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Strees Analysis in Elastic Half Space Due To a Thermoelastic Strain

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***Abstract**: The stress distribution on elastic space due to nuclei of thermo elastic strain distributed uniformly on the circumference of a circle of radius R situated in the place z= λ of the elastic semi space of Hookean model has been discussed by Nowacki: The Force stress and couple stress have been determined . The fore stress reduces to the one obtained by Nowacki for classical elasticity*.

I. Introduction:

Analysis of stress distribution in elastic space due to nuclei of thermoelastic strain distributed uniformly on the circumference of a circle of radius r situated in the plane Z = h of the elastic semi space of Hookean model has been discussed by Nowacki.

This note is an extension of the analysis of above problem for micropolar elastic semi-space. Force stress $^{\sigma}ji$ and couple stress $^{\mu}ji$ have been determined due to presence of nuclei of thermoelastic strain situated in the place Z = h inside the semi space. The force stress reduces to the one obtained by Nowascki for classical elasticity.

II. Basic Equations:

We consider a homogenous isotropic elastic material occupying the sami infinite region Z ≥ O in cylindrical polar coordinate system (r, θ, Z). It has been shown by Nowacki [64] that is in the case when the 2 macrodisplacement vector $\overrightarrow{u}$ and microrotation $\overrightarrow{w}$ depend only on r and z the basic equations of equilibrium of micro-polar theory of elasticity are decomposed into two mutually independent sets. Here we shall be concerned with the set $\overrightarrow{u}$ = (u_r, O, u_z) and the rotation vector $\overrightarrow{w}$ = (O, ϕ_θ,O):

$$(\mu+\alpha)(\nabla^2-\frac{1}{{}_r2})u_r+(\lambda+\mu-\alpha)\frac{\partial e}{\partial r}-2\alpha\frac{\partial\phi_\theta}{\partial z}=\varsigma\frac{\partial T}{\partial r}$$

$$(\mu+\alpha)(\nabla^2_{u_z}-+(\lambda+\mu-\alpha)\frac{\partial e}{\partial r}+2\alpha.\frac{1}{r}\frac{\partial}{\partial r}(r\phi_\theta)=\varsigma\frac{\partial T}{\partial z}$$

…..(6.1)

$$(\gamma+\in)(\nabla^2-\frac{1}{{}_r2}\phi_\theta+2\alpha(\frac{\partial u_r}{\partial z}-\frac{\partial u_z}{\partial r})-4\alpha\phi_\theta=0$$

Where e = $\frac{1}{r}\frac{\partial}{\partial r}(r\mu_r)+\frac{\partial u_z}{\partial z}$

∇^2 ≡ $\partial_r^2+\frac{1}{r}\partial_r+\partial_z^2$

ζ = (3λ + 2μ) $^{\alpha}t$
u_r , u_z = displacement components
ϕ_θ = Component of rotation vector
λ, μ, α, γ, ∈ = elastic constants
T (r, z) = temperature distribution
$^{\alpha}t$ = coefficient of thermal expansion.

To the displacement vector $\vec{u}$ (u_r , O, u_z) and the rotation vector $\vec{w}$ = (O, ϕ_θ, O) is ascribed the following state of force stress $^{\sigma}$ij and couple stress $^{\mu}$ij

$$^{\sigma}ij = \begin{Vmatrix} ^{\sigma}rr & 0 & ^{\sigma}rz \\ 0 & \sigma & 0 \\ ^{\sigma}zr & 0 & ^{\sigma}zz \end{Vmatrix}$$

$$^{\mu}ij = \begin{Vmatrix} 0 & ^{\mu}r\theta & 0 \\ ^{\mu}\theta r & 0 & ^{\mu}\theta z \\ 0 & ^{\mu}z\theta & 0 \end{Vmatrix}$$

III. Stress-Strain relations :

The relation between stress tensor σ_{ij} , μ_{ij} and displacement $\vec{u}$ and rotation $\vec{w}$ in the cylindrical coordinates are given by 3

$$\sigma_{rr} = 2\mu\frac{\partial u_r}{\partial r}+\lambda e-T$$

$$\sigma_{\theta\theta} = 2\mu\frac{u_r}{r}+\lambda e-T$$

$$\sigma_{zz} = 2\mu\frac{\partial u_z}{\partial z}+\lambda e-T$$

$$\sigma_{rz} = \mu(\frac{\partial u_{zz}}{\partial r}+\frac{\partial u_r}{\partial z})-\alpha(\frac{\partial u_r}{\partial z}+\frac{\partial u_z}{\partial r})+2\alpha\phi_\theta$$

$$\sigma_{zr} = \mu(\frac{\partial u_z}{\partial r}+\frac{\partial u_r}{\partial z})+\alpha(\frac{\partial u_r}{\partial z}-\frac{\partial u_z}{\partial r})-2\alpha\phi_\theta$$

$$\mu_{r\theta} = \gamma(\frac{\partial \phi_\theta}{\partial r}-\frac{\phi_\theta}{r})+\in(\frac{\partial \phi_\theta}{\partial r}+\frac{\phi_\theta}{r})$$

$$\mu_{\theta r} = \gamma(\frac{\partial \phi_\theta}{\partial r}-\frac{\phi_\theta}{r})+\in(\frac{\partial \phi_\theta}{\partial r}+\frac{\phi_\theta}{r}) \quad \text{....(6.2)}$$

$$\mu_{\theta z} = (\gamma - \in)\frac{\partial \phi_\theta}{\partial z},$$

$$\mu_{z\theta} = (\gamma - \in)\frac{\partial \phi_\theta}{\partial z}$$

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Following Nowacki [108], we introduce displacement potentials ϕ, Ψ and rotation potential V such that

$$\mu_r = \frac{\partial \phi}{\partial r}+\frac{\partial^2 \psi}{\partial_r \partial z}$$

$$\mu_z = \frac{\partial \phi}{\partial z}-\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial \psi}{\partial r}) \quad \text{.... (6.3)}$$

$\phi_\theta = \dfrac{\partial v}{\partial r}$

Substituting (6.3) in (6.2) we get

$$(\lambda+2\mu)\frac{\partial}{\partial r}(\nabla^2\theta)+\frac{\partial^2}{\partial z\partial r}\left[(\mu+\alpha)\nabla^2\psi-2\alpha v\right] \;|=\varsigma\frac{\partial T}{\partial r} \quad \ldots\ldots(6.4)$$

$$(\lambda+2\mu)\frac{\partial}{\partial r}(\nabla^2\theta)-(\nabla^2-)\frac{\partial^2}{\partial z^2}\left[(\mu+\alpha)\nabla^2\psi-2\alpha v\right]=\varsigma\frac{\partial T}{\partial r}$$

$$\frac{\partial}{\partial r}\left[(\gamma+\in)\nabla^2-4\alpha\right]v+2\alpha=\frac{\partial}{\partial r}\nabla 2\Psi=0$$

The above equations are satisfied if

$$\nabla^2\,\nabla^2\,\phi = m\,\nabla^2 T$$

$$\nabla^2\,((^2\nabla^2-1)\,V=0 \quad \ldots.(6.5)$$

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Where $£^2 = \dfrac{(\mu+\alpha)\gamma+\in)}{4\alpha\mu}$, m = $\dfrac{\varsigma}{\lambda+2\mu}$, and V and Ψ are related by

$$\nabla^2\Psi = -2\left[(\frac{\gamma+\in}{4\alpha})\nabla^2-1\right] V \quad \ldots.(6.6)$$

To solve (6.5) we write

$$\phi = \phi' + \phi'' \quad \ldots..(6.7)$$

$$V = V' + V''$$

Where ϕ' and V' are particular integrals for non-homogeneous part and ϕ'', V'' are general solutions of homogeneous part. Now for particular integral we have

$$\nabla^2\phi' = mT \quad \ldots(6.8)$$

and $V' = 0$

and for general solution we have

$$\nabla^2\,\nabla^2\,\phi'' = 0$$

$$\nabla^2(\ell^2\,\nabla^2-1)\,V'' = 0 \quad \ldots(6.9)$$

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IV. Solution of the title problem :

We consider nuclei of thermo elastic strain distributed uniformly on the circumference of a circle of radius r and situated in the plane z = h inside the elastic half space. The stress distribution σ_{ij} can be considered as sum of two stress systems $|\bar{S}|$ and $|\bar{\bar{S}}|$. The system $|\bar{S}|$ constitute stress distribution $\sigma'ij$ of infinite elastic space containing two nuclei of thermoelastic strains situated in the planes z = h and z = -h distributed uniformly along the circumferences of the circles, each of radius r. The second system $|\bar{\bar{S}}|$ constitutes stress distribution σ_{ij} corresponding to elastic semi-space in the isothermal state. The stress σ''_{ij} is so chosen that the boundary conditions on the plane z = O.

$$\sigma_{zz} = 0, \quad \sigma_{zr} = 0, \quad \mu_{z\theta} = 0$$

are satisfied.

The thermoelastic displacement potential ϕ' corresponding to $\sigma'ij$ satisfies the equation

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$$\nabla^2\phi = m\delta(R^r - R)\,[\delta(z-h) - \delta(z+h)] \quad \ldots.(6.10)$$

Where $r^2 = x^2 + y^2$ and $\delta(x)$ represents Dirac – delta function.

Representing the right hand side of the equations (6.10) by the Fourier Integral mδ (r-R) [δ (z-h) - δ (z + h]

$$= \frac{mR}{\pi} \int_0^\infty \int_0^\infty \xi J_o(\xi r) J_o(\xi R)\left[Cosr(z-h) - Cosr(z+h\right] d\xi dr$$

The solution of (6.10) is represented by the integral

$$\phi' = -\frac{mR}{2} \int_o^\infty J_o(\xi R) J_o(\xi r)\left[e^{-}\xi(z-h) -_e^{-} \xi(z+h)\right] d.$$

$$\xi, |z| - h > 0$$

…..6.11)

$$= -\frac{mR}{2} \int_o^\infty J_o(\xi R) J_o(\xi r)\left[e^{-}\xi(z-h) -_e^{-} \xi(z+h)\right] d\xi, |z| - h \le 0$$

…..6.12)

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The stress distribution for the system ($\bar{S}$) is obtained

σ'$_{rr}$ = 2μ $\left[(\frac{\partial^2 \phi'}{\partial r^2}) - \nabla^2 \phi' \right]$

= mμR $\int_o^\infty \xi^2 J_o(\xi R)\left[J_o(\xi R) + \frac{1}{\xi r} J_1(\xi r) \right]\left[e^{(\xi(z-h)} - e^{-\xi(z+h)} \right] d\xi$

σθθ' = 2μ $(\frac{1}{r}\frac{\partial \phi'}{\partial r} - \nabla^2 \phi')$ = -2μ $(\frac{\partial^2 \phi'}{\partial r^2} + \frac{\partial^2 \phi'}{\partial z^2})$

= mμR $\int_o^\infty \xi^2 J_o(\xi R)\left[J_o(\xi r) + J_o''(\xi r) \right]\left[e^{(\xi(z-h)} - e^{-\xi(z+h)} \right] d\xi$

V. General Solution for Homogeneous Equations:

Applying Kankel transform to equation (6.9), the general solution for half space is given by

ϕ" = $\int_o^\infty \xi (A + B\xi z) e^{-\xi z} Jo(\xi r) d\xi$ … (6.14)

and V" = $\int_o^\infty \xi (L_e^{-\xi z} + M e^{-\sigma z}) J_o(\xi r) d\xi$ …. (6.15)

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where $\sigma^2 = \xi^2 + \frac{1}{\ell^2}$ and L,M,A, B are some functions of ξ, to be determined by boundary conditions.

Equations (6.4) give

L = $-\frac{\lambda + 2\mu}{\mu} \xi B.$ …(6.16)

Knowing the functions ϕ" , Ψ" and V" the force stresses and couple stresses are calculated by the relations

σ"$_{rr}$ = 2μ $\frac{\partial u_r}{\partial r} + \lambda e = 2\mu \frac{\partial^2}{\partial r^2}(\phi'' + \frac{\partial \psi''}{\partial z}) + \lambda \nabla^2 \phi''$

$$\sigma''_{\theta\theta} = 2\mu \quad \frac{1}{r}\frac{\partial}{\partial r}(\phi'' + \frac{\partial \psi''}{\partial z}) + \lambda\nabla^2\phi''$$

$$\sigma''_{zz} = 2\mu \quad \frac{\partial}{\partial z}\left[\frac{\partial \phi''}{\partial z} - (\nabla^2 - \frac{\partial^2}{\partial z^2})\psi''\right] + \lambda\nabla^2\phi''$$

$$\sigma''_{zr} = \frac{\partial}{\partial r}\left[\mu\left\{2\frac{\partial \phi''}{\partial z} - (\nabla^2 - 2\frac{\partial^2 z}{\partial z^2})\psi''\right\}\right.$$

$$\left. + \alpha\nabla^2\psi'' - 2\alpha V''\right]$$

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$$\mu''_{r\theta} = (\gamma+\in)\frac{\partial^2 V''}{\partial_r^{\ 2}} - (\gamma-\in)\frac{1}{r}\frac{\partial V''}{\partial_r}$$

$$\mu''_{\theta r} = (\gamma-\in)\frac{\partial^2 V''}{\partial_r^{\ 2}} - (\gamma+\in)\frac{1}{r}\frac{\partial V''}{\partial_r}$$

$$\mu''_{z\theta} = (\gamma+\in)\frac{\partial^2 V''}{\partial r\partial z}$$

$$\mu''_{\theta z} = (\gamma-\in)\frac{\partial^2 V''}{\partial r\partial z}$$

Since the bounding surface z = 0 is free from tractions, we have on z = O, $|S| + |\overset{=}{S}| = O$
Thus

$$\sigma_{zz} = \sigma'_{zz} + \sigma''_{zz} = O$$

$$\sigma_{zr} = \sigma'_{zr} + \sigma''_{zr} = O$$

$$\mu_{z\theta} = \mu'_{z\theta} + \mu''_{z\theta} = O$$

Since $\mu'_{z\theta}$ = O, we get $\mu''_{z\theta}$ = O from (6.18)

This gives $L = -- M \frac{\sigma}{\xi}$ ……………(.6.19)

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$$L = --\left(\frac{\lambda+2\mu}{\mu}\right)\xi B$$

Also, from (6.16) we get

$$M = --L\frac{\xi}{\sigma} = \left(\frac{\lambda+2\mu}{\mu}\right)\left(\frac{\xi^2}{\sigma}\right)B$$

The solution of equation

$$\nabla^2\psi'' = -\frac{1}{2\alpha}\left[(\gamma+\in)\nabla^2 - 4\alpha\right]V''$$

Is obtained as

$$\psi'' = \frac{\lambda+\mu}{\mu}\int_o^\infty B\left(\frac{\lambda+2\mu}{\lambda+\mu}\xi z e^{-\xi z} + 2a_o\frac{\xi 3}{\sigma}e_{-}^{-\sigma z}\right)J_o(\xi r)d\xi$$

Where $a_O \quad \frac{(\lambda+\in)(\lambda+2\mu}{4\mu(\lambda+\mu)}$

Boundary conditions (6.18) 1, 2 yield

A = 4 ao $\xi^2 P(\xi)$

$$B = \frac{(2\mu}{(\lambda+\mu)} P(\xi) \qquad \ldots (6.20)$$

Where P (ξ) $= \dfrac{mR\xi J_o(\xi R)e^{-\xi h}}{1+2a_o\xi 2(1-\xi/\sigma)}$

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Substituting expressions for ϕ", Ψ" and V" with values of A and B in (6.20), we obtain σ''_{ij} and μ''_{ij} with the help of the relations (6.17)

$$\sigma''_{zz} = 3\mu \int_o^\infty \left[4a_0\xi^2 - \frac{2\mu}{\lambda+\mu}(2-\xi z \right] P(\xi)\xi^3 e-^{-\xi z} J_o(\xi r)d\xi$$

$$+2\mu \int_o^\infty \left[(1+\frac{\mu}{\lambda+\mu})(1-\xi z)e^{-\xi z} - 2a_o\xi^2 e^{-\sigma z} \right]\xi^3 P(\xi)J_o(\xi r)d\xi$$

$$-\frac{4\mu}{\lambda+\mu}\int_o^\infty \xi^3 e^{-\xi z} p(\xi)J_o(\xi r)d\xi$$

$$\sigma''_{zr} = 2\mu\int_o^\infty \left[\frac{2\mu}{\lambda+\mu}(1-\xi z) - 4a_o\xi^2 \right] P(\xi)e^{-\xi z} J_o{}'(\xi r)d\xi$$

$$+4(\mu-\alpha)\int_o^\infty \left[1+\frac{\mu}{\lambda+\mu}e^{-\xi z} + a_o\xi^3(1/\sigma-\sigma)e^{-\sigma z} \right] P(\xi)\xi^3 J_o^{'}(\xi r)d(\xi)$$

$$+4\mu\int_o^\infty \left[1+\frac{\mu}{\lambda+\mu}(\xi z-2)e^{-\xi z} + 2a_o\xi\sigma e^{-\sigma z} \right]\xi^3 P(\xi)J_o^{'}(\xi r)d\xi$$

$$+\frac{4\alpha\ (\lambda+2\mu}{\lambda+\mu}\int_o^\infty \xi^3(e^{-\xi z} - \frac{\xi}{\sigma}e^{-\sigma z})P(\xi)J_o^{'}(\xi r)d\xi$$

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$$\mu''_{r\theta} = \frac{-2(\lambda+2\mu)}{\lambda+\mu}\int_o^\infty (e^{-\xi z} - \frac{\xi}{\sigma}e^{-\sigma z})\left[(\gamma+\in)J_o^{''}(\xi r) - (\gamma-\in).\frac{1}{r}J_o^{'}(\xi r) \right] x\xi^3 P(\xi)$$

$$\mu''_{z\theta} = \frac{2(\gamma+\in)(\lambda+2\in)}{\lambda+\mu}\int_o^\infty (e^{-\xi z} - e^{-\sigma z})\xi^4 P(\xi)J_o^{'}(\xi r)d\xi \qquad \ldots. (6.21)$$

Stress distribution in the elastic half space is obtained by adding (6.13) and (6.21)

Thus

$$\sigma_{rr} = \sigma_{rr}^{'} + \sigma_r{}''$$

$$= m\mu R\int_o^\infty \left[J_o(\xi r) + \frac{1}{\xi r}J_1(\xi r) \right]\left[e^{\xi(z-h)} - e^{-\xi(z-h)} \right]\xi^2 J_o(\xi R)d\xi$$

$$+2\mu \int_o^\infty \left[4a_o\xi^2 + \frac{2\mu}{\lambda+\mu}\xi zP \right]\left[\frac{1}{\xi r}J_1(\xi r) - J_o(\xi r) \right]\xi^3 P(\xi)e^{-\xi z}d\xi$$

$$+ 4\mu \int_o^\infty \left[(1+\frac{\mu}{\lambda+\mu})(1-\xi z)e^{-\xi z} - 2a_o\xi^2 e^{-\sigma z} \right]\left[\frac{1}{\xi r}J_1(\xi r) - J_o(\xi r) \right]\xi^3 P(\xi)d\xi$$

$$-- \frac{4\mu\lambda}{\lambda+\mu}\int_o^\infty \xi^3 e^{-\xi z} p(\xi) j_0(\xi r)d\xi$$

$$\sigma_{\theta\theta} = \sigma_{\theta\theta}' + \sigma_{\theta\theta}''$$

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$$= m\mu R \int_o^\infty \left[e^{\xi(z-h)} - e^{-\xi(z+h)} \right] \frac{1}{\xi r} . J_1(\xi r) J_o(\xi R) d\xi$$

$$+2\mu \int_o^\infty \left[4a_o\xi^2 + \frac{2\mu}{\lambda+\mu}\xi z \right] \frac{\xi^2}{r} P(\xi) e^{-\xi z} J_1(\xi r) d\xi$$

$$-- \ 4\mu \int_o^\infty \left[(1+\frac{\mu}{\lambda+\mu})(1-\xi z)e^{-\xi z} - 2ao\xi^2 e^{-\sigma z} \right] \frac{\xi 2}{r} P(\xi) J_1(\xi r) d\xi$$

$$-\frac{4\lambda\mu}{\lambda+\mu} \int_o^\infty \xi^3 e^{-\xi z} P(\xi) J_o(\xi r) d\xi$$

$$\sigma_{zz} = \sigma_{zz}' + \sigma_{zz}''$$

$$= -m\mu R \int_o^\infty \left[e^{-\xi(z-h)} - e^{-\xi(z+h)} \right] \xi^2 J_o(\xi R) J_1(\xi r) d\xi$$

$$+2\mu \int_o^\infty \left[4a_o\xi^2 - \frac{2\mu}{\lambda+\mu}(2-\xi z) \right] \xi^3 e^{-\xi z} P(\xi) J_o(\xi r) d\xi$$

$$+2\mu \int_o^\infty \left[(1+\frac{\mu}{\lambda+\mu})(1-\xi z)e^{-\xi z} - 2a_o\xi^2 e^{-\sigma z} \right] \xi^3 p(\xi) J_o(\xi r) d\xi$$

$$-\frac{4\lambda\mu}{\lambda+\mu} \int_o^\infty \xi^3 e^{-\xi z} P(\xi) J_o(\xi r) d\xi$$

$$\sigma_{zr} = \sigma_{zr}' + \sigma_{zr}''$$

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$$= -m\mu R \int_o^\infty \xi^2 \left[e^{\xi(z-h)} - e^{-\xi(z+h)} \right] J_o(\xi R) J_1(\xi r) d\xi$$

$$-2\mu \int_o^\infty \left[\frac{2\mu}{\lambda+\mu}(1-\xi z) - 4a_o\xi^2 \right] \xi^3 P(\xi) e^{-\xi z} J_1(\xi r) d\xi$$

$$-4(\mu-\alpha) \int_o^\infty \left[(1+\frac{\mu}{\lambda+\mu})e^{-\xi z} + a_o(\frac{1}{\sigma}-\sigma)\xi^3 e^{-\sigma z} \right] \xi^3 P(\xi) J_1(\xi r) d\xi$$

$$-4\mu \int_o^\infty \left[(1+\frac{\mu}{\lambda+\mu})(\xi z-2)e^{-\xi z} + 2a_o\xi\sigma e^{-\sigma z} \right] \xi^3 P(\xi) J_1(\xi r) d\xi$$

$$-\frac{4\alpha(\lambda+2\mu)}{\lambda+\mu} \int_o^\infty \left[(e^{-\xi z} - \frac{\xi}{\sigma} e^{-\sigma z}) \xi^3 P(\xi) J_1(\xi r) d\xi \right]$$

$$\mu_{r\theta} = \mu_{r\theta}'' = -\frac{2(\lambda+2\mu)}{\lambda+\mu} \int_\circ^\infty \left[(e^{-\xi z} - \frac{\xi}{\sigma} e^{-\sigma z}) \right] \left[\xi r J_2(\xi r) - \in \xi J_o(\xi r) \right] x P(\xi) d\xi$$

$$\mu_{z\theta} = \mu_{z\theta}'' = \frac{-2(\gamma+\in)(\lambda+2\mu)}{\lambda+\mu} \int_\circ^\infty (e^{-\xi z} - e^{-\sigma z}) \xi^4 P(\xi) J_1(\xi r) d\xi$$

$$\sigma_{rr} - \sigma_{\theta\theta} = (\sigma_{rr}^{'} + \sigma_{rr}'') - (\sigma_{\theta\theta}^{'} + \sigma_{\theta\theta}'')$$

$$= (\sigma_{rr}^{'} - \sigma_{\theta\theta}') + (\sigma_{rr}^{''} - \sigma_{\theta\theta}'')$$

$$= -m\mu R \int_{o}^{\infty} \left[e^{\xi(z-h)} - e^{-\xi(z+h)} \right] \xi^2 J_o(\xi R) J_2(\xi r) d\xi$$

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$$+2\mu R \int_{o}^{\infty} \left[(4a_o\xi^2 + \frac{2\mu}{\lambda+\mu}\xi z)e^{-\xi z} + 2(1+\frac{\mu}{\lambda+\mu})(1-\xi z)e^{-\xi z} - 2a_o\xi^2 e^{-\sigma z} \right] p(\xi)$$

$$x J_2(\xi r) d\xi \ldots \quad \ldots. (6.22)$$

For α = O, the micropolar couple stress vanishes and in that case $\gamma = \in$ = O, a_o= O, σ = 0. Thus we get from (6.22)

$$\sigma_{rr} = muR \int_{o}^{\infty} \left[J_o(\xi r) + \frac{1}{\xi r} j_1(\xi r) \right] \left[e^{\xi(z-h)} - e^{-\xi(z+h)} \right] J_o(\xi r) d\xi$$

$$+ \frac{2\mu}{1-2\upsilon} \int_{\circ}^{\infty} P(\xi) e^{-\xi z} \left[(2-\xi z) J_o(\xi r) + (2\upsilon - 2 + \xi z) \frac{J_1(\xi r)}{\xi r} \right] \xi^3 d\xi$$

$$\sigma_{\theta\theta} = muR \int_{o}^{\infty} \left[J_o(\xi r) + \frac{1}{\xi r} J_1(\xi r) \right] \left[e^{\xi(z-h)} - e^{-\xi(z+h)} \right] \xi^2 J_o(\xi R) d\xi$$

$$+ \frac{2\mu}{1-2\upsilon} \int_{\circ}^{\infty} \left[(2\upsilon J_o(\xi r) - (2\upsilon - 2\xi z) \frac{J_1(\xi r)}{\xi r} \right] P(\xi) e^{-\xi z} \xi^3 d\xi$$

$$\sigma_{zz} = -muR \int_{o}^{\infty} \left[e^{\xi(z-h)} - e^{-\xi(z+h)} \right] \xi^2 J_o(\xi R) J_1(\xi r) d\xi$$

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$$+ \frac{2\mu}{1-2\upsilon} \int_{\circ}^{\infty} P(\xi) \xi^4 e^{-\xi z} J_o(\xi r) d\xi$$

σ_{zr} = σ_{rz} = - $-muR \int_{o}^{\infty} \left[e^{\xi(z-h)} - e^{-\xi(z+h)} \right] \xi^2 J_o(\xi R) J_1(\xi r) d\xi$

$$- \frac{2\mu}{1-2\upsilon} \int_{\circ}^{\infty} (1-\xi z) P(\xi) \xi^3 e^{-\xi z} J_1(\xi r) d\xi$$

….. (6.23)

urθ = uθr = 0

where P (ξ) reduces to (1-2 υ) e^{-h} J_o(ξR).

Results in (6,23) have been obtained in for Hookean thermo elasticity.

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