

ANALYSIS OF A PLATE WITH A CIRCULAR HOLE BY FEM

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Abstract A plate with a circular hole subjected to a uniform stress, the effect of an initial stretching of a rectangular plate with a cylindrical hole on the stress and displacement distributions around the hole, which are caused by the additional loading, was studied using the finite element method. It is assumed that the initial stresses are caused by the uniformly stretching forces acting on the 2 opposite ends. It is also assumed that the cylindrical hole contained by the thick plate is between these ends and goes in parallel with them. The aim of this paper is to analyze a plate with a circular hole subjected to a uniform stress and observe the Variation in the results obtained through various meshes.

I. INTRODUCTION

The problem can be approximated as 2-dimensional since the load is applied in the plane of the plate. In a Cartesian coordinate system there are two possible assumptions to take in regard to the behavior of the structure in the third dimension: (1) the plane stress condition, in which the stress components acting out of the 2D plane are assumed to be negligible; (2) the plane strain condition, in which the strain components out of the 2D plane are assumed negligible. The plane stress condition is appropriate for solids whose third dimension is thin as in this case; the plane strain condition is applicable for solids where the third dimension is thick.

At the end of the tutorial, the user can: investigate the sensitivity of the solution to mesh resolution and mesh grading; and, increase the size of the plate in comparison to the hole to try to estimate the error in comparing the analytical solution for an infinite plate to the solution of this problem of a finite plate.

1.1. Boundary and initial conditions

Once the mesh generation is complete, load the mesh into Foam X: remember this is done by clicking the right mouse button on Mesh in the case directory tree and selecting the Read Mesh function. The names of the patches will appear in a dictionary named Patches which should be set as follows: left and down patches are both symmetry Plane; the undefined Faces are the front and back planes of the 2D geometry and should therefore be declared empty; the other patches are traction boundary conditions, set by traction boundary type.

The Fields must be set as before; here, the displacement U and temperature T. The traction boundary conditions are specified by a linear combination of: (1) a boundary traction vector; (2) a pressure that produces a traction normal to the boundary surface that is defined as negative when pointing out of the surface. The up and hole patches are zero traction so the boundary traction and pressure are set to zero.

1.2 Mechanical properties

The physical properties for the case are set in the mechanical Properties dictionary. For this problem, we need to specify the mechanical properties of steel given in table

Table 1.1 Mechanical properties for steel

PROPERTY	UNITS	KEYWORDS	VALUE
Density	Kgm ⁻³	Rho	7854
Young's modulus	Pa	E	2*10 ¹¹
Poisson's ratio	--	Nu	0.3

II. PROBLEM DEFINATION

This problems deals with, a plate with a circular hole subjected to a uniform stress $\sigma_0=320\text{psi}$. Determine the stress concentration factor through FEA by using HM pre-processor-NASTRAN

solver combination and compare the same with photo elasticity value of 2.35 and analytical value as given by the formula(6)

III. TYPES OF MESH

In this problem, we used different types of meshes, they are as follows,

3.1. Auto mesh

Auto mesh is one of the fastest, less time consuming and easy method of meshing. Any component can be mesh with help of auto mesh. This command when we use, at that time, this part will automatically generate mesh according to its shape and size. Quad, Trias, Combination of quad and trias, R-trias (Right Angled Trias), advanced type of mesh pattern we can use in this method.

3.2. Ruled mesh

In this method we can select 2 sets of node. And sequence of selecting node is very important in this method of meshing. If the sequence of selecting the node is missed then mesh we obtained is not good one, mixing of no of component is taken place. At the time of meshing we have no of options like

1. Mesh, without surfaces 2. Mesh, keep surfaces 3. Mesh, delete, surfaces and 4. Surface only, we can use any of the option, as per requirement.

3.3. Spline option

In this option the surface is created in between the selected nodes .And this surface can be mesh with help of no. of options.

3.4. Spin option

This option is largely used in circular component. Where the washer or hole is there, spin cart is used. It creates mesh around the base point we mentioned during meshing. We can use spin 0 degree to 360 degree.

3.5. Manual meshing

This is one of the time consuming, but gives good type of mesh pattern. And obviously it gives good result also. This method requires good practice. Most of the industry practice uses this type of meshing. In this first small and critical parts are meshed and then go for big or larger parts compared to all parts. This is the key rule in manual meshing.

3.6. Split element options

This option is also very important, when the size of meshing is too large or big then we can use the split element option and reduce the size of the mesh element.

1. Split all sides 2. Divide quads 3. Mid point to quads 4. Mid point to trias. These options are available in split element option.

3.7. Edit element

In edit element option we can combine no. of elements, split no. of elements, cleanup elements. Combine to quads, combine to trias options also we can use.

IV. FINITE ELEMENT ANALYSIS

Static Stress and Strain Concentration Factors

Consider the plate shown in Fig.1, loaded in tension by a force per unit area, σ . Although not drawn to scale, consider that the outer dimensions of the plate are infinite compared with the diameter of the hole $2a$. It can be shown, from linear elasticity that the tangential stress throughout the plate is given by,

$$\sigma_{\theta} = \frac{\sigma}{2} \left[1 + \frac{a^2}{r^2} - \left(1 + 3\frac{a^4}{r^4} \right) \cos 2\theta \right] \quad (2)$$

The maximum stress is $\sigma_{\theta} = 3\sigma$ at $r = a$ and $\theta = \pm 90^\circ$

Figure.2.shows how the tangential stress varies along the x and y axes of the plate. For the top (and bottom) of the hole, we see the stress gradient is extremely large compared with the nominal stress, and hence the term stress concentration applies. Along the surface of the hole, the tangential stress is $-\sigma$ at $\theta = 0^\circ$ and 180° , and increases, as θ increases, to 3σ at $\theta = 90^\circ$ and 270°

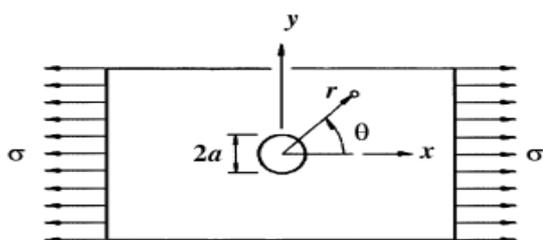


Figure 1 Circular hole in a plate loaded in tension.

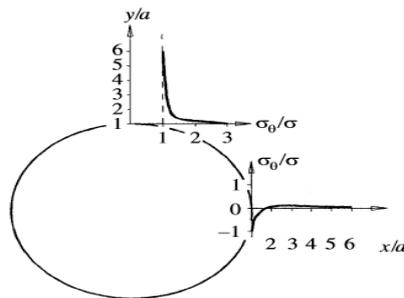


Figure 2 Tangential stress distribution for $y = 0$ and 90

The static stress concentration factor in the elastic range, K_t , is defined as the ratio of the maximum stress, σ_{max} , to the nominal stress, σ_{nom} . That is,

$$K_t = \frac{\sigma_{max}}{\sigma_{nom}} \quad (3)$$

For the infinite plate containing a hole and loaded in tension, $\sigma_{nom} = \sigma$, $\sigma_{max} = 3\sigma$, and thus $K_t = 3$.

The analysis of the plate in tension with a hole just given is for a very wide plate (infinite in the limit). As the width of the plate decreases, the maximum stress becomes less than three times the nominal stress at the zone containing the hole. Figure 3(a) shows a plate of thickness $t = 0.125$ in, width $D = 1.50$ in, with a hole of diameter $2r = 0.50$ in, and an applied uniform stress of $\sigma_0 = 320$ psi.

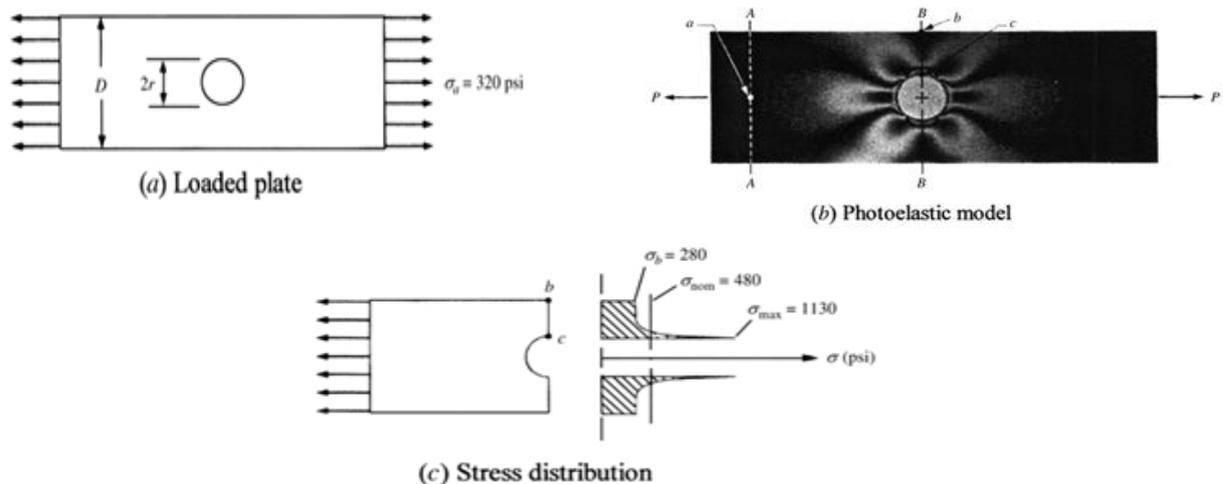


Figure 3 Stress distributions for a plate in tension containing a centrally located hole.

A photoelastic* model is shown in Fig. 3(b). From a photoelastic analysis, the stresses at points a, b, and c are found to be

zone A A: $\sigma_a = 320$ psi

zone B B: $\sigma_b = 280$ psi; $\sigma_c = 1130$ psi

The nominal stress in zone B B is

$$\sigma_{nom} = \frac{D}{D - 2r} \sigma_0 = \frac{1.50}{1.50 - 0.5} 320 = 480 \text{ psi} \quad (4)$$

If the stress was uniform from b to c, the stress would be 480 psi. However, the photoelastic analysis shows the stress to be nonuniform, ranging from 280 psi at b to a maximum stress at c of 1130 psi. Thus, for this example, the stress concentration factor is found to be

$$K_t = \frac{\sigma_{\max}}{\sigma_{\text{nom}}} = \frac{1130}{480} = 2.35 \quad (5)$$

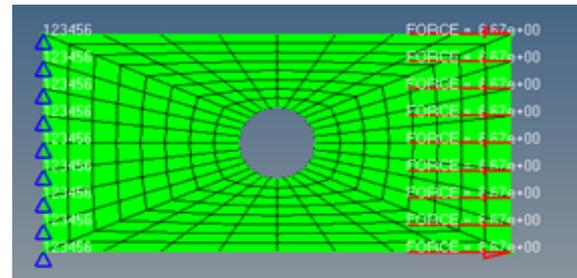
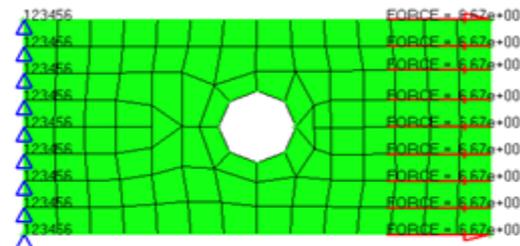
The static stress concentration factor for a plate containing a centrally located hole in which the plate is loaded in tension depends on the ratio $2r/D$ as given for case 7a of Table1. For our example here, $2r/D = 0.5/1.5 = 1/3$ The equation for K_t from Table gives

$$K_t = 3.00 - 3.13\left(\frac{1}{3}\right) + 3.66\left(\frac{1}{3}\right)^2 - 1.53\left(\frac{1}{3}\right)^3 = 2.31 \quad (6)$$

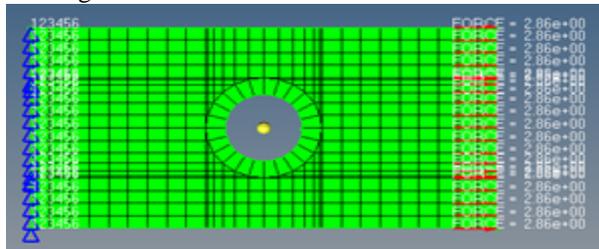
Which is within 2% of the results from the photo elastic model. Provides the means to evaluate the static stress concentration factors in the elastic range for many cases that apply to fundamental forms of geometry and loading conditions.

V. DETAILS OF MESH PLOTS

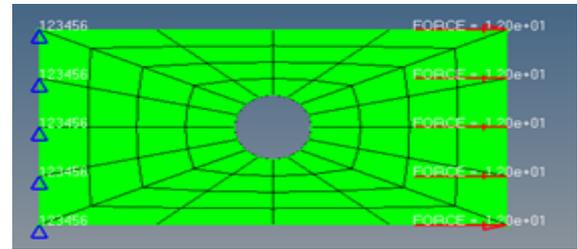
5.1. Automesh



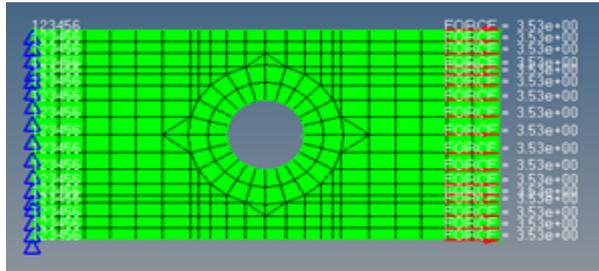
5.2. Single Washer



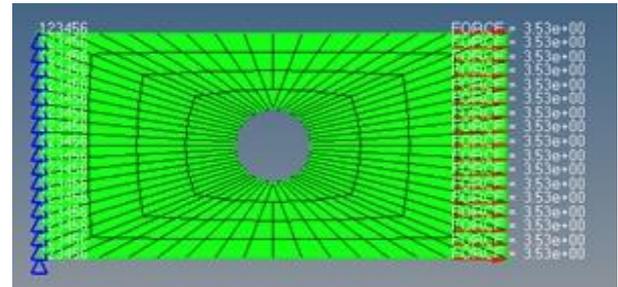
5.6. Coarse Ruled Mesh Without Washer



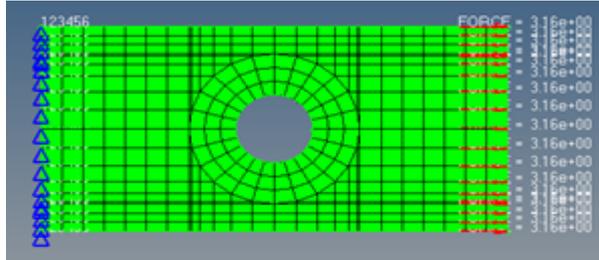
5.3. Double Washer



5.7. fine ruled mesh without washer



5.4. Three Washer

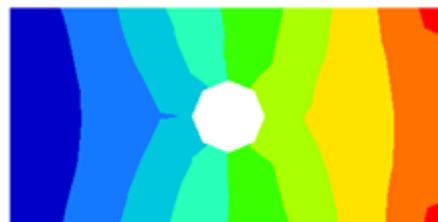


5.5. Middle Ruled Mesh Without Washer

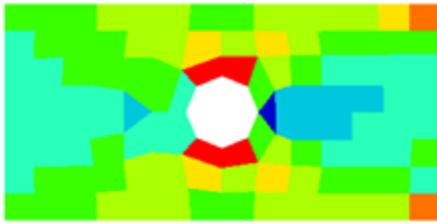
VI. PLOTS OF DISPLACEMENT AND STRESS

6.1. Automesh

Displacement plot

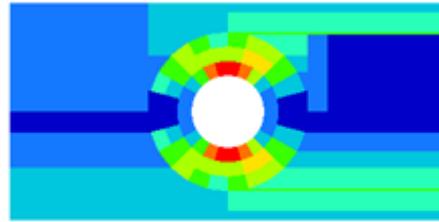


Stress plot

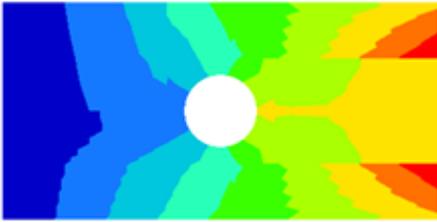


6.2. Single_Washer
Displacement plot

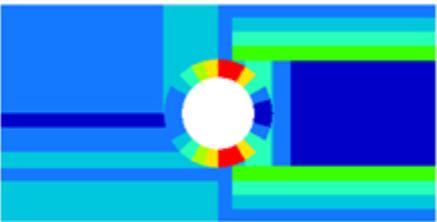
Stress plot



6.5. Middle Ruled Mesh Without Washer
Displacement plot

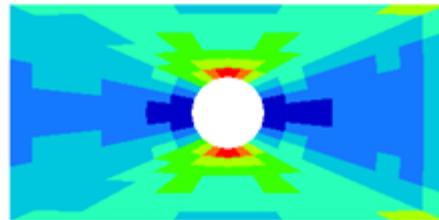


Stress plot

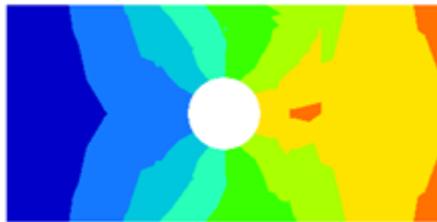


6.3. Double Washer
Displacement plot

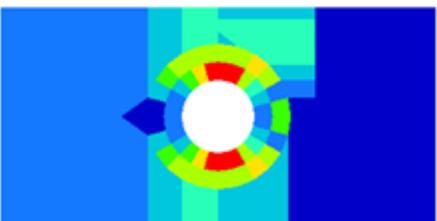
Stress plot



6.6. Coarse Ruled Mesh Without Washer
Displacement plot

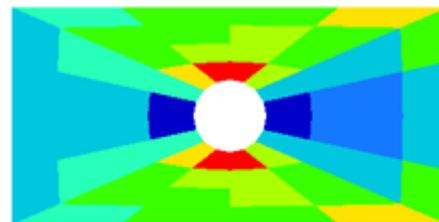


Stress plot

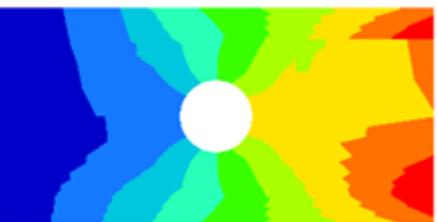


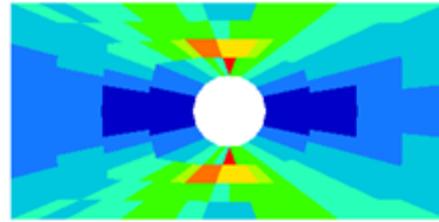
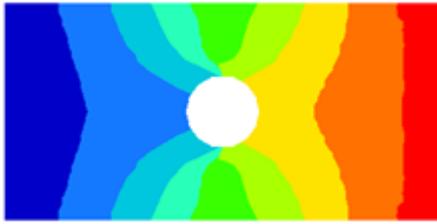
6.4. Three Washer
Displacement plot

Stress plot



6.7. fine ruled mesh without washer
Displacement plot





Stress_plot

VII. COMPARISON TABLE

sr.no.	Title	max.disp	max.stress
1	Plate With Hole Automesh	4.015e-05	5.745e+02
2	Corse Ruled Mesh Without Washer	4.086e-05	6.073e+02
3	Fine ruled mesh without washer	4.167e-05	8.209e+02
4	Middle Ruled Mesh Without Washer	4.037e-05	7.767e+02
5	Single Washer	2.401e-04	4.729e+02
6	Double washer	2.475e-05	5.309e+02
7	Three Washer	2.582e-05	5.827e+02

VIII. CONCLUSION

Thus we observe that a lot of variation in the results is obtained through various meshes. The FEA results are significantly affected by the meshing algorithm and this is reflected in the above table. The ruled mesh (Middle Ruled Mesh without Washer) which is symmetric offers a symmetric response in the plots whereas the other meshes don't offer that symmetry of solution to a desirable extent. Further scope of the work lies in investigating the mesh quality parameters impact on the results.

REFERENCES

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- [2] Warren C. Young And Richard G. Budynas, "Roark's Formulas for Stress and Strain", *McGraw-Hill, New York Chicago San Francisco Lisbon London, Madrid Mexico City Milan New Delhi San Juan Seoul, Singapore Sydney Toronto*