

## **Evaluation of Free Vibration Characteristics of Cantilever Beams Made From Different Materials**

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**Abstract:** This paper focuses on the free vibration characteristics of cantilever beams. Now a day vibration is one of the most important areas of the research, because so many failures are occur due to excess vibrations in machines or any other field like construction etc, so that we measured the vibration and reduce it or control it. Vibration is the study of oscillatory motions. It is both useful and harmful for engineering systems. To control the vibration dampers are used but they posses internal damping due to that energy dissipate in to heat. Estimating Damping is a biggest challenge in materials. The objective of the study is to find out the Free Vibrations and natural frequency of cantilever beams which are made up from different materials (Aluminium, Stainless Steel, Mild Steel and Wood) and also find out their damping ratio, the vibration characteristics of an cantilever beam are find using vibscanner and Accelerometer sensor. All the theoretical values are compared with experiment value and also find out percentage error between them.

**Keywords:-** Accelerometer sensor, Cantilever Beam, Clamp, Damping ratio, Free vibration, Vibscanner.

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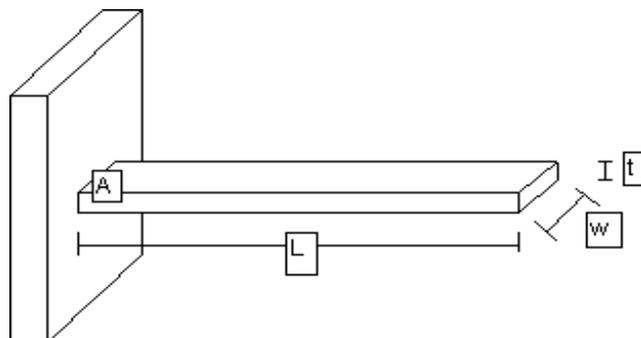
### **I INTRODUCTION**

Vibration analysis is one of the vital tasks in designing of structural and mechanical system. The effect of vibration absorber on the rotating machineries, vehicle suspension system and the dynamic behaviour of machine tool structures due to excitation are the important information that design engineer wants to obtain. This information helps to design system to control the excessive amplitude of the vibration. But in case of cantilever beams, a straight, horizontal cantilever beam under a vertical load will deform into a curve. When this force is removed, the beam will return to its original shape; however, its inertia will keep the beam in motion. Thus, the beam will vibrate at its characteristic frequencies [1]. Pawar, R.S, Sawant, S.H.(2014) this paper focuses on the study of the vibration analysis of cracked cantilever beam subjected to free and harmonic excitation at the base. The objective of the study is to identify the effect of non-linearity namely material, geometric, and damping on the natural frequency and mode shapes of cracked cantilever beam by theoretical, numerical and experimental methods [2] . In the Numerical verification of vibration analysis of cracked cantilever beam with non-linear parameters and evaluation of natural frequency and mode shapes with MATLAB/ANSYS software for both Free and Forced vibration, and the theoretical values are compared with it. Chopade, J.P., Barjibhe, R.B. (2013) this paper focuses on the theoretical analysis of transverse vibration of fixed free beam and investigates the mode shape frequency. All the theoretical values are analyzed with the numerical approach method by using ANSYS program package and correlate the theoretical values with the numerical values to find out percentage error between them [3]. It has been found that the relative error between theoretical approach and the numerical approach are very minute. The numerical study using the ANSYS program allows investigates the free vibration of fixed free beam to find out mode shape and their frequencies with high accuracy [4]. From there it concluded that theoretical data is in good agreement with numerical results with negligible error. Singh, R., Sharma, M., Singh, V.P. (2012) In this paper eddy current damper is used to control the vibration of the cantilever beams. Eddy Current Damper works on the principle of Electromagnetic Induction. According to the theory of electromagnetic induction, a current flows in a conductor whenever a change in magnetic flux is linked with it. One of the major causes of failure of structures is vibration or dynamic loads which produces the dynamic stresses in the structural elements [5] . From the analysis it can be seen which structural parameters most affect the dynamic response so that if an improvement or change in the response is required, the structure can be modified in the most economic and appropriate way. Very often the dynamic response can only be effectively controlled by changing the damping in the structure. Eddy currents provides an efficient way of adding damping to the structure without coming in contact with the structure. Cekus, D (2012) In this paper the Lagrange multiplier formalism has been used to find a solution of free vibration problem of a cantilever tapered beam. The sample numerical calculations for the cantilever tapered

beam have been carried out and compared with experimental results to illustrate the correctness of the present method. In this the free vibration problem of the cantilever tapered Timoshenko beam has been formulated and solved on the basis of Lagrange multiplier formalism. On the basis of a comparison between numerical calculations and experimental results, the percentage of error is to be find out.

**1.1 Beam**

A beam is a structural element that is capable of with standing load primarily by resisting bending. The bending force induced into the material of the beam as a result of the external loads, own weight, span and external reactions to these loads is called a moment [6]. Beams are traditionally descriptions of building or civil engineering structural elements, but smaller structures such as truck or automobile frames, machine frames, and other mechanical or structural systems contain beam structures that are designed and analyzed in a similar fashion [7].



**Fig-1 Cantilever Beam**

A cantilever beam is one whose one end is fixed and the other end carries a point or concentrated load.

L-length

W-width

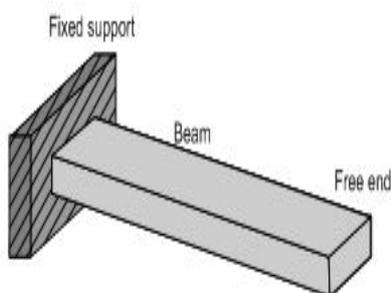
T-thickness

**1.2 Theory of Vibration (Theory of Free Vibration of Cantilever Beams)**

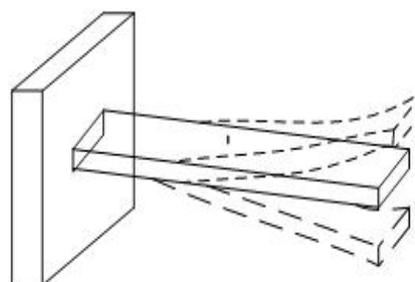
For a cantilever beam subjected to free vibration, and the system is considered as continuous system in which the beam mass is considered as distributed along with the stiffness of the shaft, the equation of motion can be written as:-

$$\frac{d^2}{dx^2} \{EI(x) \frac{d^2 Y(x)}{dx^2}\} = \omega_n^2 m(x) Y(x)$$

Where,  $E$  is the modulus of rigidity of beam material,  $I$  is the moment of inertia of the beam cross-section,  $Y(x)$  is displacement in  $y$  direction at distance  $x$  from fixed end,  $\omega_n$  is the circular natural frequency,  $m$  is the mass per unit length,  $m = \rho A(x)$ ,  $\rho$  is the material density,  $x$  is the distance measured from the fixed end [8].



**Fig.2. A cantilever beam**



**Fig.3. The beam under free vibration**

Figure-2 shows of a cantilever beam with rectangular cross section, which can be subjected to bending vibration by giving a small initial displacement at the free end; and Fig.-3 depicts of cantilever beam under the free vibration. The natural frequency is related with the circular natural frequency as

$$f_{nf} = \frac{\omega_{nf}}{2\pi} \text{ Hz}$$

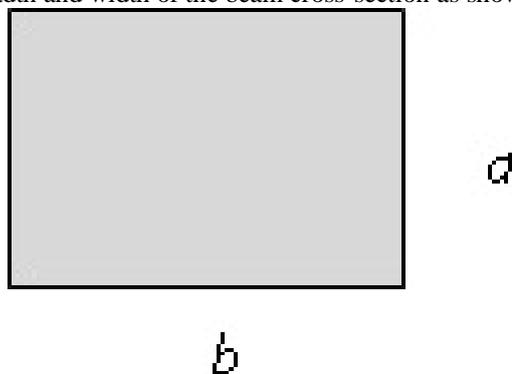
where  $I$ , the moment of inertia of the beam cross-section, for a circular cross-section it is given as

$$I = \frac{\pi}{64} d^4$$

Where,  $d$  is the diameter of cross section and for a rectangular cross section

$$I = \frac{bd^3}{12}$$

Where  $b$  and  $d$  are the breadth and width of the beam cross-section as shown in the Fig.



**Fig.4. Cross-section of the cantilever beam**

### 1.3 Euler Bernoulli Beam Theory

Euler Bernoulli's Beam Theory also known as engineer's beam theory or classical beam theory is a simplification of the linear theory of elasticity which provides a means of calculating the load carrying and deflection characteristics of beams . It covers the case for small deflections of a beam which is subjected to lateral loads only. It is thus a special case of Timoshenko beam theory. For a cantilever beam subjected to free vibration, and the system is considered as continuous system in which the beam mass is considered as distributed along with the stiffness of the shaft, the equation of motion can be written as:-

$$\frac{d^2}{dx^2} \{ EI(x) \frac{d^2 Y(x)}{dx^2} \} = \omega_n^2 m(x) Y(x)$$

Following are the boundary conditions for a cantilever beam:-

$$x = 0, Y(x) = 0, \frac{dY(x)}{dx} = 0$$

$$x = l, \frac{d^2 Y(x)}{dx^2} = 0, \frac{d^3 Y(x)}{dx^3} = 0$$

$$\frac{d^4 Y(x)}{dx^4} - \beta^4 Y(x) = 0$$

$$\beta^4 = \frac{\omega_n^2 m}{EI}, \beta_n L = \alpha_n$$

$$\omega_{nf} = \alpha_n^2 \sqrt{\frac{EI}{mL^4}}$$

### 1.4 Timoshenko Theory Of Beams

The Timoshenko beam theory was suitable for describing the behaviour of short beams, sandwich composite beams or beams subject to high-frequency excitation when the wavelength approaches the thickness of the beam. In static Timoshenko beam theory ,for a linear elastic, isotropic, homogeneous beam of constant cross-section these two equations can be combined to give:-

$$EI \frac{\partial^4 \omega}{\partial x^4} + m \frac{\partial^2 \omega}{\partial t^2} - (\rho I + \frac{E I m}{KAG}) \frac{\partial^2 \omega}{\partial x^2 \partial t^2} + \frac{Jm}{KAG} \frac{\partial^4 \omega}{\partial t^4} = q(x,t) + \frac{\rho I}{KAG} \frac{\partial^2 q}{\partial t^2} - \frac{EI}{KAG} \frac{\partial^2 q}{\partial x^2}$$

## II OBJECTIVE

- The main objective of the study is to find the free vibration and natural frequency of a cantilever beams made of Aluminium, Stainless Steel, Mild Steel and Wood. The natural frequencies is measured and compared by both theoretical and experimental techniques by varying the parameters like thickness and length but keeping the width same. The percentage of error between theoretical and experimental techniques has been determined.
- The free vibration of the cantilever beam is to be recorded with the help of Vibscanner.
- The damping ratio of a cantilever beam is to be find out with the help of Damping measurement method i.e. half power band width method, the half power band width is 0.707 and compared it with other materials.

## III. EXPERIMENTAL SETUP

Experimental Analysis plays a vital role in the research work. Experimental Analysis is being carried out to justify the validation of theoretical analysis and experiment analysis or different intelligent techniques.

Vibscanner is that mechanical instrument which are used to measure the vibrations, it also measure R.P.M and temperature. Vibscanner is also used for measurement the natural frequency of the cantilever beams, the main component of the Vibscanner are portable data collectors, accelerometer data, and acquisition system. Material of cantilever beams for experimentation:

- 1) Aluminium (AL)
- 2) Stainless steel (SS)
- 3) Mild steel (MS)
- 4) Wood(WD)

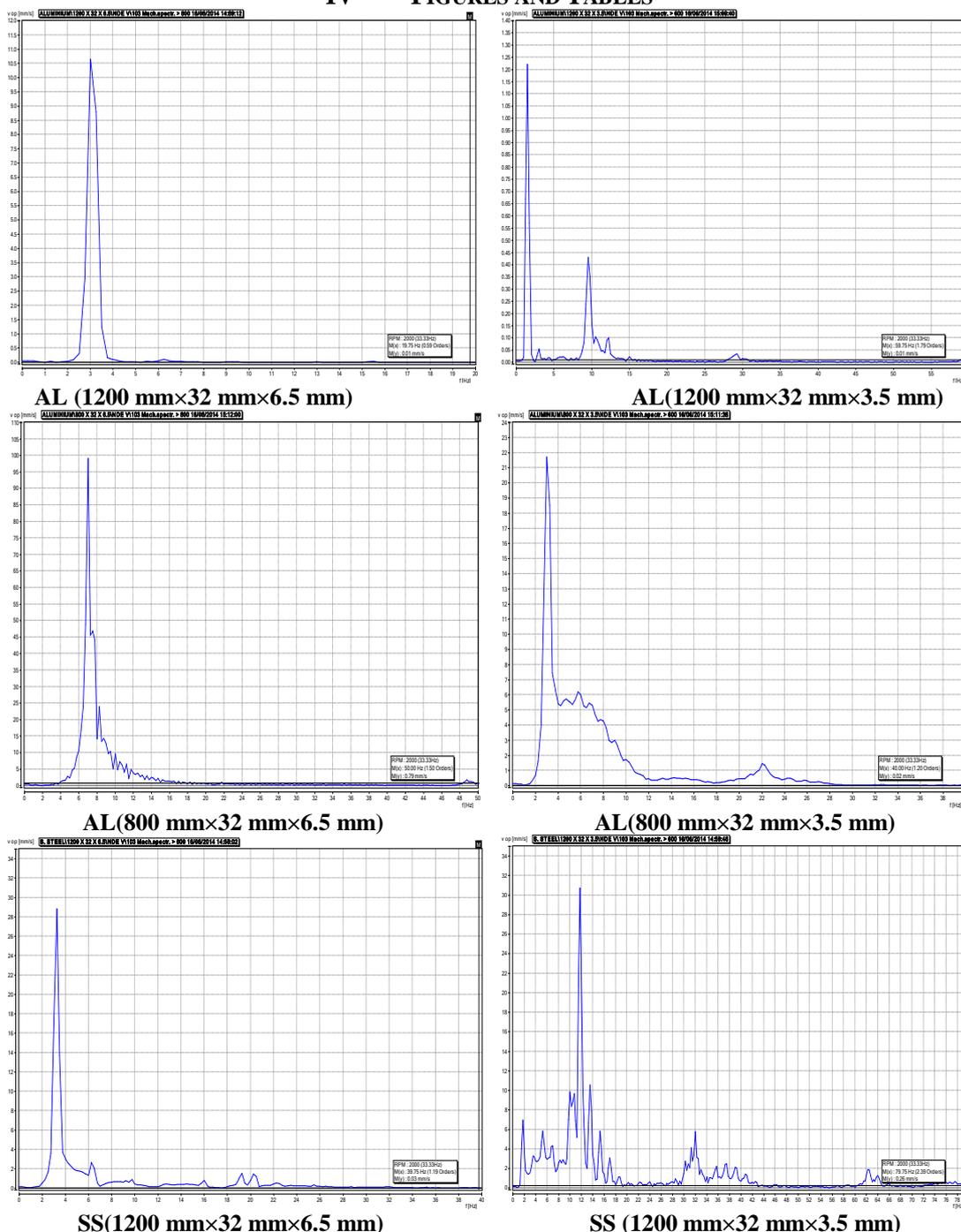
Cantilever Beam's Specifications are shown in Table 1.

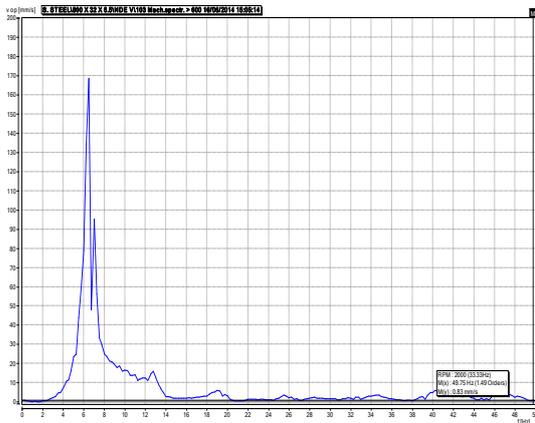
**Table.1. Beam's Specifications**

Material	Mild Steel	Alumi num	Stain less Steel.	Wood
Flexural Member	Beam	Beam	Beam	Beam
Length (mm)	1200, 800	1200, 800	1200, 800	1200, 800
Width(mm)	32,32	32,32	32,32	32,32
Thickness (mm)	6.5, 3.5	6.5, 3.5	6.5, 3.5	6.5, 3.5
Young's modulus (Gpa)	200	70	180	12.28
Density (kg/m <sup>3</sup> )	7850	2700	7750	650

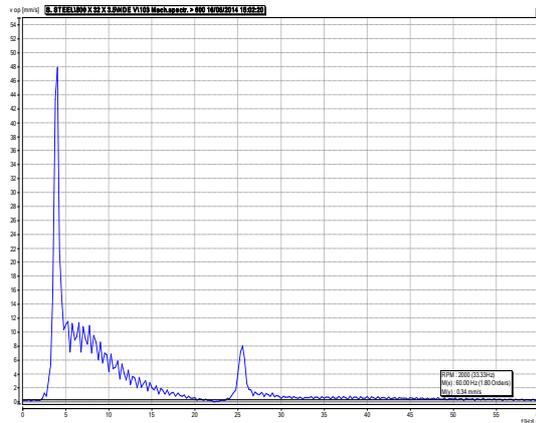
With the help of vibscanner and accelerometer sensor calculate the natural frequency of the cantilever beams these graph show the relation between the amplitude in terms of displacement and excitation frequency. The peak value shows maximum amplitude in terms of displacement corresponding to fundamental natural frequency i.e. resonant frequency, it means that when the frequency of external excitation is equal to the natural frequency of vibrating body, the amplitude of vibration becomes excessively large. This is known as resonance.

#### IV FIGURES AND TABLES

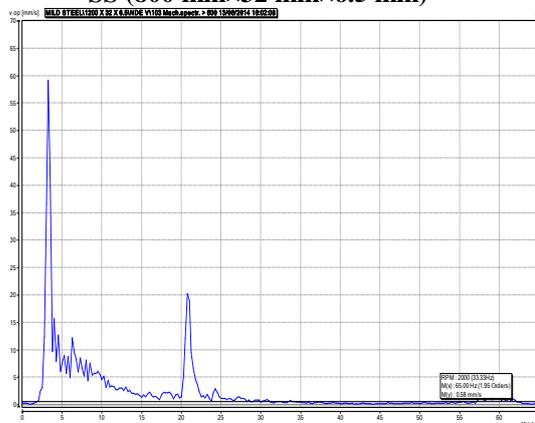




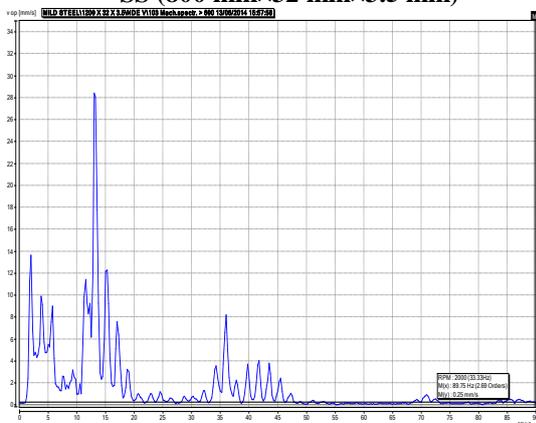
**SS (800 mm×32 mm×6.5 mm)**



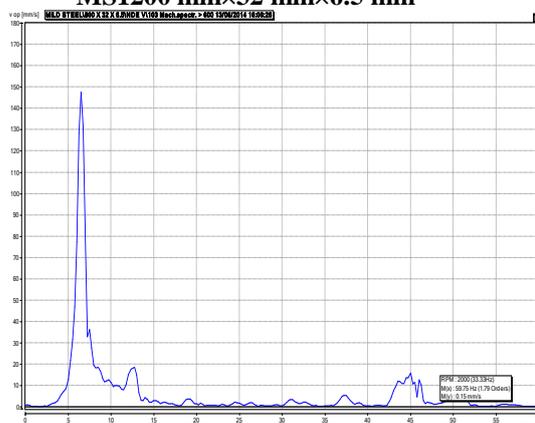
**SS (800 mm×32 mm×3.5 mm)**



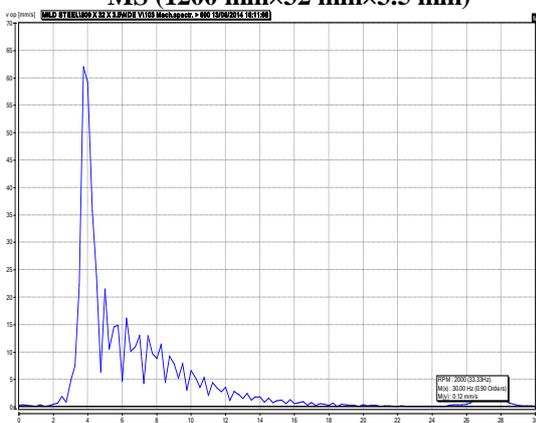
**MS1200 mm×32 mm×6.5 mm**



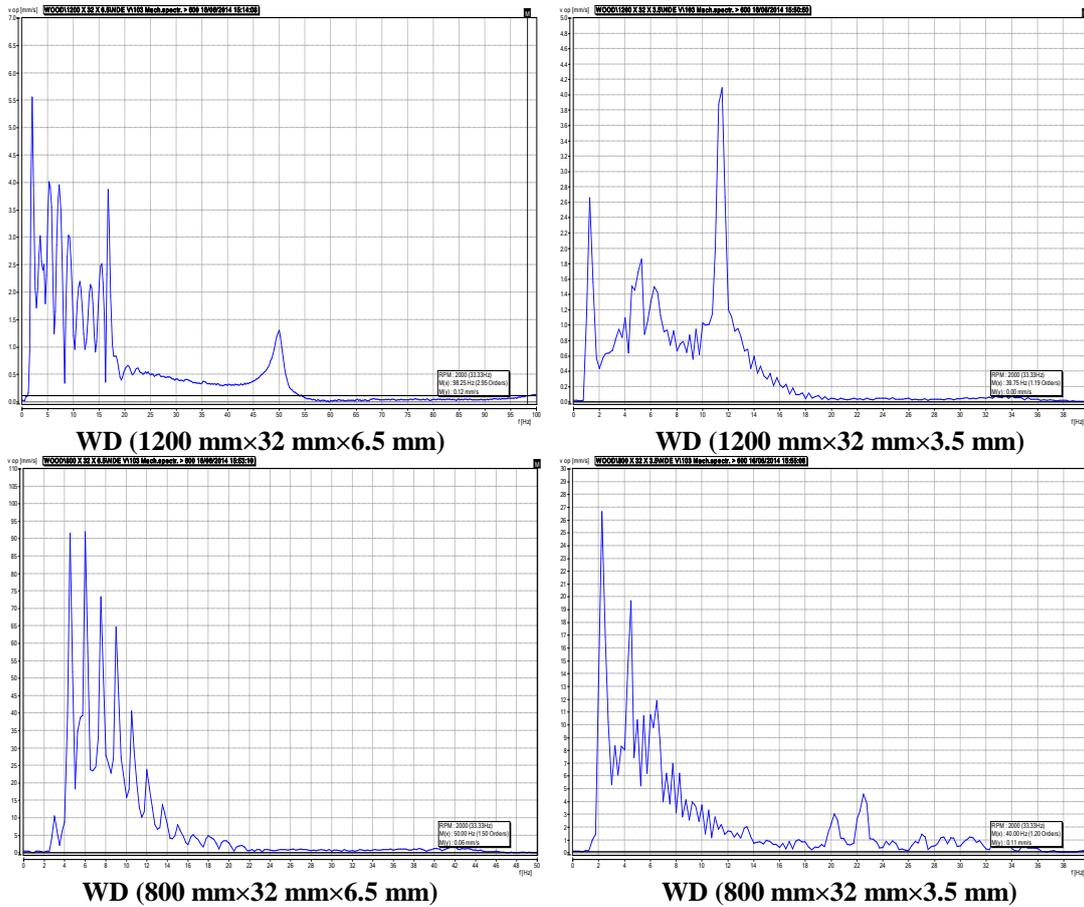
**MS (1200 mm×32 mm×3.5 mm)**



**MS (800 mm×32 mm×6.5 mm)**



**MS (800 mm×32 mm×3.5 mm)**



Notation Used :  
 1-1200mm×32mm×6.5mm  
 2-1200mm×32mm×3.5mm  
 3-800mm×32mm×6.5mm  
 4-800mm×32mm×3.5mm

**Table.2. Experimental Natural Frequency and Damping Ratio**

S. No.	Specimen	Experimental frequency (Hz)	Damping ratio (ζ)
1	AL1	3	0.0833
2	AL2	1.8	0.1944
3	AL3	6.9	0.0362
4	AL4	3.1	0.1129
5	SS1	3.3	0.0757
6	SS2	11.8	0.0211
7	SS3	6.5	0.0384
8	SS4	4.1	0.0609
9	MS1	3.5	0.0857
10	MS2	13.3	0.0300
11	MS3	6.5	0.0538
12	MS4	3.8	0.0921
13	WD1	2.4	0.1041
14	WD2	11.5	0.0260

15	WD3	6	0.0416
16	WD4	2.2	0.0909

### V THEORETICAL CALCULATIONS

First of all calculate natural frequency and damping ratio with experimentally and now calculate it theoretically.

- E- Young's modulus
- I-Moment of inertia
- L- Length of beam
- Natural frequency
- K- Stiffness
- m- Mass

**Table 3 Theoretical Calculations**

S No.	Specimen	Mass (Kg)	Stiffness (N/m)	Natural freq.
1	AL1	0.6732	88.998	1.828
2	AL2	0.3628	13.894	0.983
3	AL3	0.4492	300.37	4.111
4	AL4	0.2419	46.89	2.213
5	SS1	1.9968	228.85	1.702
6	SS2	1.0752	35.72	0.916
7	SS3	1.3312	772.38	3.830
8	SS4	0.7168	120.58	2.062
9	MS1	1.9596	254.28	1.811
10	MS2	1.0550	39.699	0.975
11	MS3	1.3062	858.20	4.075
12	MS4	0.7033	133.98	2.194
13	WD1	0.1622	15.61	1.559
14	WD2	0.0873	2.437	0.839
15	WD3	0.1081	52.69	3.509
16	WD4	0.0582	1.755	0.872

#### 5.1 Comparison between Experimental and Theoretical Natural Frequencies of Beams of Different Materials

Table. 4. Comparisons between beams of length 1200mm and thickness 6.5mm

Material	AL	SS	MS	WD
Beam Dimensions	AL 1	SS 1	MS 1	WD 1
Theoretical	1.828	1.702	1.811	1.559
Experimental	3	3.3	3.5	2.4
Percentage of Error	39.06	48.42	48.25	35.04

Table.5. Comparisons between beams of length 1200mm and thickness 3.5mm

Material	AL	SS	MS	WD
Beam Dimensions	AL2	SS2	MS 2	WD 2
Theoretical	0.983	0.916	0.975	0.839
Experimental	1.8	11.8	13.3	11.5

Percentage of Error	45.38	92.23	92.66	92.70
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Table .6. Comparisons between beams of length 800mm and thickness 6.5mm

Material	AL	SS	MS	WD
Beam Dimensions	AL 3	SS 3	MS 3	WD 3
Theoretical	4.111	3.830	4.075	3.509
Experimental	6.9	6.5	6.5	6
Percentage of Error	40.42	41.07	37.30	41.51

Table.7. Comparisons between beams of length 800mm and thickness 3.5mm

Material	AL	SS	MS	WD
Beam Dimensions	AL 4	SS 4	MS 4	WD 4
Theoretical	2.213	2.062	2.194	0.872
Experimental	3.1	4.1	3.8	2.2
Percentage of Error	28.61	49.70	42.26	60.36

## 5.2 Damping Ratio

Damping ratio is calculated with the help of half power band width method

Table.8. calculate damping ratio of material 1200mm×32mm×6.5mm

Material	Beam Dimensions	Natural frequency	Damping ratio( $\zeta$ )
Aluminium	AL 1	3	0.083
Stainless steel	SS 1	3.3	0.075
Mild steel	MS 1	3.5	0.085
Wood	WD1	2.4	0.104

Table.9. calculate damping ratio of material 1200mm×32mm×3.5mm

Material	Beam Dimensions	Natural frequency	Damping ratio( $\zeta$ )
Aluminium	AL 2	1.8	0.194
Stainless steel	SS 2	11.8	0.021

Mild steel	MS 2	13.3	0.030
Wood	WD 2	11.5	0.026

Table.10. calculate damping ratio of material 800mm×32mm×6.5mm

Material	Beam Dimensions	Natural frequency	Damping ratio( $\zeta$ )
Aluminium	AL 3	6.9	0.036
Stainless steel	SS 3	6.5	0.038
Mild steel	MS 3	6.5	0.053
Wood	WD 3	6	0.041

Table.11. calculate damping ratio of material 800mm×32mm×3.5mm

Material	Beam Dimensions	Natural frequency	Damping ratio( $\zeta$ )
Aluminium	AL 4	3.1	0.112
Stainless steel	SS 4	4.1	0.060
Mild steel	MS 4	3.8	0.092
Wood	WD 4	2.2	0.090

## VI CONCLUSION

The main purpose of the present work is to study the vibration damping characteristics of four materials. The vibration analysis has been done using theoretical, experimental analysis and also to do comparison between that. In this analysis natural frequency plays an important role to find the free vibrations of a cantilever beams which are made up from different materials. The free vibrations of cantilever beams measured with the help of mechanical vibration measurement instrument i.e. Vibscanner and accelerometer who measure the vertical displacement of beams and damping ratio has been computed using half power band width method. On the basis of present study following conclusions are drawn:

- It concluded that when the thickness decreases but length same then the natural frequency decreases.
- When the length decreases but thickness same then the natural frequency goes to increase
- The natural frequency decreases with decreases in thickness. But it is increases in SS1SS2, MS1, MS2 WD1, WD2 these cases.
- All the theoretical natural frequency value decreases when thickness decreases but same length and it increases when length decreases but same thickness.
- The Experimentally measured natural frequencies are compared with theoretically measured natural frequency and find the percentage error. From there it concluded that when the thickness decreases the percentage error increases.
- The Damping Ratio is different for different Materials.
- From the experimentation we concluded that the damping ratio is higher for AL2 and lowest for SS2.
- The damping ratio increases with decrease in thickness in case of 800mm length materials.

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