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Kane's Method for Robotic Arm Dynamics: A Novel Approach

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ABSTRACT: *This paper is the result of Analytical Research work in multi-body dynamics and desire to apply Kanes Method on the Robotic Dynamics. The Paper applies Kane's method (originally called Lagrange form of d'Alembert's principle) for developing dynamical equations of motion and then prepare a solution scheme for space Robotics arms. The implementation of this method on 2R Space Robotic Arm with Mat Lab Code is presented in this research paper. It is realized that the limitations and difficulties that are aroused in arm dynamics are eliminated with this novel Approach.*

Key Words: *Dynamics, Equation of Motion, Lagrangian, Robotic arm, Space Robot.*

I. INTRODUCTION

Robotics is considered as a multi body structure, its motion analysis is worked out using D-H Method of kinematics and dynamics. Essentially all methods for obtaining equations of motion are equivalent. However, the ease of use of the various methods differs; some are more suited for multibody dynamics than others. The Newton-Euler method is comprehensive in that a complete solution for all the forces and kinematic variables are obtained, but it is inefficient. Applying the Newton-Euler method requires that force and moment balances be applied for each body taking in consideration every interactive and constraint force. Therefore, the method is inefficient when only a few of the system's forces need to be solved for. Lagrange's Equations provides a method for disregarding all interactive and constraint forces that do not perform work. The major disadvantage of this method is the need to differentiate scalar energy functions (kinetic and potential energy). This is not much of a problem for small multibody systems, but becomes an efficiency problem for large multibody systems. Kane's method offers the advantages of both the Newton-Euler and Lagrange methods without the disadvantages. With the use of generalized forces the need for examining interactive and constraint forces between bodies is eliminated. Since Kane's method does not employ the use of energy functions, differentiating is not a problem. The differentiating required to compute velocities and accelerations can obtained through the use of algorithms based on vector products. Kane's method provides an elegant means to develop the dynamics equations for multibody systems that lends itself to automated numerical computation. (Huston 1990) This paper is organized in the following manner: Section 2 gives the general form of Kane's Equation and its derivation. The systematic procedure to implement the derived Kanes Equations is described in Section 3 .The application of Kane's method to space robotic arm and its implementation are given in sections 4 and 5 respectively. Finally Section 6 draws conclusions.

II. DERIVATION OF KANE'S EQUATIONS

Consider an open-chain multibody system of N interconnected rigid bodies each subject to external and constraint forces. The external forces can be transformed into an equivalent force and torque ($\vec{F}_k$ and $\vec{M}_k$) passing through $\vec{G}_k$, the mass center of the body k (k = 1,2…N). Similar to the external forces, the constraint forces may be written as $\vec{F}_k^c$ and $\vec{M}_k^c$. Using d'Alembert's principle for the force equilibrium of body k, the following is obtained:

$$\vec{F}_k + \vec{F}_k^* + \vec{F}_k^c = 0 \tag{2.1}$$

where $\vec{F}_k^* = -m_k a_k$ is the inertia force of body k.

The concept of virtual work may be described as follows for a system of N particles with 3N degrees of freedom. The systems configuration can be described using q_r (r = 1,2,…3N) generalized coordinates with force components F_1, F_2,…, F_{3N} applied to the particles along the corresponding generalized coordinates. The virtual work is then defined as:

$$\delta W = \sum_{i=1}^{N} \vec{F}_i \cdot \delta r_i \tag{2.2}$$

Where $\vec{F}_i$ is the resultant force acting on the i[th] particle and r_i is the position vector of the particle in the inertial reference frame. δr_i is the virtual displacement, which is imaginary in the sense that it is assumed to occur without the passage of time. Now applying the concept of virtual work to our multibody system considering only the work due to the forces on the system we obtain:

$$\delta W = (\vec{F}_k + \vec{F}_k^* + \vec{F}_k^c) \cdot \delta r_k = 0 \qquad (k = 1,2,\ldots,N) \tag{2.3}$$

The constraints that are commonly encountered are known as workless constraints so…

$$\vec{F}_k^c \cdot \delta r_k = 0 \tag{2.4}$$

This simplifies the virtual work equation to:

$$\delta W = (\vec{F}_{ki} + \vec{F}_k^*) \cdot \delta r_k = 0 \qquad (k = 1,2,\ldots,N)$$

or

$$\delta W = (\vec{F}_k + \vec{F}_k^*) \cdot \frac{\partial \vec{r}_k}{\partial q_r} \delta q_r = 0 \qquad (r = 1,2,\ldots,3N) \tag{2.5}$$

The positions vector may also be written as:

$$\vec{r}_k = \vec{r}_k(q_r, t) \tag{2.6}$$

so

$$\dot{\vec{r}}_k = \frac{\partial \vec{r}_k}{\partial q_r}\frac{dq_r}{dt} + \frac{\partial \vec{r}_k}{\partial t}$$

$$= \frac{\partial \vec{r}_k}{\partial q_r}\dot{q}_r + \frac{\partial \vec{r}_k}{\partial t} \tag{2.7}$$

Taking the partial derivative of $\dot{\vec{r}}_k$ with respect to $\dot{q}_r$ yields

$$\frac{\partial \dot{\vec{r}}_k}{\partial \dot{q}_r} = \frac{\partial \vec{r}_k}{\partial q_r}$$

or

$$\frac{\partial \vec{v}_k}{\partial \dot{q}_r} = \frac{\partial \vec{r}_k}{\partial q_r} \tag{2.7}$$

Since the virtual displacement δq_r is arbitrary without violating the constraints we can write * as:

$$f_r + f_r^* = 0 \tag{2.8}$$

where f_r and f_r^* are the generalized active and inertia forces respectively and are defined as follows:

$$f_r = \vec{F}_k \cdot \frac{\partial \vec{v}_k}{\partial \dot{q}_r}$$

and

$$f_r^* = \vec{F}_k^* \cdot \frac{\partial \vec{v}_k}{\partial \dot{q}_r}$$

In a similar fashion it can be shown using virtual work that the moments can be written as:

$$M_r + M_r^* = 0 \tag{2.9}$$

where M_r and M_r^* are the generalized active and inertia moments respectively and are defined as follows:

$$M_r = \vec{T}_k \cdot \frac{\partial \vec{\omega}_k}{\partial \dot{q}_r}$$

and

$$M_r^* = -(\vec{\alpha}_k \cdot \vec{\vec{I}} + \vec{\omega}_k \times \vec{\vec{I}} \cdot \vec{\omega}_k) \cdot \frac{\partial \vec{\omega}_k}{\partial \dot{q}_r}$$

By superposition of the force and moment equations we arrive at Kane's equations:

$$F_r + F_r^* = 0 \tag{2.10}$$

where

$$F_r = f_r + M_r$$
$$F_r^* = f_r^* + M_r^*$$

III. GENERAL PROCEDURE OF KANE'S METHOD

1. Label *important* points (important points being defined as all center of mass locations and locations of applied forces with the exception of conservative constraint forces).

2. Select generalized coordinates (q_r) and generalized speeds (u_r), then generate expressions for angular velocity and acceleration of all bodies and velocity and acceleration of the important points.

3. Construct a partial velocity table of the form;

Generalized Speeds (u_r)	${}^N\vec{v}_r^{\,A}$	${}^N\vec{v}_r^{\,B}$	${}^N\vec{\omega}_r^{\,A}$	${}^N\vec{\omega}_r^{\,B}$
r = 1				
r = 2				
. . .	. . .	. . .	. . .	. . .

4. Fr + Fr* = 0

where the generalized active force, F_r, is defined as:

$$F_r = \sum_r \left(\vec{F}_A \cdot {}^N\vec{v}_r^{\,A} + \vec{T}_A \cdot {}^N\vec{\omega}_r^{\,A} + \vec{F}_B \cdot {}^N\vec{v}_r^{\,B} + \vec{T}_B \cdot {}^N\vec{\omega}_r^{\,B} \right) \tag{3.1}$$

and the generalized inertia force, F_r^*, is defined as:

$$F_r^* = \sum_r \left(-m \cdot {}^N\vec{a}^A \cdot {}^N\vec{v}_r^{\,A} - \left({}^N\vec{\alpha}^A \cdot \vec{\vec{I}} + {}^N\vec{\omega}^A \times \vec{\vec{I}} \cdot {}^N\vec{\omega}^A \right) {}^N\vec{\omega}_r^{\,A} - m \cdot {}^N\vec{a}^B \cdot {}^N\vec{v}_r^{\,B} - \left({}^N\vec{\alpha}^B \cdot \vec{\vec{I}} + {}^N\vec{\omega}^B \times \vec{\vec{I}} \cdot {}^N\vec{\omega}^B \right) {}^N\vec{\omega}_r^{\,B} \right) \tag{3.2}$$

5) Which can then be written in the form:

$$[M]\{\dot{u}\} = \{RHS(u)\} \tag{3.3}$$

IV. KANE'S METHOD APPLIED TO ROBOTIC ARMDYNAMICS

The double Linked Arm is shown in Figure-1.Three Coordinate systems have been introduced at joints and End of the arm.The joint arms are {O} and {P}.{B} is the arm end point. The direction cosines between coordinate system {O} and {P} are given in Table-1 and between {P} and {B} in Table-2. In this case we will introduce u_3 to find an expression for T_c (constraint torque about n_2). The joints at O and P are revolute. Body A and B are uniform rods with length 4L and 2L respectively. Body A has two times the mass of body B.

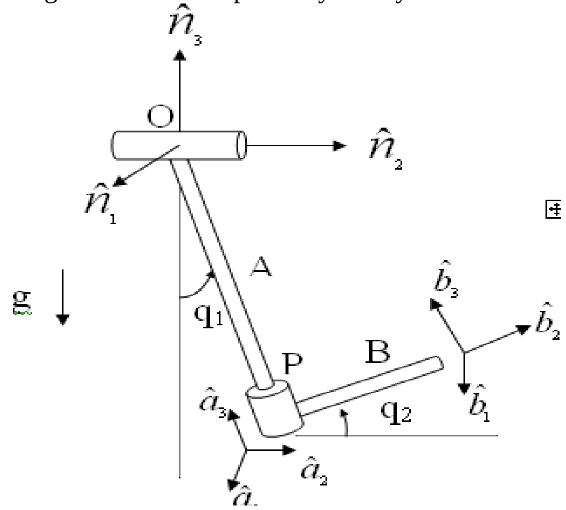

Fig.1 : Double link Robotic Arm

Table:1 DC between {O}and {P}

	$\hat{a}_1$	$\hat{a}_2$	$\hat{a}_3$
$\hat{n}_1$	C_1	0	S_1
$\hat{n}_2$	0	1	0
$\hat{n}_3$	$-S_1$	0	C_1

Table:2 DC between {A} and {B}

	$\hat{a}_1$	$\hat{a}_2$	$\hat{a}_3$
$\hat{b}_1$	C_1	0	S_1
$\hat{b}_2$	0	1	0
$\hat{b}_3$	$-S_1$	0	C_1

Step 1) Choose important points: Center of Mass of bodies A and B, and point P.

Step 2) Select generalized coordinates as shown in the figure (plus auxiliary generalized coordinate u_3) and generate velocity and acceleration expressions for the important points.

The prime in the equations below indicates that the specified quantities contain the auxiliary generalized coordinate.

Body A

$$ {}^N\vec{\omega}'^A = u_3\hat{a}_1 + u_1\hat{a}_2 \tag{4.1} $$

$$ {}^N\vec{v}'^{A*} = {}^N\vec{v}^O + {}^N\vec{\omega}'^A \times \vec{r}^{OA*} = 2L(u_3\hat{a}_2 - u_1\hat{a}_1) \tag{4.2} $$

$$ {}^N\vec{\alpha}^A = \dot{u}_1\hat{a}_2 \tag{4.3} $$

$$ {}^N\vec{a}^{A*} = -2L\dot{u}_1\hat{a}_1 + 2Lu_1^2\hat{a}_3 \tag{4.4} $$

Point P

$${}^{N}\vec{v}'^{P} = {}^{N}\vec{v}^{O} + {}^{N}\vec{\omega}'^{A} \times \vec{r}^{OP} = 4L(u_3\hat{a}_2 - u_1\hat{a}_1) \tag{4.5}$$

Body B

$${}^{N}\vec{\omega}'^{B} = {}^{N}\vec{\omega}'^{A} + {}^{A}\vec{\omega}^{B} = (u_3\hat{a}_1 + u_1\hat{a}_2) + u_2\hat{a}_3 \tag{4.6}$$

$${}^{N}\vec{v}'^{B*} = {}^{N}\vec{v}'^{P} + {}^{N}\vec{\omega}'^{B} \times \vec{r}^{PB*} = L\left[(-4u_1 - u_2c_2)\hat{a}_1 + (4u_3 - u_2s_2)\hat{a}_2 + (u_3c_2 + u_1s_2)\hat{a}_3\right] \tag{4.7}$$

$${}^{N}\vec{\alpha}^{B} = {}^{N}\vec{\alpha}^{A} + {}^{A}\vec{\alpha}^{B} + {}^{N}\vec{\omega}^{A} \times {}^{A}\vec{\omega}^{B} = \dot{u}_1\hat{a}_2 + \dot{u}_2\hat{a}_3 + u_1u_2\hat{a}_1 \tag{4.8}$$

$${}^{N}\vec{a}^{B*} = {}^{N}\vec{a}^{P} + {}^{N}\vec{\alpha}^{B} \times \vec{r}^{PB*} + {}^{N}\vec{\omega}^{B} \times ({}^{N}\vec{\omega}^{B} \times \vec{r}^{PB*})$$
$$= -4\dot{u}_1\hat{a}_1 + u_1^2\hat{a}_2 + L\left[-\dot{u}_2c_2\hat{a}_1 - \dot{u}_2s_2\hat{a}_2 + (u_1u_2c_2 + \dot{u}_1s_2)\hat{a}_3\right] + L\left[(u_1^2s_2 + u_2^2s_2)\hat{a}_1 - u_2^2c_2\hat{a}_2 + u_1u_2c_2\hat{a}_3\right] \tag{4.9}$$

Step 3) Construct a partial velocity table.

u_r	${}^{N}\vec{\omega}_r'^{A}$	${}^{N}\vec{v}_r'^{A*}$	${}^{N}\vec{\omega}_r'^{B}$	${}^{N}\vec{v}_r'^{B*}$
r = 1	$\hat{a}_2$	$-2L\hat{a}_1$	$\hat{a}_2$	$-4L\hat{a}_1 + Ls_2\hat{a}_3$
r = 2	0	0	$\hat{a}_3$	$L(-c_2\hat{a}_1 - s_2\hat{a}_2)$
r = 3	$\hat{a}_1$	$2L\hat{a}_2$	$\hat{a}_1$	$L(4\hat{a}_2 + c_2\hat{a}_3)$

Step 4) $F_r + F_r^* = 0$

$$F_r = (-2mg\hat{n}_3)\cdot {}^{N}\vec{v}_r'^{A*} + (T_c\hat{a}_3)\cdot {}^{N}\vec{\omega}_r'^{A} + (-mg\hat{n}_3)\cdot {}^{N}\vec{v}_r^{B}$$

$$F_1 = -mgL(8s_1 + s_2c_1)$$
$$F_2 = -mgLs_1c_2$$
$$F_3 = -mgLc_1c_2 + T_c$$

$$F_r^* = (-2m\cdot {}^{N}\vec{a}^{A*})\cdot {}^{N}\vec{v}_r'^{A*} - \left({}^{N}\vec{\alpha}^{A}\cdot \bar{\bar{I}}^{A/A^*} + {}^{N}\vec{\omega}^{A} \times \bar{\bar{I}}^{A/A^*}\cdot {}^{N}\vec{\omega}^{A}\right)\cdot {}^{N}\vec{\omega}_r'^{A}$$
$$+ (-m\cdot {}^{N}\vec{a}^{B*})\cdot {}^{N}\vec{v}_r'^{B*} - \left({}^{N}\vec{\alpha}^{A}\cdot \bar{\bar{I}}^{B/B^*} + {}^{N}\vec{\omega}^{B} \times \bar{\bar{I}}^{B/B^*}\cdot {}^{N}\vec{\omega}^{B}\right)\cdot {}^{N}\vec{\omega}_r'^{B}$$

$$\bar{\bar{I}}^{A/A^*} = 2m\begin{bmatrix} \frac{4}{3}L^2 & 0 & 0 \\ 0 & \frac{4}{3}L^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \frac{4}{3}L^2\hat{a}_1\hat{a}_1 + \frac{4}{3}L^2\hat{a}_2\hat{a}_2$$

$$\bar{\bar{I}}^{B/B^*} = m\begin{bmatrix} \frac{1}{3}L^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3}L^2 \end{bmatrix} = \frac{1}{3}L^2\hat{b}_1\hat{b}_1 + \frac{1}{3}L^2\hat{b}_3\hat{b}_3$$

$$F_1^* = -\left(\frac{8mL^2}{3}\right)\dot{u}_1 + (-8mL^2)\dot{u}_1 + \left(-\frac{mL^2}{3}\right)(\dot{u}_1 s_2^2 + u_1 u_2 s_2 c_2)$$
$$+ (-mL^2)\left[4(\dot{u}_1 + \dot{u}_2 c_2 - s_2(u_1^2 + u_2)^2) + s_2(2u_1u_2c_2 + \dot{u}_1 s_2)\right]$$

$$F_2^* = -\left(\frac{mL^2}{3}\right)(\dot{u}_2 - s_2c_2u_1^2) + mL^2\left[(-\dot{u}_1 - \dot{u}_2c_2 + s_2(u_1^2 + u_2^2))c_2 + s_2(4u_1^2 - \dot{u}_2 s_2 - u_2^2)\right]$$

$$F_3^* = -\left(\frac{mL^2}{3}\right)(\dot{u}_1 s_2 c_2 + 2u_1u_2c_2^2) - mL^2\left[4(4u_1^2 - \dot{u}_2 s_2 - u_2^2) + c_2(2u_1u_2c_2 + \dot{u}_1 s_2)\right]$$

Step 5) Assemble $F_r + F_r^* = 0$

$$\begin{bmatrix} -\frac{8mL^2}{3} - 12mL^2 - \frac{4mL^2}{3}s_2^2 & -4mL^2c_2 & 0 \\ -mL^2c_2 & -\frac{mL^2}{3} - mL^2 & 0 \\ -\frac{mL^2}{3}s_2c_2 - mL^2 s_2 & 4mL^2 s_2 & 1 \end{bmatrix} \begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \\ T_c \end{bmatrix} = \begin{bmatrix} \frac{mL^2}{3}u_1u_2s_2c_2 + mL^2(4s_2(u_1^2 + u_2)^2 + 2s_2c_2u_1u_2) + mgl(8s_1 + s_2c_1) \\ \frac{mL^2}{3}u_1^2 s_2 c_2 - mL^2 s_2 c_2(u_1^2 + u_2^2) + mgls_1c_2 - mL^2 s_2(4u_1^2 + u_2^2) \\ \frac{2mL^2}{3}u_1u_2c_2^2 + 16mL^2u_1^2 + mglc_2c_1 + 2mL^2c_2^2u_1u_2 - 16mL^2u_2^2 \end{bmatrix} \quad (4.10)$$

V. SIMULATION STUDIES

The simulation studies , to kow the acceleration of End point of arm and Torque characterstic at joint1, is carried out with the parametersgiven in Table:3.

Table:3 Parameters taken for Simulation

Parameters	Values
M	5 Kg
L	1.25m
U_1	1.0 m/s
U2	0.25 m/s
q for joint O	0 -45^o
q for joint A	0 -30^0
End effector travel time	0 -10Sec

The dynamic behaviour of robotic arm due to uniform angular motions at joints O and A is studied interms of the acceleration of end effector (B) and the major Joint Torque at joint O and illustrated in Fig.2 and Fig. 3. The Fig. 3 illustrates the variation of acceleration computed with the proposed Kanes method as well as Leagrangian method.There is a slight deviation between the both response. This is due to approximation made in the non linear terms of the method.

Morever , an acceptable correlation between the two torque responses with the both methods Lagrangian and Kanes, can be seen in Fig. 3. This is due to fact that the proposed model allows analysing variations in the torque with the conventional linear model.

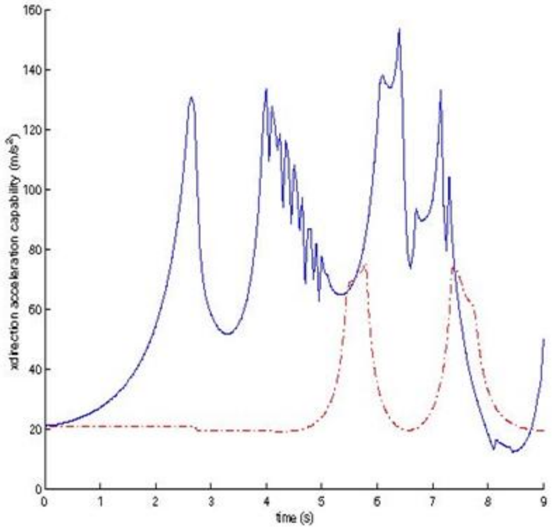

Fig. 2 The acceleration of end effector

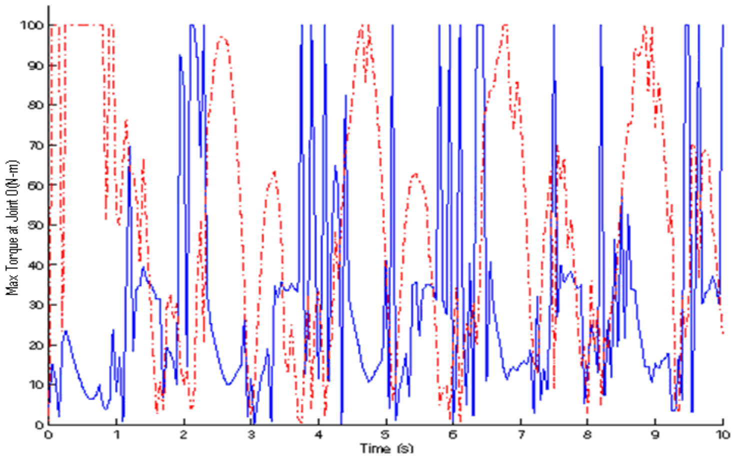

Fig. 3 Torque variation at joint O

VI. CONCLUSIONS

Although the applications of Kane's method to multibody systems in this paper has been old one, We have shown that it is a powerful technique in robot dynamics. It offers the advantages of both the Newton-Euler and Lagrange methods in that it can be comprehensive and efficient. As part of implimenting Kane's method , a dynamics software package (MATLAB) has been used. The generelasised procedure of Kane's method to develop equations of motion and perform simulations is presented in this paper. We have also successfully built dynamic models using lagrangian method to R-R Robotic Arms and published in journals. In this paper , the earlier published work is compared with Kanes method. It is also concluded that the developed Kanes model is very useful in Robotics when its complexity is increased by the way adding joints/additional degrees of freedom.

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Notation:

Notation that will be used throughout the rest of the paper:

q_r - generalized coordinate

u_r - generalized speed, typically equal to $\frac{dq_r}{dt}$

${}^N\vec{v}^P$ - velocity of P with respect to the Newtonian (inertial) reference frame

${}^N\vec{v}^{A*}$ - velocity of the center of mass of body A

${}^N\vec{v}_r^A$ - partial velocity, equal to $\frac{\partial\, {}^N\vec{v}^A}{\partial u_r}$

C_n or S_n – Cos(n) or Sin(n)