The governing heat conduction equation has been solved by using integral transform method and the results are obtained in series form in terms of Bessel's functions and the results for temperature change and stresses have been computed numerically and illustrated graphically.*

Keywords: *Thick plate* $(M \neq 0)$, *Thin plate* $(M = 0)$, *internal heat generation, thermal stresses.*

I. Introduction

During the last century the theory of elasticity has found of considerable applications in the solution of engineering problems. Thermoelasticity contains the generalized theory of heat conductions, thermal stresses. A considerable progress in the field of air-craft and machine structures, mainly with gas and steam turbines and the emergence of new topics in chemical engineering have given rise to numerous problems in which thermal stresses play an important role and frequently even a primary role.

Nowacki (1957) has determined the temperature distribution on the upper face, with zero temperature on the lower face and the circular edge thermally insulated. Roy Choudhary (1972) has determined the quasi-static thermal stresses in thin circular plate. Sharma et al. (2004) studied the behavior of thermoelastic thick plate under lateral loads. Gogulwar and Deshmukh (2005) determined the thermal stresses in thin circular plate with heat sources. Kulkarni and Deshmukh (2008) studied the thermoelastic behavior of thick circular plate with the help of arbitrary initial heat supply on the upper surface. Most recently Bhongade and Durge (2013) considered thick circular plate and discuss the effect of Michell function on steady state behavior of thick circular plate.

In this paper thick $(M \neq 0)$ and thin $(M = 0)$ circular plate is considered and discussed its thermoelasticity with the help of the Goodier's thermoelastic displacement potential function and the Michell's function. To obtain the temperature distribution integral transform method is applied. The results are obtained in series form in terms of Bessel's functions and the temperature change, displacement function and stresses have been computed numerically and illustrated graphically. Here we compute the effect of Michell function on the thickness of circular plate with internal heat generation. A mathematical model has been constructed for thick $(M \neq 0)$ and thin $(M = 0)$ circular plate with the help of numerical illustration by considering copper (pure) circular plate. No one previously studied such type of problem. This is new contribution to the field.

The direct problem is very important in view of its relevance to various industrial mechanics subjected to heating such as the main shaft of lathe, turbines and the role of rolling mill, base of furnace of boiler of a thermal power plant, gas power plant.

II. Formulation of the problem

Consider a thick$(M \neq 0)$ circular plate of radius a and thickness h defined by$0 \leq r \leq a, \frac{-h}{2} \leq z \leq \frac{h}{2}$. Let the plate be subjected to external arbitrary heat supply is applied on the upper surface, the lower surface is insulated and heat convection is maintained at fixed circular edge ($r = a$). Assume the circular boundary of a thick circular plate is free from traction. Under these prescribed conditions, the quasi-static thermal transient stresses, temperature and displacement in a thick circular plate with internal heat generation are required to be determined.

The differential equation governing the displacement potential function $\phi(r, z, t)$is given in Noda (2003) as,

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = K\tau \tag{1}$$

Where K is the restraint coefficient and temperature change$\tau = T - T_{i,}T_i$is initial temperature. Displacement function ϕ is known as Goodier's thermoelastic displacement potential.

Temperature of the plate at time *t* satisfying heat conduction equation as follows,

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{q(r,z,t)}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{2}$$

with the boundary conditions

$$T = 0 \; at \; t = 0, \; 0 \le r \le a \tag{3}$$

$$q(r,z,t) = 0 \; at \; t = 0, \tag{4}$$

$$T = f(r,t) at \; z = \frac{h}{2}, \; 0 \le r \le a \tag{5}$$

$$\frac{\partial T}{\partial z} = 0 \; atz = -\frac{h}{2}, \; 0 \le r \le a \tag{6}$$

$$h_1 T + \frac{\partial T}{\partial r} = 0 \; at \; r = a \tag{7}$$

whereα is the thermal diffusivity of the material of the plate, k is the thermal conductivity of the material of the plate, q is the internal heat generation and h_1 is heat transfer coefficient.

The Michell's function *M* must satisfy

$$\nabla^2 \nabla^2 M = 0 \tag{8}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \tag{9}$$

The components of the stresses are represented by the thermoelastic displacement potential ϕ and Michell's function *M* as

$$\sigma_{rr} = 2G\left\{\frac{\partial^2 \phi}{\partial r^2} - K\tau + \frac{\partial}{\partial z}\left[v\nabla^2 M - \frac{\partial^2 M}{\partial r^2}\right]\right\} \tag{10}$$

$$\sigma_{\theta\theta} = 2G\left\{\frac{1}{r}\frac{\partial \phi}{\partial r} - K\tau + \frac{\partial}{\partial z}\left[v\nabla^2 M - \frac{1}{r}\frac{\partial M}{\partial r}\right]\right\} \tag{11}$$

$$\sigma_{zz} = 2G\left\{\frac{\partial^2 \phi}{\partial z^2} - K\tau + \frac{\partial}{\partial z}\left[(2-v)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}\right]\right\} \tag{12}$$

and

$$\sigma_{rz} = 2G\left\{\frac{\partial^2 \phi}{\partial r \partial z} + \frac{\partial}{\partial r}\left[(1-v)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}\right]\right\} \tag{13}$$

where*G* and *v* are the shear modulus and Poisson's ratio respectively.

The boundary conditions on the traction free surfaces of circular plate are

$$\sigma_{rr} = \sigma_{rz} = 0 \text{ at } r = a \tag{14}$$

Equations (1) to (14) constitute mathematical formulation of the problem.

III. The Solution

To obtain the expression for temperature $T(r, z, t)$, we introduce the finite Hankel transform over the variable r and its inverse transform (defined in Ozisik (1968))are

$$\bar{T}(\beta_m, z, t) = \int_{r'=0}^{a} r' K_0(\beta_m, r') \; T(r', z, t) \, dr' \tag{15}$$

$$T(r,z,t) = \sum_{m=1}^{\infty} K_0(\beta_m, r) \, \bar{T}(\beta_m, z, t) \tag{16}$$

where

$$K_0(\beta_m, r) = \frac{\sqrt{2}}{a}\frac{\beta_m}{({h_1}^2 + {\beta_m}^2)^{\frac{1}{2}}}\frac{J_0(\beta_m r)}{J_0(\beta_m a)} \tag{17}$$

and $\beta_1, \beta_2, \ldots$ are the positive roots of the transcendental equation

$$\beta_m J_0{}'(\beta_m a) + h_1 J_0(\beta_m a) = 0 \tag{18}$$

where$J_n(x)$ is the Bessel function of the first kind of order n.

The Hankel transform-H, defined in Eq. (15), satisfies the relation

$$H\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right] = -{\beta_m}^2 \bar{T}(\beta_m, z, t) \tag{19}$$

On applying the finite Hankel transform defined in the Eq. (15) , its inverse transform defined in Eq. (16) and applying Laplace transform and its inverse by residue method successively to the Eq. (2), one obtains the expression for temperature as

$$T(r,z,t) = \sum_{m=1}^{\infty}\left\{\left(\frac{\sqrt{2}}{a}\frac{\beta_m}{({h_1}^2 + {\beta_m}^2)^{\frac{1}{2}}}\frac{J_0(\beta_m r)}{J_0(\beta_m a)}\right)\right.$$

$$\times \sum_{n=1}^{\infty} \left[\begin{array}{c} \frac{(2n-1)\pi\alpha}{(-1)^n h^2} cos\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right] \\ \times \left(\frac{-F(\beta_m,\ t)\left[e^{-\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]t}-1\right]}{\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]^2} - f_1(t) \right) \\ + \frac{2\alpha}{(-1)^n h} sin\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right] f_2(t) + C_1(\beta_m, z, t) \end{array} \right] \Bigg\} \tag{20}$$

where $C_1(\beta_m, z, t) = L^{-1}[A_1(\beta_m, z, s)]$,
$A_1(\beta_m, z, t)$is particular integral of differential Eq.(2).

$$f_1(t) = \int_0^t A_1(\beta_m, \frac{h}{2}, t-u) e^{-\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]u} du$$

and$f_2(t) = -\int_0^t A_1(\beta_m, -\frac{h}{2}, t-u) e^{-\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]u} du$

3.1 Michell's Function *M*
We select M which satisfy Eq. (8) is given by

$$M = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} [B_{mn}\, J_0(\beta_m r) + C_{mn}\, \beta_m r J_1(\beta_m r)]\, cosh[\beta_m(z+\frac{h}{2})] \tag{21}$$

whereB_{mn} andC_{mn} are arbitrary functions, which can be determined by using condition Eq. (14).

3.2 GoodiersThermoelastic Displacement Potential and Thermal Stresses
Displacement function $\phi(r, z, t)$ which satisfies Eq. (1) as

$$\phi(r,z,t) = \sum_{m=1}^{\infty} \left[\frac{\sqrt{2}}{a} \frac{K\,\beta_m}{(h_1{}^2+\beta_m{}^2)^{\frac{1}{2}}} \frac{J_0(\beta_m r)}{J_0(\beta_m a)}\right]$$

$$\times \sum_{n=1}^{\infty} \left[\begin{array}{c} \frac{-(2n-1)\pi\alpha}{(-1)^n h^2} cos\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right] \times \left(\frac{-F(\beta_m,\ t)\ [e^{-\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]t}-1]}{\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]^2} - f_1(t) \right) \\ - \frac{2\alpha}{(-1)^n h} sin\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right] \frac{f_2(t)}{\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]} \\ - \frac{(2n-1)\pi\alpha f_2(t) e^{[\sqrt{K+\beta_m{}^2}\left(z+\frac{h}{2}\right)]}}{(-1)^n h^2 \sqrt{K+\beta_m{}^2}\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]} \end{array} \right] \tag{22}$$

Now using Eqs. (20), (21) and (22) in Eqs. (10), (11), (12) & (13), one obtains the expressions for stresses respectively as
For convenience setting

$$g_1(r) = \left[\frac{-J_1{}'(\beta_m r)\beta_m}{\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]} + J_0(\beta_m r)\right],$$

$$g_2(r) = \left[\frac{-J_1(\beta_m r)\beta_m}{r\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]} + J_0(\beta_m r)\right],$$

$$A = \left[\frac{-J_1{}'(\beta_m a)\beta_m{}^2}{\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]} + J_0(\beta_m a)\right]$$

and$R = \left\{ \begin{array}{l} [(2v-1)\beta_m{}^3 J_0(\beta_m a) + a\,J_0(\beta_m a)\beta_m{}^4] J_1(\beta_m a)\beta_m{}^3 \\ -[a\,J_0(\beta_m a)\beta_m{}^4 - 2(1-v)\beta_m{}^3 J_1(\beta_m a)] J_1{}'(\beta_m a)\beta_m{}^3 \end{array} \right\}$

$$\frac{\sigma_{rr}}{K} = 2G \sum_{m=1}^{\infty} \left\{ \left[-\frac{\sqrt{2}}{a} \frac{\beta_m}{\sqrt{h^2{}_1+\beta_m{}^2}} \frac{1}{J_0(\beta_m a)} \right] \right.$$

$$\times \sum_{n=1}^{\infty}\left(\begin{bmatrix}\frac{(2n-1)\alpha\pi}{(-1)^n h^2}\cos\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right] \\ \times\left(\frac{-F(\beta_m,t)\left[e^{-\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]t}-1\right]-f_1(t)\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]}{\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]^2}\right) \\ +\frac{2\alpha}{(-1)^n h}\sin\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right]f_2(t)\end{bmatrix}\times g_1(r) \\ +\frac{(2n-1)\pi\alpha}{(-1)^n h^2}\frac{\beta_m{}^2 J_1'(\beta_m r)f_2(t)}{\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]}\frac{e^{[\sqrt{K+\beta_m{}^2}\left(z+\frac{h}{2}\right)]}}{\sqrt{K+\beta_m{}^2}}+J_0(\beta_m r)C_1(\beta_m,z,t)\right)$$

$$\left.+\sum_{m=1}^{\infty}\begin{bmatrix}B_{mn}\,J_1'(\beta_m r) \\ +C_{mn}\,\langle\beta_m r J_1(\beta_m r)+(2v-1)J_0(\beta_m r)\rangle\end{bmatrix}\beta_m{}^3\sinh\left[\beta_m\left(z+\frac{h}{2}\right)\right]\right\} \tag{23}$$

$$\frac{\sigma_{\theta\theta}}{K}=2G\sum_{m=1}^{\infty}\left\{\left[-\frac{\sqrt{2}}{a}\frac{\beta_m}{\sqrt{h^2{}_1+\beta_m{}^2}}\frac{1}{J_0(\beta_m a)}\right]\right.$$

$$\times \sum_{n=1}^{\infty}\left(\begin{bmatrix}\frac{(2n-1)\alpha\pi}{(-1)^n h^2}\cos\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right] \\ \times\left(\frac{-F(\beta_m,\ t)\left[e^{-\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]t}-1\right]-f_1(t)\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]}{\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]^2}\right) \\ +\frac{2\alpha}{(-1)^n h}\sin\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right]f_2(t)\end{bmatrix}\times g_2(r) \\ +\frac{(2n-1)\pi\alpha}{(-1)^n h^2}\frac{\beta_m{}^2 J_1'(\beta_m r)f_2(t)}{\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]}\frac{e^{[\sqrt{K+\beta_m{}^2}\left(z+\frac{h}{2}\right)]}}{\sqrt{K+\beta_m{}^2}}+J_0(\beta_m r)C_1(\beta_m,z,t)\right)$$

$$\left.+\sum_{m=1}^{\infty}[\ C_{mn}\,\beta_m{}^3 J_0(\beta_m r)\ +\ B_{mn}\ \frac{\beta_m{}^2}{r}J_1(\beta_m r)]\sinh\left[\beta_m\left(z+\frac{h}{2}\right)\right]\right\} \tag{24}$$

$$\frac{\sigma_{zz}}{K}=2G\sum_{m=1}^{\infty}\left\{\left[\frac{\sqrt{2}}{a}\frac{\beta_m}{\sqrt{h^2{}_1+\beta_m{}^2}}\frac{J_0(\beta_m r)}{J_0(\beta_m a)}\right]\right.$$

$$\times \sum_{n=1}^{\infty}\left(\begin{bmatrix}\frac{(2n-1)\pi\alpha}{(-1)^n h^2}\left[\frac{(2n-1)^2\pi^2}{4h^2[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}]}-1\right]\cos\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right] \\ \times\left(\frac{-F(\beta_m,\ t)\left[e^{-\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]t}-1\right]-f_1(t)\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]}{\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]^2}\right) \\ +\frac{2\alpha}{(-1)^n h}\sin\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right]f_2(t)\end{bmatrix} \\ +\frac{(2n-1)\pi\alpha}{(-1)^n h^2}\frac{\sqrt{K+\beta_m{}^2}f_2(t)}{\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]}e^{[\sqrt{K+\beta_m{}^2}\left(z+\frac{h}{2}\right)]}+C_1(\beta_m,z,t)\right)$$

$$\left.+\begin{bmatrix}C_{mn}\ \langle\,2(1-v)+\beta_m r J_1(\beta_m r)\rangle \\ -B_{mn}\,J_0(\beta_m r)\end{bmatrix}\beta_m{}^3\ \sinh\left[\beta_m\left(z+\ \frac{h}{2}\right)\right]\right\} \tag{25}$$

$$\frac{\sigma_{rz}}{K}=2G\sum_{m=1}^{\infty}\left\{\left[\frac{-\sqrt{2}}{a}\frac{\beta_m{}^2}{\sqrt{h^2{}_1+\beta_m{}^2}}\frac{J_1(\beta_m r)}{J_0(\beta_m a)}\right]\right.$$

$$\times \sum_{n=1}^{\infty}\left(\begin{bmatrix} \frac{(2n-1)^2\pi^2\alpha}{2(-1)^n h^3} sin\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right] \\ \times\left(\frac{-F(\beta_m\,,\ t)\left[e^{-\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]t}-1\right]-f_1(t)\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]}{\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]^2}\right) \\ -\frac{(2n-1)\alpha\pi}{(-1)^n h^2} cos\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right]\frac{f_2(t)}{\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]} \end{bmatrix} \\ +\frac{(2n-1)\pi\alpha}{(-1)^n h^2}\frac{f_2(t)}{\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]} e^{\left[\sqrt{K+\beta_m{}^2}\left(z+\frac{h}{2}\right)\right]}\right)$$

$$+\sum_{m=1}^{\infty}\begin{bmatrix} C_{mn}\ \langle(2v-2)J_1(\beta_m r)+r\beta_m J_0(\beta_m r)\rangle \\ +B_{mn}\ J_1(\beta_m r)\end{bmatrix}\beta_m{}^3 cosh\left[\beta_m\left(z+\frac{h}{2}\right)\right]\Bigg\} \qquad (26)$$

In order to satisfy condition Eq. (14), solving equations Eqs.(23) and (26) for B_{mn} andC_{mn} one obtains

$$B_{mn} = \sum_{m=1}^{\infty}\left(\frac{\sqrt{2}}{Ra}\frac{K\beta_m}{\sqrt{h^2{}_1+\beta_m{}^2}}\frac{1}{J_0(\beta_m a)}\right)\Bigg\{\frac{[(2v-2)J_1(\beta_m a)+a\beta_m J_0(\beta_m a)]}{sinh[\beta_m(z+\frac{h}{2})]}$$

$$\times \sum_{n=1}^{\infty}\left(\begin{bmatrix} \frac{(2n-1)\pi\alpha}{(-1)^n h^2} cos\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right] \\ \times\left(\frac{-F(\beta_m\,,\ t)\left[e^{-\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]t}-1\right]-f_1(t)\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]}{\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]^2}\right) \\ +\frac{2\alpha}{(-1)^n h}\sin\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right]\ f_2(t) \end{bmatrix}\times A \\ +\frac{(2n-1)\pi\,\alpha}{(-1)^n h^2}\frac{\beta_m{}^2\ J_1{}'(\beta_m a)\ f_2(t)}{\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]}\frac{e^{[\sqrt{K+\beta_m{}^2}(z+\frac{h}{2})]}}{\sqrt{K+\beta_m{}^2}}+J_0(\beta_m a)\ C_1(\beta_m,z\,,t)\right)$$

$$-\frac{\beta_m\ J_1(\beta_m a)}{J_0(\beta_m a)}\frac{\left[(2v-1)\ \beta_m{}^3 J_0(\beta_m a)+\ a\,J_1(\beta_m a)\beta_m{}^4\right]}{cosh[\beta_m(z+\frac{h}{2})]}$$

$$\times \sum_{n=1}^{\infty}\begin{bmatrix} \frac{(2n-1)^2\pi^2\alpha}{2(-1)^n h^3} sin\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right]\times\left(\frac{-F(\beta_m,t)[e^{-\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]t}-1]}{\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]^2}-f_1(t)\right) \\ -\frac{(2n-1)\pi\alpha}{(-1)^n h^2} cos\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right]\frac{f_2(t)}{\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]} \\ +\frac{(2n-1)\pi\alpha\ f_2(t)e^{[\sqrt{K+\beta_m{}^2}(z+\frac{h}{2})]}}{(-1)^n h^2\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]} \end{bmatrix}\Bigg\}$$

(27)

$$C_{mn} = \sum_{m=1}^{\infty}\left(\frac{\sqrt{2}}{a\,R}\frac{K\,\beta_m}{\sqrt{h^2{}_1+\beta_m{}^2}}\frac{1}{J_0(\beta_m a)}\right)\Bigg\{\frac{\boldsymbol{\beta_m{}^3 J_1(\beta_m a)}}{\boldsymbol{sinh\,[\beta_m(z+\frac{h}{2})]}}$$

$$\times \sum_{n=1}^{\infty} \left(\left[\begin{array}{c} \frac{(2n-1)\pi\alpha}{(-1)^n h^2} cos\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right] \\ \times \left(\frac{-F(\beta_m\,,\ t\,)\left[e^{-\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]t}-1\right]-\ f_1(t)\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]}{\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]^2} \right) \\ +\frac{2\alpha}{(-1)^n h} sin\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right] f_2(t) \end{array} \right] \times A \\ + \frac{(2n-1)\pi\,\alpha}{(-1)^n h^2} \frac{\beta_m{}^2 J_1{}'(\beta_m a)\, f_2(t)}{\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]} \frac{e^{\,[\sqrt{K+\beta_m{}^2}\left(z+\frac{h}{2}\right)]}}{\sqrt{K+\beta_m{}^2}} + J_0(\beta_m a)\, C_1(\beta_m, z\,, t) \right)$$

$$-\frac{J_1{}'(\beta_m a)\ J_1(\beta_m a)\beta_m{}^4}{\cosh\ [\beta_m(z+\frac{h}{2})]}$$

$$\times \sum_{n=1}^{\infty} \left[\begin{array}{c} \frac{(2n-1)^2\pi^2\alpha}{2(-1)^n h^3} sin\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right] \times \left(\frac{-F(\beta_m,t\,)\ [e^{-\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]t}-1]}{\alpha\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]^2} - f_1(t) \right) \\ -\frac{(2n-1)\pi\alpha}{(-1)^n h^2} cos\left[\frac{(2n-1)\pi}{2h}\left(z+\frac{h}{2}\right)\right] \frac{f_2(t)}{\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]} \\ +\frac{(2n-1)\pi\alpha\, f_2(t) e^{\,[\sqrt{K+\beta_m{}^2}\left(z+\frac{h}{2}\right)]}}{(-1)^n 2h^2\left[\beta_m{}^2+\frac{(2n-1)^2\pi^2}{4h^2}\right]} \end{array} \right] \Bigg\} \tag{28}$$

IV. Special case and Numerical calculations

Setting

(1) $f(r,t) = \delta(r-r_1)e^{-t}, r_1 = 0.5m.$

where $\delta(r)$ is well known dirac delta function of argument r.

$$F(\beta_m,t) = \frac{\sqrt{2}}{a} \frac{\beta_m}{\sqrt{\beta_m{}^2 + h_1{}^2}} \frac{r_1\, J_0(\beta_m r_1)}{J_0(\beta_m a)} e^{-t}$$

(2) $q(r,z,t) = \delta(r-r_0)\ t\ cos(\frac{2\pi z}{h}), r_0 = 0.5m.$

$$\bar{q}\,(\beta_m, z, t) = \frac{\sqrt{2}}{a} \frac{\beta_m}{\sqrt{\beta_m{}^2 + h_1{}^2}} \frac{cos\left(\frac{2\pi z}{h}\right)\, r_0\, J_0(\beta_m r_0)\ t}{J_0(\beta_m a)}$$

$$\bar{\bar{q}}\,(\beta_m, z, s) = \frac{\sqrt{2}}{a} \frac{\beta_m}{\sqrt{\beta_m{}^2 + h_1{}^2}} \frac{cos\left(\frac{2\pi z}{h}\right)\, r_0\, J_0(\beta_m r_0)}{J_0(\beta_m a)} \frac{1}{s^2}$$

For thick plate $h = 0.25m$ and for thin plate $h = 0.2m$.

4.1 Material Properties

The numerical calculation has been carried out for a copper (pure) circular plate with the material properties defined as,

Thermal diffusivity α = 112.34$\times 10^{-6} m^2 s^{-1}$,
Specific heat $c_\rho = 383 J/kg$,
Thermal conductivity k = 386 W/m K,
Shear modulus $G = 48\ G\ pa$,
Poisson ratio $\vartheta = 0.3$.

4.2 Roots of Transcendental Equation

The $\beta_1 = 1.7887,\ \beta_2 = 4.4634, \beta_3 = 7.4103,\ \beta_4 = 10.4566,\ \beta_5 = 13.543, \beta_6 = 16.64994$ are the roots of transcendental equation $\beta_m J_0'(\beta_m a)\ + h_1 J_0(\beta_{m,} a) = 0$.The numerical calculation and the graph has been carried out with the help of mathematical software Matlab.

V. Discussion

In this paper a thick $(M \neq 0)$ and thin $(M = 0)$ circular plate is considered and determined the expressions for stresses due to internal heat generation within it and we compute the effect of Michell function on the thickness of circular plate with internal heat generation along the radial directionby substituting $M = 0$ in Eqs.(22), (23), (24), (25), (26), (27) and (28) and we compare the results for $M = 0$ and $M \neq 0$.. As a special case mathematical model is constructed for considering copper (pure) circular plate with the material properties specified above.

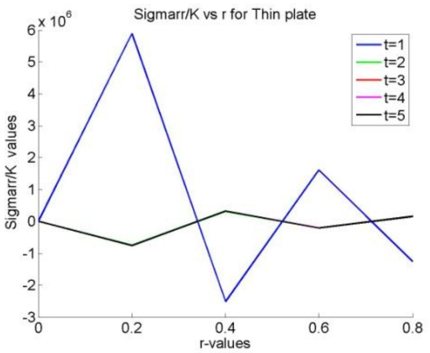


Fig. 1 Radial stress function$\frac{\sigma_{rr}}{K}$ for thin plate(M=0).

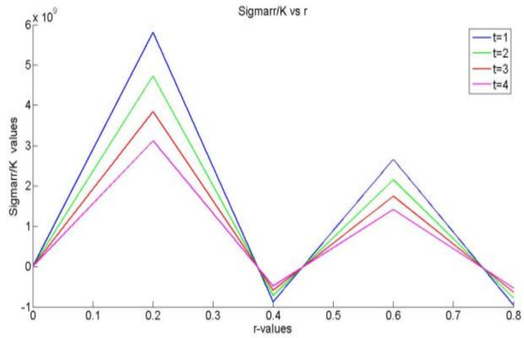


Fig. 2 Radial stress function$\frac{\sigma_{rr}}{K}$ for thick plate ($M \neq 0$).

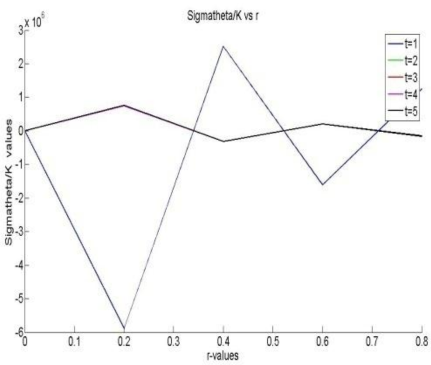


Fig. 3 Angular stress function$\frac{\sigma_{\theta\theta}}{K}$ for thin plate(M=0).

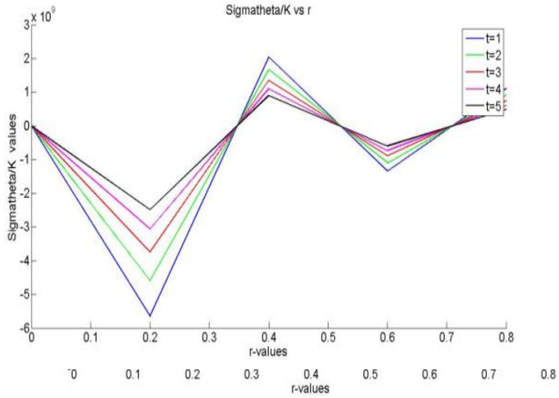
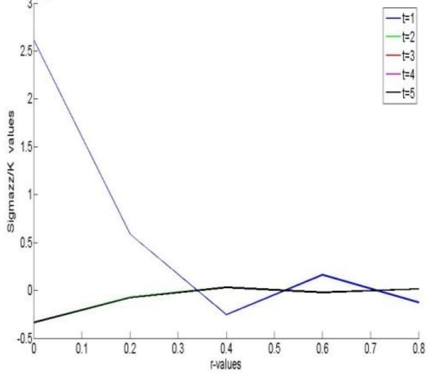


Fig. 4 Angular stress function$\frac{\sigma_{\theta\theta}}{K}$ for thick plate($M \neq 0$).



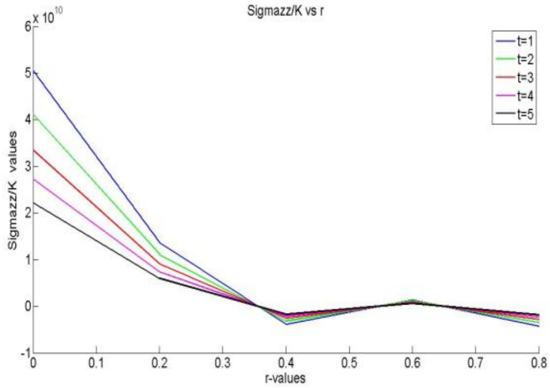
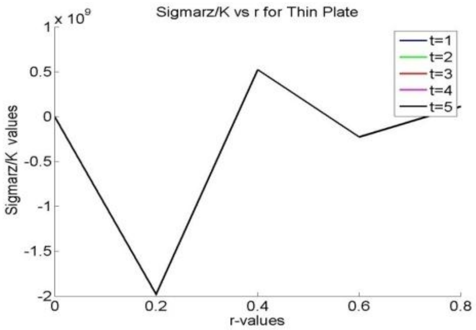


Fig.5 Axial stress function$\frac{\sigma_{zz}}{K}$ for thin plate(M=0).

Fig.6 Axial stress function$\frac{\sigma_{zz}}{K}$ for thick plate($M \neq 0$).



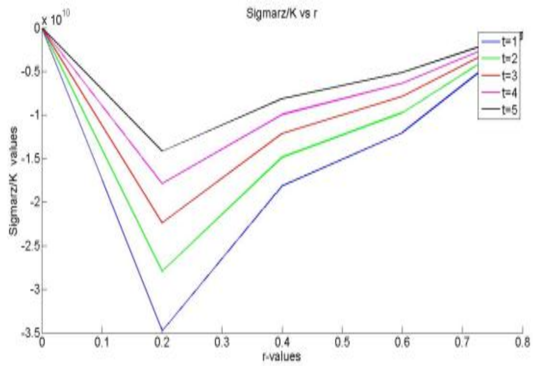


Fig.7 Stress function$\frac{\sigma_{rz}}{K}$ for thin plate(M=0).

Fig.8 Stress function$\frac{\sigma_{rz}}{K}$ for thick plate($M \neq 0$).

From figure 1 and 2, it is observed that due to Michell function the radial stress functionσ_{rr} decreases towards the lateral surface of thick circular plate along radial direction, it develops compressive stress in the radial direction.
From figure 3 and 4, it is observed that due to Michell function the angular stress function$\sigma_{\theta\theta}$ increases towards the lateral surface of thick circular plate along radial direction, it develops tensile stress in the radial direction.
From figure 5 and 6, it is observed that due to Michell function the axial stress functionσ_{zz} decreases towards the lateral surface of thick circular plate along radial direction, it develops compressive stress in the radial direction.
From figure 7 and 8, it is observed that due to Michell function the stress functionσ_{rz} increases towards the lateral surface of thick circular plate along radial direction, it develops tensile stress in the radial direction.

VI. Conclusion

We can summarize that the Michell function act as a stimulant. The radial stress functionσ_{rr} and the axial stress functionσ_{zz} decreases towards the lateral surface of thick circular plate along radial direction. Also the angular stress function$\sigma_{\theta\theta}$ and the stress functionσ_{rz} increases towards the lateral surface of thick circular plate along radial direction.

The results obtained here are useful in engineering problems particularly in the determination of state of stress in thick circular plate and base of furnace of boiler of a thermal power plant and gas power plant.

References

[1]. Nowacki, The state of stresses in a thick circular disk due to temperature field, Bull. Acad. Polon. Sci., Ser. Sci. Techn., 5, 227, (1957) .
[2]. Roy Choudhary S. K., A note of quasi static stress in a thin circular plate due to transient temperature applied alongthe circumference of a circle over the upper face, Bull Acad. PolonSci, Ser, Scl, Tech., 20-21, (1972).
[3]. J. N. Sharma, P. K. Sharma and R. L. Sharma, Behavior of thermoelastic thick plate under lateral loads, Journal of Thermal Stresses, 27, 171-191, (2004).
[4]. V. S. Gogulwar and K. C. Deshmukh, Thermal stresses in a thin circular plate with heat sources, Journal of IndianAcademy of Mathematics, 27, (1), (2005).
[5]. V. S. Kulkarni and K. C. Deshmukh, Quasi-static transient thermal stresses in thick circular plate,Journal of Brazilian Society of Mechanical Sciences and Engineering,, 30, no.2, 172-177, (2008).
[6] Bhongade C. M. and Durge M. H., Effect of Michell function on steady state behavior of thick circular plate, IOSR Journal of Mathematics,8(2), 55-60,(2013).
[6]. Naotake Noda, Richard B Hetnarski and YoshinobuTanigawa ,Thermal stresses 2 ndedn., 259-261, Taylor and FrancisNew York, (2003).
[7]. M. N. Ozisik, Boundary value problems of heat conduction International Text Book Company, Scranton, Pennsylvania,(1968).