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Simulation and Analysis of Full Car Model for various Road profile on a analytically validated MATLAB/SIMULINK model

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ABSTRACT *: Suspension system design is a challenging task for the automobile designers in view of multiple control parameters, complex objectives and stochastic disturbances. For vehicle, it is always challenging to maintain simultaneously a high standard of ride comfort, vehicle handling under all driving conditions. The objective of this paper is to develop a* MATLAB/SIMULINK *model of full car to analyze the ride comfort and vehicle handling. As well as the detail study of mathematical modeling with step by step formation of state space matrix are to be developed and validation of simulink model with analytical solution of state space matrix is to be done elaborately on this paper.*

Keywords *– Vehicle dynamics, Full car model, State space equation, Ride comfort, Matlab/Simulink*

I. INTRODUCTION

Present automotive industry is witnessing a neck to neck fight among the automotive companies so as to produce highly developed models for better performance. One of the performance requirements is advanced suspension systems to give better vehicle handling for smooth drive leading to passenger comfort. Most of research activities during last decades have been directed to vibration control of vehicle, which are influenced by the harmful effects of vibrations caused by road irregularities on driver's comfort. Griffin et al [1], have shown that the interior vibration of a vehicle has a significant effect on comfort and road holding capability. To reduce this type of vibration, manufacturer's efforts have led to a suspension system installed between road excitation and vehicle body. Gundogdu [2] presented an optimization of a four-degree of freedom quarter car seat and suspension system using genetic algorithms to determine a set of parameters to achieve the best performance of the driver's seat. Wong [3] through his elaborative research has established the role of road surface irregularities, ranging from potholes to random variations of the surface elevation profile, acts as a major source that excites the vibration of the vehicle body through the tyre/wheel assembly and the suspension system. Thite [4] has developed and analyzed refined quarter car suspension model, which includes the effect of series stiffness, to estimate the response at higher frequencies; Governing equations of motion are manipulated to calculate the effective stiffness and damping values. State space model is arranged in a novel form to find eigenvalues. Agharkakli [5] have obtained a mathematical model for the passive and active suspensions systems for quarter car model and offered a compromise between two conflicting criteria, good road handling and improve passenger comfort are Simulated model for quarter car by using MATLAB/SIMULINK software. The literature mainly focuses on the effect of road irregularities on ride comfort and road holding of quarter-car and half-car models and governing equations are also formed to develop a SIMULINK model, however there remains ample scope for further studies such as validation of SIMULINK model with analytical models. The present work aims at developing a details analytical formation of governing equations for a full car model. At first a mathematical full car model considering seven degrees of freedom has been developed using passive suspension and then through state space matrix, analytical solution for the displacement of the vehicle body has obtained. This paper also discusses the development of Simulink model for 7-DOF full car model and a validation of that model with analytical solution. Further, this validated Simulink model can be used to study the various parameters sets involved for optimization of ride comfort and road holding as per ISO: 2631-1, 1997 [8] for different standard Road profile specified in, IRC-99-1988 [5] and Traffic advisory Leaflet 10/00 [6].

II. MATHEMATICAL MODEL

Fig.1 shows a full car model with seven degrees of freedom system considered for analysis. It is consisting of sprung mass, M_s referring to the part of the car that is supported on springs and unsprung mass which refers to the mass of wheel assembly. The tyre has been replaced with its equivalent stiffness and tyre damping is neglected as it's a negligible compare to tyre stiffness. In the vehicle model sprung mass is considered to have 3 DOF i.e. bounce, pitch and roll.

M_s	Mass of vehicle body in Kg	K_{sr1}, K_{sl1}	Spring stiffness of Front right & left suspension respectively in N/mm
M_{wr1}, M_{wl1}	Mass of Front right & left wheel respectively in Kg	K_{sr2}, K_{sl2}	Spring stiffness of Rear right & left suspension respectively in N/m
M_{wr2}, M_{wl2}	Mass of Rear right & left wheel respectively in Kg	C_{sr1}, C_{sl1}	Damping coefficient of Front right & left damper respectively in N-s/m
Z_{cg}	Displacement of CG of Vehicle body in m	C_{sr2}, C_{sl2}	Damping coefficient of Rear right & left damper respectively in N-s/m
ϕ =	Roll angle of the Body at CG in degree.	K_{wr1}, K_{wl1}	Spring stiffness of Front right & left tyre respectively in N/mm
θ	Pitch angle of the Body at CG in degree	K_{wr2}, K_{wl2}	Spring stiffness of Rear right & left tyre respectively in N/m
a,b	Distance from CG to Front & Rear Wheel respectively in m	Z_{wr1}, Z_{wl1}	Displacement of Front right & left wheel respectively in m
c,d	Distance from CG to Left & Right Wheel respectively in m	Z_{wr2}, Z_{wl2}	Displacement of Rear right & left wheel respectively in m
I_{xx}, I_{yy}	M.I @ X-X axis & Y-Y axis respectively in kg-m^2	Z_{rr1}, Z_{rl1}	Road input to Front right & left wheel respectively in m
		Z_{rr2}, Z_{rl2}	Road input to Rear right & left wheel respectively in m

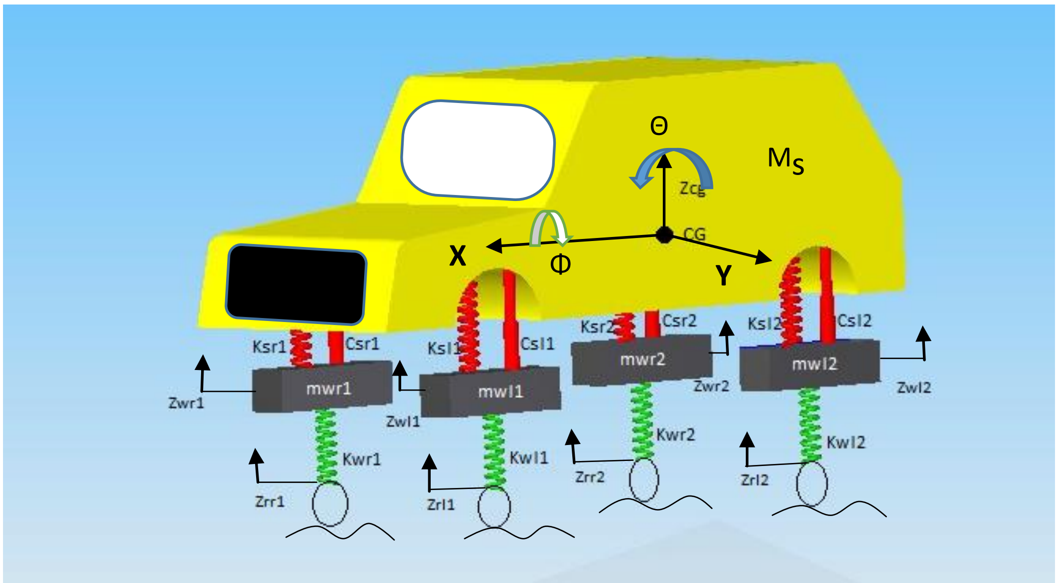

Fig.1 Mathematical model of full car

Using the Newton's second law of motion and free-body diagram concept, the following seven equations [Eq.1-Eq.7] of motion are derived.

For vehicle body bounce motion (Sprung Mass):

$$M_s\ddot{Z}_{cg} = -K_{sr1}(Z_{cg} - \theta a - \phi c - Z_{wr1}) - K_{sl1}(Z_{cg} - \theta a + \phi d - Z_{wl1})$$
$$-K_{sr2}(Z_{cg} + \theta b - \phi c - Z_{wr2}) - K_{sl2}(Z_{cg} + \theta b + \phi d - Z_{wl2})$$
$$-C_{sr1}(\dot{Z}_{cg} - \dot{\theta} a - \dot{\phi} c - \dot{Z}_{wl2}) - C_{sl1}(\dot{Z}_{cg} - \dot{\theta} a + \dot{\phi} d - \dot{Z}_{wl1})$$
$$-C_{sr2}(\dot{Z}_{cg} + \dot{\theta} b - \dot{\phi} c - \dot{Z}_{wl2}) - C_{sl2}(\dot{Z}_{cg} + \dot{\theta} b - \dot{\phi} d - \dot{Z}_{wl2})$$

$$M_s\ddot{Z}_{cg} = (-K_{sr1} - K_{sl1} - K_{sr2} - K_{sl2})Z_{cg} + (-C_{sr1} - C_{sl1} - C_{sr2} - C_{sl2})\dot{Z}_{cg}$$
$$+(K_{sr1}a + K_{sl1}a - K_{sr2}b - K_{sl2}b)\theta + (C_{sr1}a + C_{sl1}a - C_{sr2}b - C_{sl2}b)\dot{\theta}$$
$$+(K_{sr1}c - K_{sl1}d - K_{sr2}c + K_{sl2}d)\phi + (C_{sr1}c - C_{sl1}d + C_{sr2}c - C_{sl2}d)\dot{\phi}$$
$$+(K_{sr1})Z_{wr1} + (K_{sl1})Z_{wl1} + (K_{sr2})Z_{wr2} + (K_{sl2})Z_{wl2}$$
$$+(C_{sr1})\dot{Z}_{wr1} + (C_{sl1})\dot{Z}_{wl1} + (C_{sr2})\dot{Z}_{wr2} + (C_{sl2})\dot{Z}_{wl2} -------------------- (1)$$

For Vehicle Body pitching motion (Sprung Mass):

$$I_{yy}\ddot{\theta} = K_{sr1}(Z_{cg} - \theta a - \phi c - Z_{wr1})a + K_{sl1}(Z_{cg} - \theta a + \phi d - Z_{wl1})a$$
$$-K_{sr2}(Z_{cg} + \theta b - \phi c - Z_{wr2})b - K_{sl2}(Z_{cg} + \theta b + \phi d - Z_{wl2})b$$
$$-C_{sr1}(\dot{Z}_{cg} - \dot{\theta} a - \dot{\phi} c - \dot{Z}_{wr1})a + C_{sl1}(\dot{Z}_{cg} - \dot{\theta} a + \dot{\phi} d - \dot{Z}_{wl1})a$$
$$-C_{sr2}(\dot{Z}_{cg} + \dot{\theta} b - \dot{\phi} c - \dot{Z}_{wr2})b - C_{sl2}(\dot{Z}_{cg} + \dot{\theta} b + \dot{\phi} d - \dot{Z}_{wl2})b$$

$$Iyy\ddot{\theta} = (K_{sr1}a + K_{sl1}a - K_{sr2}b - K_{sl2}b)Z_{cg} + (C_{sr1}a + C_{sl1}a - C_{sr2}b - C_{sl2}b)\dot{Z}_{cg}$$
$$+(-K_{sr1}a^2 - K_{sl1}a^2 - K_{sr2}b^2 - K_{sl2}b^2)\theta + (-C_{sr1}a^2 - C_{sl1}a^2 - C_{sr2}b^2 - C_{sl2}b^2)\dot{\theta}$$
$$+(-K_{sr1}ac + K_{sl1}ad + K_{sr2}bc - K_{sl2}bd)\phi + (-C_{sr1}ac + C_{sl1}ad + C_{sr2}bc - C_{sl2}bd)\dot{\phi}$$
$$+(-K_{sr1}a)Z_{wr1} + (-K_{sl1}a)Z_{wl1} + (K_{sr2}b)Z_{wr2} + (K_{sl2}b)Z_{wl2}$$
$$+(-C_{sr1}a)\dot{Z}_{wr1} + (-C_{sl1}a)\dot{Z}_{wl1} + (C_{sr2}b)\dot{Z}_{wr2} + C_{sl2}b)\dot{Z}_{wl2} ---------------------- (2)$$

For Vehicle Body rolling motion (Sprung Mass):

$$I_{xx}\ddot{\phi} = K_{sr1}(Z_{cg} - \theta a - \phi c - Z_{wr1})c - K_{sl1}(Z_{cg} - \theta a + \phi d - Z_{wl1})d$$
$$+K_{sr2}(Z_{cg} + \theta b - \phi c - Z_{wr2})c - K_{sl2}(Z_{cg} + \theta b + \phi d - Z_{wl2})d$$
$$+C_{sr1}(\dot{Z}_{cg} - \dot{\theta} a - \dot{\phi} c - \dot{Z}_{wr1})c - C_{sl1}(\dot{Z}_{cg} - \dot{\theta} a + \dot{\phi} d - \dot{Z}_{wl1})d$$
$$+C_{sr2}(\dot{Z}_{cg} + \dot{\theta} b - \dot{\phi} c - \dot{Z}_{wr2})c - C_{sl2}(\dot{Z}_{cg} + \dot{\theta} b + \dot{\phi} d - \dot{Z}_{wl2})d$$

$$I_{xx}\ddot{\phi} = (K_{sr1}c - K_{sl1}d + K_{sr2}c - K_{sl2}d)Z_{cg} + (C_{sr1}c - C_{sl1}d + C_{sr2}c - C_{sl2}d)\dot{Z}_{cg}$$
$$+(-K_{sr1}ac + K_{sl1}ad + K_{sr2}bc - K_{sl2}bd)\theta + (-C_{sr1}ac + C_{sl1}ad + C_{sr2}bc - C_{sl2}bd)\dot{\theta}$$
$$+(-K_{sr1}c^2 - K_{sl1}d^2 - K_{sr2}c^2 - K_{sl2}d^2)\phi + (-C_{sr1}c^2 - C_{sl1}d^2 - C_{sr2}c^2 - C_{sl2}d^2)\dot{\phi}$$
$$+(-K_{sr1}c)Z_{wr1} + (K_{sl1}d)Z_{wl1} + (-K_{sr2}c)Z_{wr2} + (K_{sl2}d)Z_{wl2}$$
$$+((-C_{sr1}c)\dot{Z}_{wr1} + (C_{sl1}d)\dot{Z}_{wl1} + (-C_{sr2}c)\dot{Z}_{wr2} + (C_{sl2}d)\dot{Z}_{wl2}) ------------------ (3)$$

For Front right wheel (Unsprung Mass):

$$M_{wr1}\ddot{Z}_{wr1} = (K_{sr1})Z_{cg} + C_{sr1}\dot{Z}_{cg} + (-K_{sr1}a)\theta + (-C_{sr1}a)\dot{\theta} + (-K_{sr1}c)\phi$$
$$+(-C_{sr1}c)\dot{\phi} + (-K_{sr1} - K_{wr1})Z_{wr1} + (-C_{sr1})\dot{Z}_{wr1} + K_{wr1}Z_{rr1}$$

$$M_{wr1}\ddot{Z}_{wr1} = K_{sr1}(Z_{cg} - \theta a - \phi c - Z_{wr1}) - K_{wr1}(Z_{wr1} - Z_{rr1})$$
$$+C_{sr1}(\dot{Z}_{cg} - \dot{\theta} a - \dot{\phi} c - \dot{Z}_{wr1}) ---------------------------(4)$$

For Front left wheel (Unsprung Mass):

$$M_{wl1}\ddot{Z}_{wl1} = (K_{sl1})Z_{cg} + C_{sl1}\dot{Z}_{cg} + (-K_{sl1}a)\theta + (-C_{sl1}a)\dot{\theta} + (K_{sl1}d)\phi$$
$$+(C_{sl1}d)\dot{\phi} + (-K_{sl1} - K_{wl1})Z_{wl1} + (-C_{sl1})\dot{Z}_{wl1} + K_{wl1}Z_{rl1}$$

$$M_{wl1}\ddot{Z}_{wl1} = K_{sl1}(Z_{cg} - \theta a - \phi d - Z_{wl1}) - K_{wl1}(Z_{wl1} - Z_{rl1})$$
$$+C_{sl1}(\dot{Z}_{cg} - \dot{\theta} a + \dot{\phi} d - \dot{Z}_{wl1}) ---------------------------------(5)$$

For Rear right wheel (Unsprung Mass):

$$M_{wr2}\ddot{Z}_{wr2} = (K_{sr2})Z_{cg} + C_{sr2}\dot{Z}_{cg} + (K_{sr2}b)\theta + (C_{sr2}b)\dot{\theta} + (-K_{sr2}c)\phi$$
$$+(-C_{sr2}c)\dot{\phi} + (-K_{sr2} - K_{wr2})Z_{wr2} + (-C_{sr2})\dot{Z}_{wr2} + K_{wr2}Z_{rr2}$$

$$M_{wr2}\ddot{Z}_{wr2} = K_{sr2}(Z_{cg} + \theta b - \phi c - Z_{wr2}) - K_{wr2}(Z_{wr2} - Z_{rr2})$$
$$+C_{sr2}(\dot{Z}_{cg} + \dot{\theta} b - \dot{\phi} c - \dot{Z}_{wrr2}) --------------------------------(6)$$

For Rear left wheel (Unsprung Mass):

$$M_{wl2}\ddot{Z}_{wl2} = (K_{sl2})Z_{cg} + C_{sl2}\dot{Z}_{cg} + (K_{sl2}b)\theta + (C_{sl2}b)\dot{\theta} + (K_{sl2}d)\phi$$
$$+(C_{sl2}d)\dot{\phi} + (-K_{sl2} - K_{wl2})Z_{wl2} + (-C_{sl2})\dot{Z}_{wl2} + K_{wl2}Z_{rl2}$$

$$M_{wl2}\ddot{Z}_{wl2} = K_{sl2}(Z_{cg} + \theta b + \phi d - Z_{wl2}) - K_{wl2}(Z_{wl2} - Z_{rl2})$$
$$+C_{sl2}(\dot{Z}_{cg} + \dot{\theta} b + \dot{\phi} c - \dot{Z}_{wl2}) ----------------------------(7)$$

Parameters for Simulation of full car model

The fixed parameters mentioned in various research papers are taken for the simulation study. Suspension spring stiffness are considering 55000 N/m, 25000 N/m and damping coefficient 4000 N-s/m, 1000 N-s/m respectively. The permutation and combination of stiffness and damping coefficient are studied in simulation. The fixed parameters of full car model are shown in Table1.

Table 1 Fixed parameters of full car model

M_s =1200 Kg.	$K_{wr1} = K_{wl1}$ =30,000 N/m.	a = b =1.5 m
$M_{wr1} = M_{wl1}$ =60 Kg.	$K_{wr2} = K_{wl2}$ =30,000 N/m	C = d =1 m
$M_{wr2} = M_{wl2}$ =60 Kg.	$I_{xx,}$ = 4000 Kg-m^2	I_{yy} = 950 Kg-m^2

Assuming following state variables

$Z_{cg} = X_1$ $\dot{Z}_{cg} = X_2$ $\theta = X_3$ $\dot{\theta} = X_4$ $\phi = X_5$ $\dot{\phi} = X_6$ $Z_{wr1} = X_7$

$\dot{Z}_{wr1} = X_8$ $Z_{wl1} = X_9$ $\dot{Z}_{wl1} = X_{10}$ $Z_{wr2} = X_{11}$ $\dot{Z}_{wr2} = X_{12}$ $Z_{wl2} = X_{14}$ $\dot{Z}_{wl2} = X_{14}$

Substituting above variables in Eq.(1-7) and writing the equations in state space matrix form,

$$[\dot{X}] = [A][X] + [B][U]$$
$$[Y] = [C][X] + [D][U]$$

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$$[A] = [A1 \quad A2 \quad A3 \quad A4 \quad A5 \quad A6 \quad A7 \quad A8 \quad A9 \quad A10 \quad A11 \quad A12 \quad A13 \quad A14]$$

$$[B] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{K_{wr1}}{M_{wr1}} & 0 & \frac{K_{wl1}}{M_{wl1}} & 0 & \frac{K_{wr2}}{M_{wr2}} & 0 & \frac{K_{wl1}}{M_{wl1}} \end{bmatrix}^T$$

Input matrix U will depends on the road bump input

$$[U] = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad Z_{rr1} \quad 0 \quad Z_{rl1} \quad 0 \quad Z_{rr2} \quad 0 \quad Z_{rl2}]^T$$

Output matrix C will be depending on the output variable to be found out as follows.

$$[C] = [1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \text{-------for } Z_{cg}$$
$$[C] = [0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \text{-------for } \theta$$
$$[C] = [0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \text{-------for } \phi$$

Direct transmission matrix D will be as follows.

$$[D] = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$$

State Space matrix A will be as follows

$$A1 = [0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$$
$$A3 = [0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$$
$$A5 = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$$
$$A7 = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$$
$$A9 = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0]$$
$$A11 = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0]$$
$$A13 = [0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1]$$

$$A2=\begin{bmatrix} \frac{(-K_{sr1}-K_{sl1}-K_{sr2}-K_{sl2})}{m_s} \\ \frac{(-C_{sr1}-C_{sl1}-C_{sr2}-C_{sl2})}{m_s} \\ \frac{(K_{sr1}a+K_{sl1}a-K_{sr2}b-K_{sl2}b)}{m_s} \\ \frac{(C_{sr1}a+C_{sl1}a-C_{sr2}b-C_{sl2}b)}{m_s} \\ \frac{(K_{sr1}c-K_{sl1}d+K_{sr2}c-K_{sl2}d)}{m_s} \\ \frac{(C_{sr1}c-C_{sl1}d+C_{sr2}c-C_{sl2}d)}{m_s} \\ \frac{K_{sr1}}{m_s} \\ \frac{C_{sr1}}{m_s} \\ \frac{K_{sl1}}{m_s} \\ \frac{C_{sl1}}{m_s} \\ \frac{K_{sr2}}{m_s} \\ \frac{C_{sr2}}{m_s} \\ \frac{K_{sl2}}{m_s} \\ \frac{C_{sl2}}{m_s} \end{bmatrix}^T \quad A4=\begin{bmatrix} \frac{(K_{sr1}a+K_{sl1}a-K_{sr2}b-K_{sl2}b)}{I_{yy}} \\ \frac{(C_{sr1}a+C_{sl1}a-C_{sr2}b-C_{sl2}b)}{I_{yy}} \\ \frac{(-K_{sr1}a^2-K_{sl1}a^2-K_{sr2}b^2-K_{sl2}b^2)}{I_{yy}} \\ \frac{(-C_{sr1}a^2-C_{sl1}a^2-C_{sr2}b^2-C_{sl2}b^2)}{I_{yy}} \\ \frac{(-K_{sr1}ac+K_{sl1}ad+K_{sr2}bc-K_{sl2}bd)}{I_{yy}} \\ \frac{(-C_{sr1}ac-C_{sl1}ad-C_{sr2}bc-C_{sl2}bc)}{I_{yy}} \\ \frac{-K_{sr1}a}{I_{yy}} \\ \frac{-C_{sr1}a}{I_{yy}} \\ \frac{-K_{sl1}a}{I_{yy}} \\ \frac{-C_{sl1}a}{I_{yy}} \\ \frac{K_{sr2}b}{I_{yy}} \\ \frac{C_{sr2}b}{I_{yy}} \\ \frac{K_{sl2}b}{I_{yy}} \\ \frac{C_{sl2}b}{I_{yy}} \end{bmatrix}^T \quad A6=\begin{bmatrix} \frac{(K_{sr1}c-K_{sl1}d+K_{sr2}c-K_{sl2}d)}{I_{xx}} \\ \frac{(C_{sr1}c-C_{sl1}d+C_{sr2}c-C_{sl2}d)}{I_{xx}} \\ \frac{(-K_{sr1}ac+K_{sl1}ad+K_{sr2}bc-K_{sl2}bd)}{Ixx} \\ \frac{(-C_{sr1}ac-C_{sl1}ad-C_{sr2}bc-C_{sl2}bd)}{I_{xx}} \\ \frac{(-K_{sr1}c^2-K_{sl1}d^2-K_{sr2}c^2-K_{sl2}d^2)}{I_{xx}} \\ \frac{(-C_{sr1}c^2-C_{sl1}d^2-C_{sr2}c^2-C_{sl2}d^2)}{I_{xx}} \\ \frac{-K_{sr1}c}{I_{xx}} \\ \frac{C_{sr1}c}{I_{xx}} \\ \frac{K_{sl1}d}{I_{xx}} \\ \frac{C_{sl1}d}{I_{xx}} \\ \frac{-K_{sr2}c}{I_{xx}} \\ \frac{-C_{sr2}c}{I_{xx}} \\ \frac{K_{sl2}d}{I_{xx}} \\ \frac{C_{sl2}d}{I_{xx}} \end{bmatrix}^T$$

Fig. 2 shows the plot of variation of displacement of Z_{CG} with time obtained by simulation of SIMULINK model developed from the governing equations (1-7). The simulation result is in good agreement with analytical solution using the above State-Space Matrix solved using MATLAB coding and output plot is shown in Fig.3 as variation of displacement Z_{CG} with respect to time as analytical solution.

$$A10 = \begin{bmatrix} \frac{K_{sl1}}{m_{wl1}} \\ \frac{C_{sl1}}{m_{wl1}} \\ \frac{-K_{sla}}{m_{wl1}} \\ \frac{-C_{sl1}a}{m_{wl1}} \\ \frac{K_{sl1}d}{m_{wl1}} \\ \frac{C_{sl1}d}{m_{wl1}} \\ 0 \\ 0 \\ \frac{(-K_{sl1}-K_{wl1})}{m_{wl1}} \\ \frac{-C_{sl1}}{m_{wl1}} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T \quad A12 = \begin{bmatrix} \frac{K_{sr2}}{m_{wr2}} \\ \frac{C_{sr2}}{m_{wr2}} \\ \frac{K_{sr2}b}{m_{wr2}} \\ \frac{C_{sr2}b}{m_{wr2}} \\ \frac{-K_{sr2}c}{m_{wr2}} \\ \frac{-C_{sr2}c}{m_{wr2}} \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{(-K_{sr2}-K_{wr1})}{m_{wr2}} \\ \frac{-C_{sr2}}{m_{wr2}} \\ 0 \\ 0 \end{bmatrix}^T \quad A14 = \begin{bmatrix} \frac{K_{sl2}}{m_{wl2}} \\ \frac{C_{sl2}}{m_{wl2}} \\ \frac{K_{sl2}b}{m_{wl2}} \\ \frac{C_{sl2}b}{m_{wl2}} \\ \frac{K_{sl2}d}{m_{wl2}} \\ \frac{C_{sl2}d}{m_{wl2}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{(-K_{sl2}-K_{wl2})}{m_{wl2}} \\ \frac{-C_{sl2}}{m_{wl2}} \end{bmatrix}^T \quad A8 = \begin{bmatrix} \frac{K_{sr1}}{m_{wr1}} \\ \frac{C_{sr1}}{m_{wr1}} \\ \frac{-K_{sr1}a}{m_{wr1}} \\ \frac{-C_{sr1}a}{m_{wr1}} \\ \frac{-K_{sr1}c}{m_{wr1}} \\ \frac{-C_{sr1}c}{m_{wr1}} \\ \frac{(-K_{sr1}-K_{rr1})}{m_{wr1}} \\ \frac{-C_{sr1}}{m_{wr1}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T$$

III. VALIDATIONS OF FULL CAR MODEL

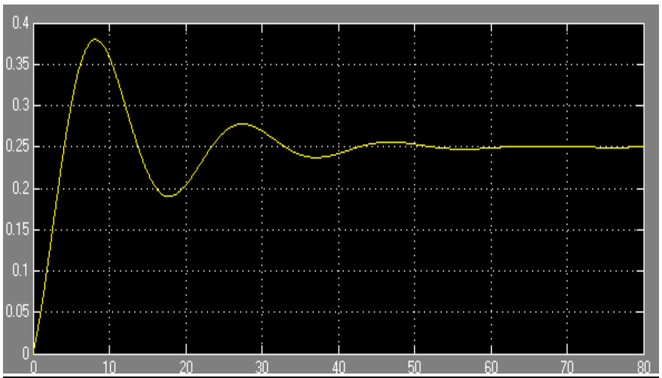

Fig. 2 Sprung Mass Displacement (Z_{CG}) vs. Time In Simulink model

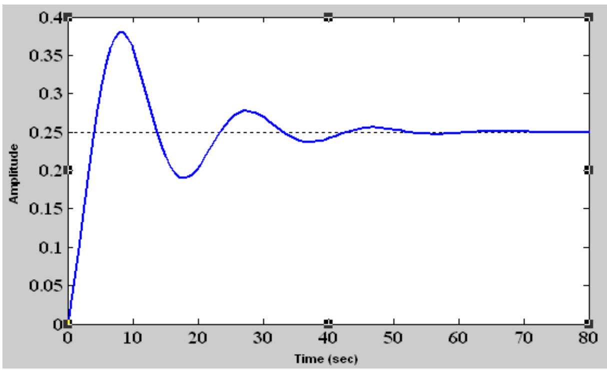

Fig.3 Sprung Mass Displacement (Z_{CG}) Vs. Time using Analytical solutions (State space)

Analysis of validated Simulink Model

The performance characteristics which are of utmost interest while designing the vehicle suspension includes passenger ride comfort ($\ddot{Z}_{cg}$) and road holding (Z_w-Z_r). As per ISO: 2631-1-1997, the passenger feels highly comfortable if the weighted RMS acceleration is below 0.315 m/s2,but to cross the bump speed of the vehicle must be less than 10 kmph which can be fairly comfortable to human body. For the proper Suspension travel minimum of 5 inches (0.127 m) of suspension travel must be available in order to absorb a bump acceleration of one-half "g" without hitting the suspension stops (Gillespie, 2003). For the proper road holding relative displacement between wheel and road must be in the range of 0.0508 m (Gillespie, 2003). In this paper, analysis of validated full car simulation model is conducted to study the effect of suspension spring and damping coefficient on ride comfort and road holding is tabulated in Table.1. A standard highway road profile (table top half sine wave, width 3.7m & amplitude 10cm) as per IRC-99-1988 is used to study the simulation for different vehicle speed such as 40, 25 and 10 kmph, shown in Fig. 4. Also the road profile of city (half sine wave, width 0.3 m & amplitude 10cm) used for analysis to find the performance characteristics of vehicle.

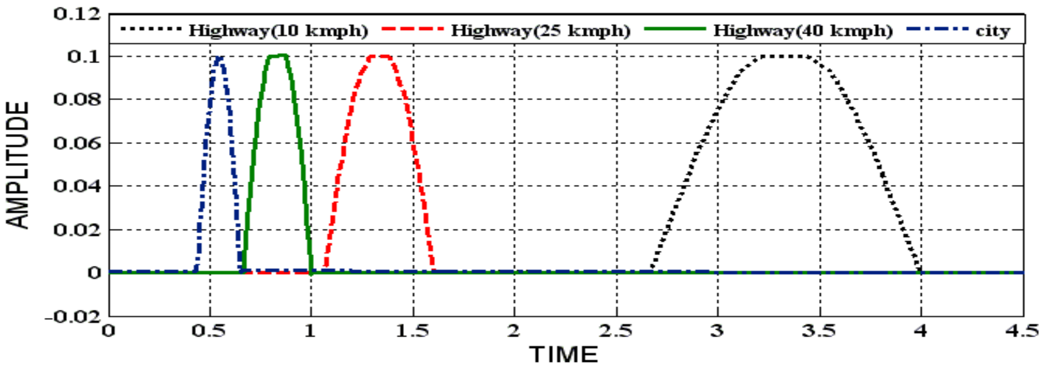

Fig.no 4 Road profile simulink model

IV. RESULTS

Table 1: Effect of Stiffness and Damping Coefficient on Ride Comfort and Road Holding

Suspension Stiffness, K (N/m)	Damping Coefficient, C (N-s/m)	velocity of vehicle (m/s)	Ride Comfort RMS accl. (m/s^2)	Suspension travel (m)	Road Holding (m)	Satlling Time (Sec)
Highway Bump of width 3.7 mts. and Height 10 cms.						
55000	1000	40	2.397	0.082	0.0196	5
		25	2.444	0.084	0.01111	6
		10	0.4529	0.054	0.004937	6.2
55000	4000	40	2.567	0.07	0.015	1.8
		25	1.396	0.065	0.0085	2
		10	0.353	0.054	0.003	3.5
25000	1000	40	1.39	0.07	0.02011	4.5
		25	1.07	0.075	0.00834	5
		10	0.311	0.062	0.00351	6
25000	4000	40	1.9541	0.06	0.0149	1.4
		25	1.158	0.061	0.00858	1.8
		10	0.304	0.055	0.003	3.5
City Bump of width 0.3 mts. and Height 10 cms.						
55000	4000	5	3.048	0.06	0.01665	1
		10	5.45	0.04	0.03447	1.2

Vehicle CG Bounce

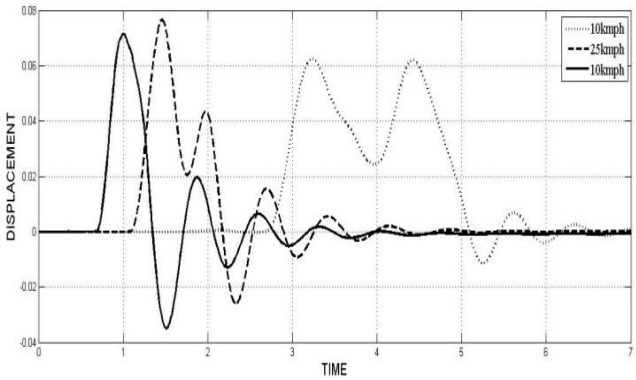

Fig.5 Displacement Vs Time for K=25000,C=1000

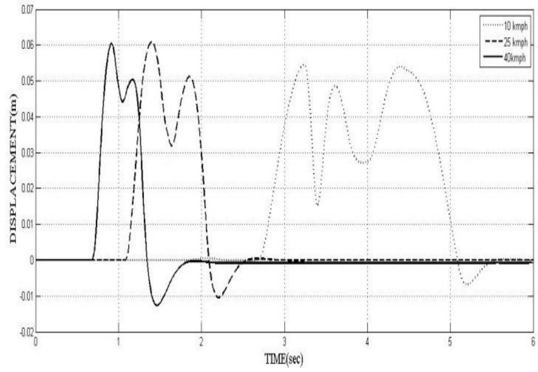

Fig.6 Displacement Vs Time for K=25000,C=4000

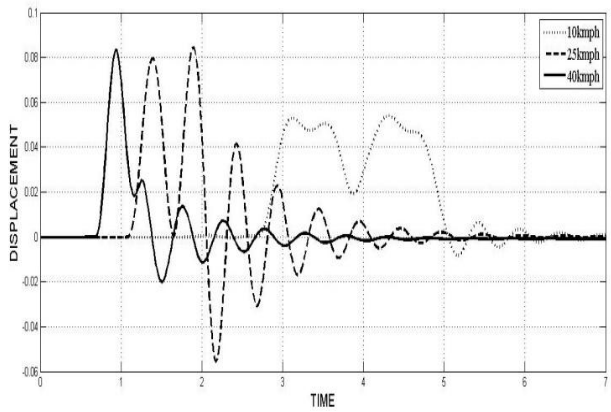

Fig.7 Displacement Vs Time for K=55000,C=1000

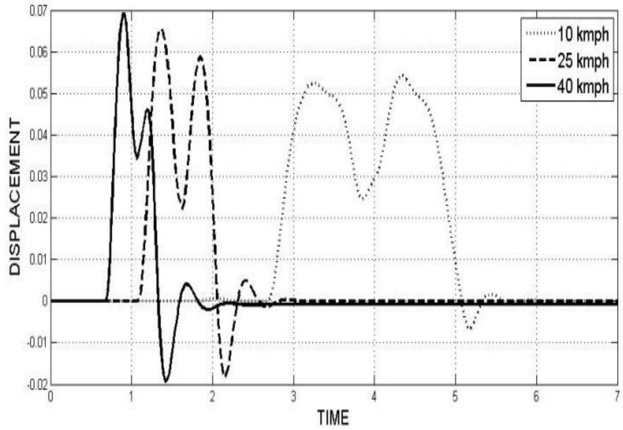

Fig.8 Displacement Vs Time for K=55000,C=4000

Vehicle Pitching and Rolling

Pitching

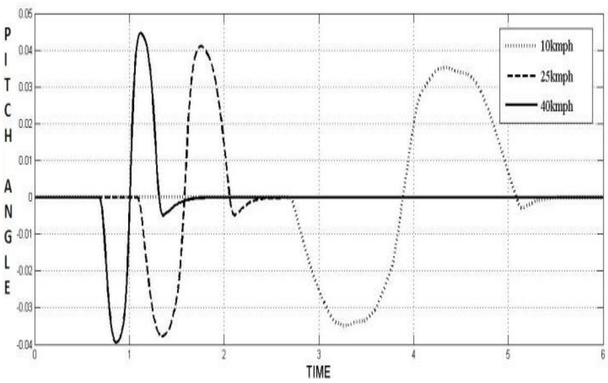

Fig. 9 Pitch angle Vs Time for K=25000,C=4000

Rolling

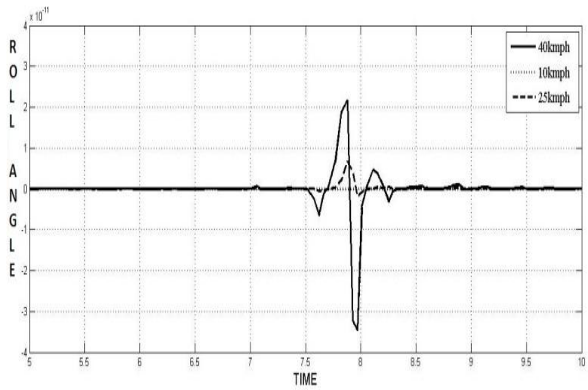

Fig.10 Roll angle Vs Time for K=25000,C=4000

V. CONCLUSION

In this work the methodology was developed to design a passive suspension for a passenger car for satisfying the two conflicting criteria viz. Ride comfort and Road holding as per ISO-2631-1, 1997. Mathematical modeling has been also performed using a seven degree-of-freedom model of the full car for passive system. The solution of analytical methods is validated with the Simulink model. This validated simulation model is used as a platform to analyze the performance of vehicle dynamics for different road profile. Table 1 shows the ride comfort and road holding for different standard road profiles. From this table one can easily conclude that speed range of 5 to 10 kmph must be an optimum speed to cross the bump without affecting the Human tolerance zone of 0.315 m/s^2 to 0.625 m/s^2 as per ISO standard. Presently, the effect of synthetic type bump are used in city area, which are more dangerous for human health, as vehicle body acceleration is very high, even at velocity of 10 kmph. The effect of bump of same amplitude nearly has no effect on pitch angle and roll angle of the vehicle, as shown in Fig.9 and 10. As per results spring stiffness, damping coefficient as 25000 N/m and 4000 N-s/m may provide better comfort.
The outcome of this paper using the validated simulink model of full car with detailed steps for further study, analysis and optimization of the other suspension parameters in automotive system designs.

VI. Future Scope

There is tremendous amount of scope for further studies of this topic, as one can compare the Semi-active, Active suspension system with Passive system. Some evolutionary optimization techniques, like Genetic Algorithm will be used to optimize the multiobjective functions.

REFERENCES

[1] M J Griffin, Handbook of Human Vibration. (1996), Academic press, London Publication, ISBN-10: 0123030412.

[2] O. Gundogdu, "Optimal seat and suspension design for a quarter car with driver model using genetic algorithms, *International Journal of Industrial Ergonomics, 37(4), (2007), 327-332.*

[3] J. Y. Wong, Theory of Ground Vehicles, John Wiley & Sons, New York, (2008). ISBN-10: 0470170387.

[4] A. N. Thite, Development of a Refined Quarter Car Model for the Analysis of Discomfort due to Vibration, *Advances in Acoustics and Vibration, Hindawi Publishing Corporation, Article ID 863061,(2012) doi:10.1155/2012/863061.*

[5] A. Agharkakli, G. S. Sabet, A. Barouz, Simulation and Analysis of Passive and Active Suspension System Using Quarter Car Model for Different Road Profile, *International Journal of Engineering Trends and Technology, 3(5),* (2012), 636-644.

[6] IRC-99-1988: "Tentative guidelines on the provision of speed breakers for control of vehicular speeds on minor roads*" published by The Indian Road Congress.*

[7] Traffic advisory Leaflet 10/00, Road humps: discomfort, noise, and ground-borne vibration (2000), *TM Division, Department of Transport, 2/06 Great Minster House, 76 Marsham Street, London SW1P 4DR.*

[8] ISO: 2631-1, 1997, "Mechanical vibration and shock - Evaluation of human exposure to whole-body vibration"

[9] P.Y. Zhu, J.P. Hessling, D.S. Liu, Optimal road hump for comfortable speed reduction, *Fourth International Symposium on Precision Mechanical Measurements, Proc. of SPIE Vol. 7130, 71304L,* (2008).

[10] A. Shirahatt , P.S.S. Prasad , P. Panzade, M.M. Kulkarni, Optimal design of passenger car suspension for ride and road holding, *Journal of the Brazilian Society of Mechanical Science & Engineering. 30(1), (2008), 66-76.*

Appendix:
14 X 14 State Space matix A

$$
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{(-K_{sr1}-K_{sf1}-K_{sr2}-K_{sf2})}{m_s} & \frac{(-C_{sr1}-C_{sf1}-C_{sr2}-C_{sf2})}{m_s} & \frac{(K_{sr1}a+K_{sf1}a-K_{sr2}b-K_{sf2}b)}{m_s} & \frac{(C_{sr1}a+C_{sf1}a-C_{sr2}b-C_{sf2}b)}{m_s} & \frac{(K_{sr1}c-K_{sf1}d+K_{sr2}c-K_{sf2}d)}{m_s} & \frac{(C_{sr1}c-C_{sf1}d+C_{sr2}c-C_{sf2}d)}{m_s} & \frac{K_{sr1}}{m_s} & \frac{C_{sr1}}{m_s} & \frac{K_{sf1}}{m_s} & \frac{C_{sf1}}{m_s} & \frac{K_{sr2}}{m_s} & \frac{C_{sr2}}{m_s} & \frac{K_{sf2}}{m_s} & \frac{C_{sf2}}{m_s} \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{(K_{sr1}a+K_{sf1}a-K_{sr2}b-K_{sf2}b)}{I_{yy}} & \frac{(C_{sr1}a+C_{sf1}a-C_{sr2}b-C_{sf2}b)}{I_{yy}} & \frac{(-K_{sr1}a^2-K_{sf1}a^2-K_{sr2}b^2-K_{sf2}b^2)}{I_{yy}} & \frac{(-C_{sr1}a^2-C_{sf1}a^2-C_{sr2}b^2-C_{sf2}b^2)}{I_{yy}} & \frac{(-K_{sr1}ac+K_{sf1}ad+K_{sr2}bc-K_{sf2}bd)}{I_{yy}} & \frac{(-C_{sr1}ac-C_{sf1}ad-C_{sr2}bc-C_{sf2}bd)}{I_{yy}} & \frac{-K_{sr1}a}{I_{yy}} & \frac{-C_{sr1}a}{I_{yy}} & \frac{-K_{sf1}a}{I_{yy}} & \frac{-C_{sf1}a}{I_{yy}} & \frac{K_{sr2}b}{I_{yy}} & \frac{C_{sr2}b}{I_{yy}} & \frac{K_{sf2}b}{I_{yy}} & \frac{C_{sf2}b}{I_{yy}} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{(K_{sr1}c-K_{sf1}d+K_{sr2}c-K_{sf2}d)}{I_{xx}} & \frac{(C_{sr1}c-C_{sf1}d+C_{sr2}c-C_{sf2}d)}{I_{xx}} & \frac{(-K_{sr1}ac+K_{sf1}ad+K_{sr2}bc-K_{sf2}bd)}{Ixx} & \frac{(-C_{sr1}ac-C_{sf1}ad-C_{sr2}bc-C_{sf2}bd)}{I_{xx}} & \frac{(-K_{sr1}c^2-K_{sf1}d^2-K_{sr2}c^2-K_{sf2}d^2)}{I_{xx}} & \frac{(-C_{sr1}c^2-C_{sf1}d^2-C_{sr2}c^2-C_{sf2}d^2)}{I_{xx}} & \frac{-K_{sr1}c}{I_{xx}} & \frac{C_{sr1}c}{I_{xx}} & \frac{K_{sf1}d}{I_{xx}} & \frac{C_{sf1}d}{I_{xx}} & \frac{-K_{sr2}c}{I_{xx}} & \frac{-C_{sr2}c}{I_{xx}} & \frac{K_{sf2}d}{I_{xx}} & \frac{C_{sf2}d}{I_{xx}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{K_{sr1}}{m_{wr1}} & \frac{C_{sr1}}{m_{wr1}} & \frac{-K_{sr1}a}{m_{wr1}} & \frac{-C_{sr1}a}{m_{wr1}} & \frac{-K_{sr1}c}{m_{wr1}} & \frac{-C_{sr1}c}{m_{wr1}} & \frac{(-K_{sr1}-K_{tr1})}{m_{wr1}} & \frac{-C_{sr1}}{m_{wr1}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\frac{K_{sf1}}{m_{wf1}} & \frac{C_{sf1}}{m_{wf1}} & \frac{-K_{sf}a}{m_{wf1}} & \frac{-C_{sf1}a}{m_{wf1}} & \frac{K_{sf1}d}{m_{wf1}} & \frac{C_{sf1}d}{m_{wf1}} & 0 & 0 & \frac{(-K_{sf1}-K_{tf1})}{m_{wf1}} & \frac{-C_{sf1}}{m_{wf1}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
\frac{K_{sr2}}{m_{wr2}} & \frac{C_{sr2}}{m_{wr2}} & \frac{K_{sr2}b}{m_{wr2}} & \frac{C_{sr2}b}{m_{wr2}} & \frac{-K_{sr2}c}{m_{wr2}} & \frac{-C_{sr2}c}{m_{wr2}} & 0 & 0 & 0 & 0 & \frac{(-K_{sr2}-K_{tr1})}{m_{wr2}} & \frac{-C_{sr2}}{m_{wr2}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\frac{K_{sf2}}{m_{wf2}} & \frac{C_{sf2}}{m_{wf2}} & \frac{K_{sf2}b}{m_{wf2}} & \frac{C_{sf2}b}{m_{wf2}} & \frac{K_{sf2}d}{m_{wf2}} & \frac{C_{sf2}d}{m_{wf2}} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{(-K_{sf2}-K_{tf2})}{m_{wf2}} & \frac{-C_{sf2}}{m_{wf2}}
\end{bmatrix}
$$

Full Car Simulink Model: