IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE)
e-ISSN: 2278-1684,p-ISSN: 2320-334X, Volume 17, Issue 3 Ser. IV (May - June 2020), PP 48-54
www.iosrjournals.org

A note on Stokes drags in the perspective of Newtonian and Micro-polar fluid

Deepak Kumar Srivastava
Associate Professor, Department of Mathematics
B.S.N.V. Post Graduate College
Station Road, Charbagh, Lucknow-226001, U.P., India
dksflow@hotmail.com

Abstract- *A note on Stokes drag on axi-symmetric small particle placed in axial and transverse flow of Newtonian and micro-polar fluid is written. The results are based on Stokes drag(Datta and Srivastava, 1999) for Newtonian fluid and for micro-polar fluid (Srivastava and Srivastava, 2016). The general relationships for Stokes drag on small axi-symmetric particle in axial and transverse flow are given for both Newtonian and micro-polar fluid. The relationship is then tested for sphere and spheroid for general validity. Further, the linear relationship between moment and drag is exploited for writing transverse drag in terms of moment of body rotating about axis of symmetry. The same is tested and verified for sphere.*

Key words- *Stokes drag, axial flow, transverse flow, Newtonian fluid, micro-polar fluid.*
AMS Subject Classification*- 76D07*

Date of Submission: 12-06-2020 Date of Acceptance: 29-06-2020

I. Introduction

The study of steady slow viscous incompressible fluid past axi-symmetric particle governed by Stokes equations is a classical problem in fluid dynamics. This type of motion is also known as creep motion of small particle through the fluid which is very common in bio-engineering, chemical engineering and naval engineering.Stokesdrag is important for understanding the swimming of microorganisms and sperm; also, the sedimentation of small particles and organisms in water, under the force of gravity.Golden syrup, oil and polymeric solutions are few examples of such type of fluid. In all body-fluid interaction problems, the important quantity to calculate is drag experienced by body by surrounding fluid. Newtonian fluid is the fluid which follows Newton's law of motion. The drag values on sphere and spheroid for various aspect ratio are presented in their book by Kim and Karilla(2005).The theory of micro polar fluid was first introduced by Eringen(1966). The micro-polar fluid differs with Newtonian fluid by micro-inertia and micro rotation in nature i.e. solitary particle can rotate independently from the rotation and motion of fluid in complete form.Complex fluids like polymeric suspensions, animal blood, liquid crystals, lubricants, colloidal suspensions, bubbly fluids, granular fluids are few examples of micro polar fluids.

Payne and Pell(1960) has provided the general expression of Stokes drag on axi-symmetric particle in terms of stream function for Newtonian fluid. Ramkissoon and Majumdar(1976) gave the general expression of Stokes drag on axi-symmetric particle in terms of stream function placed in micro-polar fluid which immediately reduces to the expression of Stokes drag established by Payne and Pell on taking micro-polarity parameter k=0. Datta and Rathore(1983) concluded that the Stokes drag on spheroid placed in micro-polar fluid differs with Stokes drag on spheroid in Newtonian fluid only by replacing μ+k insteadμ, where μ is viscosity coefficient and k is small micro-polarity parameter. Datta and Srivastava(1999) gave general expression of Stokes drag on axi-symmetric particle of revolution about axis of symmetry in meridional plane with the condition of continuously turning tangent at every point on curve in terms of geometric variables related to body profile. The results are termed as DS-conjecture for Stokes drag and expressions of drag are given for both axial and transverse flows. By catching this idea, Srivastava and Srivastava(2016) provided the general expressions of Stokes drag on small axi-symmetric particle in axial and transverse cases for micro-polar fluid.Erdogan(1972) discussed the complete dynamics of polar fluid at micro level. Ariman (1973) provided the complete review overmicro continuum fluid mechanics. There exists some related work on Stokes flow past axially symmetric bodies of micro polar fluid mainly by Ramkissoon and O'Neill (1983), Iyengar and Charya (1993), Hayakawa (2000), Palaniappan and Ramkissoon (2005), Hoffmann et al. (2007), Shu and Lee (2008), Sherief et al. (2010), Deo and Shukla (2012) etc.

In present work, the general relationships for Stokes drag on small axi-symmetric particle in axial and transverse flow are given first for Newtonian fluid and then for micro-polar fluid. The relationship is then tested for sphere and spheroid for general validity.

II. Stokes drag on axially symmetric particle in Newtonian fluid

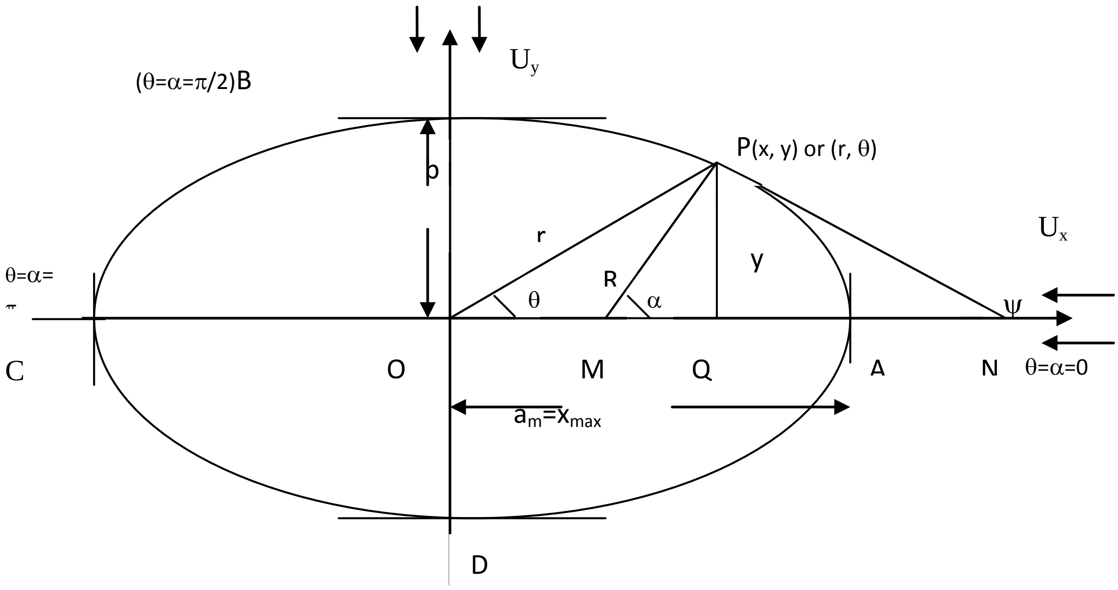

Figure 1. Geometry of axially symmetric body of revolution of curve in meridional plane

Datta and Srivastava(1999) gave the expressions of Stokes drag on axially symmetric particle of revolution of curve in meridional plane with the condition of continuously turning tangent(figure 1) in axial flow(along x-axis) as

$$F_x = \frac{1}{2}\frac{\lambda b^2}{h_x} = \frac{4}{3}\frac{\lambda b^2}{\int_0^{\pi} R \sin^3\alpha \, d\alpha}, \text{ where } \lambda = 6\pi\mu U_x,$$

$$= \frac{8\pi\mu b^2 U_x}{\int_0^{\pi} R \sin^3\alpha \, d\alpha}. \qquad (2.1)$$

Datta and Srivastava(1999) gave the expressions of Stokes dragon axially symmetric particle of revolution of curve in meridional plane with the condition of continuously turning tangent(figure 1)in transverse flow(perpendicular to axis of symmetry, y-axis) as

$$F_y = \frac{1}{2}\frac{\lambda b^2}{h_y}, \text{ where } \lambda = 6\pi\mu\, U_y,$$

$$= \frac{16\pi\mu b^2 U_y}{\int_0^{\pi} R \left(2\sin\alpha - \sin^3\alpha\right) d\alpha}. \qquad (2.2)$$

In the above formulae, variable R is the intercepting length of normal between axis of symmetry and point on body curve in meridional plane and α is the angle made by normal from axis of symmetry. Also, $h_x$and $h_y$represents the height of centre of gravity of force system in both the flow configurations and 'b' is the semi-transverse length of body curve in merdional plane xy and 'μ' is the viscosity coefficient(Datta and Srivastava, 1999). In every body-fluid interaction problems in fluid dynamics, the important quantity to evaluate is the drag experienced by body which in traditional approach always found as a solution of linear Stokes equation under no-slip boundary condition(Happel and Brenner, 1964).

The relation between $h_x$and h_y(by using equations 2.1 and 2.2) is

$$h_y = \frac{3}{8}\int_0^{\pi} R \sin\alpha \, d\alpha - \frac{h_x}{2}. \tag{2.3}$$

The relation between Stokes drag in both axial flow and transverse flow cases(by using equations 2.1 and 2.2 and U_x=U_y=U) of Newtonian fluid is

$$\frac{1}{F_y} + \frac{1}{2F_x} = \frac{1}{8\pi\mu U b^2}\int_0^{\pi} R \sin\alpha \, d\alpha. \tag{2.4}$$

III. Stokes drag on axially symmetric particle in micro polar fluid

Srivastava and Srivastava(2016) gave the expressions of Stokes drag on axially symmetric particleof revolution of curve in meridional plane with the condition of continuously turning tangent(figure 1) in axial flow(along x-axis) as

$$\begin{aligned} F_x &= \frac{1}{2}\frac{\lambda b^2}{h_x} = \frac{4}{3}\frac{\lambda b^2}{\int_0^{\pi} R \sin^3\alpha \, d\alpha}, \text{ where } \lambda = 6\pi(\mu + k)U_x, \\ &= \frac{8\pi(\mu+k) b^2 U_x}{\int_0^{\pi} R \sin^3\alpha \, d\alpha}. \end{aligned} \tag{3.1}$$

Srivastava and Srivastava(2016) gave the expressions of Stokes dragon axially symmetric particle of revolution of curve in meridional plane with the condition of continuously turning tangent(figure 1) in transverse flow(perpendicular to axis of symmetry, y-axis) as

$$\begin{aligned} F_y &= \frac{1}{2}\frac{\lambda b^2}{h_y}, \text{ where } \lambda = 6\pi(\mu + k)\, U_y, \\ &= \frac{16\pi(\mu+k) b^2 U_y}{\int_0^{\pi} R \left(2\sin\alpha - \sin^3\alpha\right) d\alpha}. \end{aligned} \tag{3.2}$$

In the above formulae, variable R is the intercepting length of normal between axis of symmetry and point on body curve in meridional plane and α is the angle made by normal from axis of symmetry and h_x and $h_y$represents the height of centre of gravity of force system in both the flow configurations and 'b' is the semi-transverse length of body curve in meridional plane xy, 'μ' is the viscosity coefficient and k is the micro-polarity parameter.In every body-fluid interaction problems in fluid dynamics, the important quantity to evaluate is the drag experienced by body which in traditional approach always found as a solution of linear Stokes equation under no-slip and no-spin boundary condition(Srivastava and Srivastava, 2016).

The relation between $h_x$and h_y(by using equations 2.1 and 2.2) is

$$h_y = \frac{3}{8}\int_0^{\pi} R\sin\alpha\, d\alpha - \frac{h_x}{2}. \qquad (3.3)$$

Equation (3.3) clearly indicates that the relationship between h_x and h_y is independent to the viscosity of fluid and micro-polarity of micro-polar fluid but depends on variables R and α related to axi-symmetric particle. The relation between Stokes drag in both axial flow and transverse flow cases(by using equations 2.1 and 2.2) of micro-polarfluid is

$$\frac{1}{F_y} + \frac{1}{2F_x} = \frac{1}{8\pi(\mu+k)Ub^2}\int_0^{\pi} R\sin\alpha\, d\alpha. \qquad (3.4)$$

It is interesting to note that the relationship between centre of gravity of force systems in Newtonian fluid(eq. 2.3) and micro-polar fluid(eq. 3.3) remains same. It means that the relationship is independent to the viscosity and micro-polarity of either fluid but the drag relationship(eq. 3.4) in micro-polar fluid differs only with $\mu+k$ in place of μ in the drag relationship in Newtonian fluid(Srivastava and Srivastava, 2016).

IV. Sphere

Let us consider the parametric equation of sphere of revolution of circle about x-axis having radius 'a' as

$$x = a\cos t,\quad y = a\sin t,\quad 0 \le t \le \pi. \qquad (4.1)$$

After careful calculation, the expressions of h_x and h_y and axial and transverse Stokes drag on this sphere placed in Newtonian fluid and micro-polar fluid, by using (eq. 2.1, 3.1) and (eq. 2.2, 3.2) with fact $b=a, U_x=U_y=U$,comes out to be

$h_x=a/2$, $h_y=a/2$ (Newtonian and micro-polar fluid both)(4.2)

$$F_x = F_y = 6\pi\mu U a. \quad \text{(Newtonian fluid)} \qquad (4.3)$$

$$F_x = F_y = 6\pi(\mu+k) U a. \quad \text{(Micro-polar fluid)} \qquad (4.4)$$

For the above values, relationship (eq. 2.3 and 2.4) are satisfied for classical Newtonian fluid and, relationship (eq. 3.3 and 3.4) are satisfied for classical micro-polar fluid.

V. Spheroid

Prolate spheroid: Let us consider the prolate spheroid of revolution of ellipse about x-axis. The parametric equation of ellipse is

$x = a\cos t$, $y = b\sin t$, $0\le t \le\pi$.(5.1)

After careful calculation, the expressions of h_x and h_y and axial and transverse Stokes drag on this prolatespheroid placed in Newtonian fluid and micro-polar fluid, by using (eq. 2.1, 3.1) and (eq. 2.2, 3.2),with the fact $U_x= U_y=U$, comes out to be

$$h_x = \frac{3}{16}\frac{b^2}{ae^3}\left[\left(1+e^2\right)L-2e\right], \qquad (5.2a)$$

(forNewtonian and micro-polar fluid both)

$$h_y = \frac{3}{32}\frac{b^2}{ae^3}\left[\left(3e^2-1\right)L+2e\right], \qquad (5.2b)$$

$$F_x = 16\pi\mu U a e^3\left[\left(1+e^2\right)L - 2e\right]^{-1}, \quad (5.3a)$$

(Newtonian fluid)

$$F_y = 32\pi\mu U a e^3\left[\left(3e^2-1\right)L - 2e\right]^{-1}, \quad (5.3b)$$

$$F_x = 16\pi\left(\mu+k\right)U a e^3\left[\left(1+e^2\right)L - 2e\right]^{-1}, \quad (5.4a)$$

(Micro-polar fluid)

$$F_y = 32\pi\left(\mu+k\right)U a e^3\left[\left(3e^2-1\right)L - 2e\right]^{-1}. \quad (5.4b)$$

In all the above formulas, L= ln(1 + e)/(1 - e) with 'e' as eccentricity of ellipse. For the above values, relationship (eq. 2.3 and 2.4) for prolate spheroid is satisfied for classical Newtonian fluid and, relationship (eq. 3.3 and 3.4) are satisfied for classical micro-polar fluid.

Oblate spheroid: Let us consider the oblate spheroid of revolution of ellipse about x-axis. The parametric equation of ellipse is

$$x = b\cos t,\ y = a\sin t,\ 0 \le t \le \pi. \quad (5.5)$$

After careful calculation, the expressions of h_x and h_y and axial and transverse Stokes drag on this prolate spheroid placed in Newtonian fluid and micro-polar fluid, by using (eq. 2.1, 3.1) and (eq. 2.2, 3.2) with fact$U_x= U_y=U$, comes out to be

$$h_x = \frac{3}{8}\frac{a}{e^3}\left[e\sqrt{1-e^2} - \left(1-2e^2\right)\sin^{-1} e\right], \quad (5.5a)$$

(forNewtonian and micro-polar fluid both)

$$h_y = \frac{3}{16}\frac{a}{e^3}\left[\left(1+2e^2\right)\sin^{-1} e - e\sqrt{1-e^2}\right], \quad (5.5b)$$

$$F_x = 8\pi\mu U a e^3\left[e\sqrt{1-e^2} - \left(1-2e^2\right)\sin^{-1} e\right]^{-1}, \quad (5.6a)$$

(Newtonian fluid)

$$F_y = 16\pi\mu U a e^3\left[\left(1+2e^2\right)\sin^{-1} e - e\sqrt{1-e^2}\right]^{-1}, \quad (5.6b)$$

$$F_x = 8\pi\left(\mu+k\right)U a e^3\left[e\sqrt{1-e^2} - \left(1-2e^2\right)\sin^{-1} e\right]^{-1}, \quad (5.7a)$$

(micro-polar fluid)

$$F_y = 16\pi\left(\mu+k\right)U a e^3\left[\left(1+2e^2\right)\sin^{-1} e - e\sqrt{1-e^2}\right]^{-1}. \quad (5.7b)$$

In all the above formulae, 'e' stands for eccentricity of ellipse. For the above values, relationship (eq. 2.3 and 2.4) for prolate spheroid is satisfied for classical Newtonian fluid and, relationship (eq. 3.3 and 3.4) are satisfied for classical micro-polar fluid. Further, the proposed relationship of drag values may be checked and verified for other axially symmetric bodies like deformed sphere, egg-shaped body, cycloidal oval, hypocycloidal body of continuously turning tangent with largest mid cross-section constant radius described by Datta and Srivastava[1999].

VI. Relationship between drag and moment

The moment of axially symmetric particle rotating about axis of symmetry(x-axis) with angular velocity Ω was derived by Datta and Srivastava[1999] in terms of axial drag as

$$M_x = \frac{2}{3} a^2 \left(1 - e^2\right) \frac{\Omega}{U} F_x \text{ , (6.1)}$$

whereF_x is axial Stokes drag, 'e' is the eccentricity of spheroid and 'a' is the semi-major axis length. Now, from eq. 6.1, the linear relationship between moment and drag, we can easily write F_x in terms of M_x

$$F_x = \frac{3U}{2a^2\left(1 - e^2\right)\Omega} M_x . \qquad (6.2)$$

Now, by utilizing the relation (2.4) and (3.4), we may write another new relation between transverse drag and moment for both Newtonian fluid and micro-polar fluid as

$$\frac{1}{F_y} = \frac{1}{8\pi\mu U b^2} \int_0^\pi R \sin\alpha d\alpha - \frac{a^2\left(1 - e^2\right)\Omega}{3UM_x} \text{ , [Newtonian fluid]} \qquad (6.3)$$

$$\frac{1}{F_y} = \frac{1}{8\pi\left(\mu + k\right) U b^2} \int_0^\pi R \sin\alpha d\alpha - \frac{a^2\left(1 - e^2\right)\Omega}{3UM_x} \text{.[micro-polar fluid]} \quad (6.4)$$

In equations 6.3 and 6.4, 'b' stands for semi-transverse axis length, 'a' stands for semi-major axis length and k is micro-polar parameter.

VII. Sphere

For sphere, e=0, b=a, the moment of sphere with radius 'a' rotating with angular velocity 'Ω' about x-axis is given by[Kim and Karilla, 2005]

$M_x = 4\pi\mu\Omega a^3$, [Newtonian fluid] (7.1)

and

$M_x = 4\pi(\mu+k)\Omega a^3$, [micro-polar fluid].(7.2)

On substituting the value of M_x from eq. 7.1 in relation 6.3 and value of M_x from 7.2 in relation 6.4, after careful calculation, we get the values of Stokes drag on sphere in transverse flow both in Newtonian fluid as well as micro-polar fluid as

$F_y = 6\pi\mu Ua$, [Newtonian fluid] (7.3)

and

$F_y = 6\pi(\mu+k)Ua$, [micro-polar fluid] (7.4)

Which are in confirmation with classical values of drags in transverse flow of Newtonian fluid and micro-polar fluid[Kim and Karilla, 2005]. In the same manner, the relation 6.3 and 6.4 may be verified for spheroid and other axially symmetric small particles like spheroid, deformed spheroid, egg-shaped body, cycloidal oval, hypocycloidal body described in the paper of Datta and Srivastava (1999).

VIII. Conclusion

In the present note, it is found that the relation between h_x and h_y depends only on body profile(figure 1) variables R and $\alpha$and independent to the viscosity μ and micro-polar parameter k. The general relationship between axial Stokes drag and transverse Stokes drag is established for Newtonian and micro-polar fluid and tested for sphere, prolate spheroid and oblate spheroid. Further, the linear relationship between moment and drag is exploited for writing transverse drag in terms of moment of body rotating about axis of symmetry. The same is tested and verified for sphere. Author claims that for the accuracy of closed form analytic Stokes drag results for proposed class of axially symmetric bodies, these general relationships must be satisfied.

References

[1]. Payne, L.E., and Pell, W.H. (1960). The Stokes flow problem for a class of axially symmetric bodies, J. Fluid Mech., Vol. 7, No. 4, pp. 529-549.

[2]. Happel, J., and Brenner, H.(1964, 1983). Low Reynolds Number Hydrodynamics, Nijhoff, Dordrecht, TheNederlands.

[3]. Eringen, A. C.(1966). Theory of micropolar fluids, J. Math. Mech., Vol. 16, pp. 1-18.

[4]. Erdogan, M. E. (1972). Dynamics of Polar Fluids, ActaMechanica, Vol. 15, pp. 233-253.

[5]. Ariman, T., Turk, M.A. and Sylvester, N.D. (1973). Micro continuum Fluid Mechanics-a review, Vol. 11, pp. 905-930.

[6]. Ramkissoon, H., and Majumdar, S.R. (1976). Drag on an axially symmetric body in the Stokes flow of micro polar fluid, Physics of Fluids, Vol. 19, pp. 16-21.

[7]. Ramkissoon, H., and O'Neill, M.E.(1983). Stokes force in microcontinuum fluid mechanics, ActaMechanica, Vol. 46, pp. 255-263.

[8]. Datta, S., and Rathore, S.K. Singh. (1983). Slow uniform flow of a micro polar fluid past a prolate spheroid, Indian J. Pure Appl. Math., Vol. 14, No. 12, pp. 1474-1483.

[9]. Iyengar, T.K.V., and Charya, D. Srinivasa. (1993). Stokes flow of an incompressible micropolar fluid past an approximate sphere, Int. J. Engng Sci., Vol. 31, No. 1, pp. 115-123.

[10]. Datta, S., and Srivastava, D.K. (1999). Stokes drag on axially symmetric bodies: a new approach, Proceedings Math. Sci., Ind. Acad. Sci., Vol. 109, No. 4, pp. 441-452.

[11]. Kim, Sangtae, Karrila, Seppo J. (2005). Microhydrodynamics: Principles and Selected Applications, Courier Corporation.

[12]. Palaniappan, D., and Ramkissoon, H.(2005). A drag formula revisited, Int. J. Engng. Sci., Vol. 43, pp. 1498-1501.

[13]. Hoffmann, Karl-Heinz, Marx, David, and Botkin, Nikolaid D.(2007). Drag on spheres in micropolar fluids with nonzero boundary conditions for microrotations, J. Fluid Mech., Vol. 590, pp. 319-330.

[14]. Shu, Jian-Jhu, and Lee, JennShiun.(2008). Fundamental solutions for micropolar fluids, J. Engng. Math., Vol. 61, pp. 69-79.

[15]. Sherief, H.H.,Faltas, M.S. and Ashmawy, E.A.(2010).Axi-symmetric translational motion of an arbitrary solid prolate body in a micropolar fluid, Fluid Dynamics, Vol. 42, 065504(18 pages).

[16]. Deo, Satya and Shukla, Pankaj. (2012). Creeping flow of micro polar fluid past a fluid sphere with nonzero spin boundary condition, Int. J. Engng. Tech., Vol. 1, No. 2, pp. 67-76.

[17]. Srivastava, D.K., Yadav, R.R., and Srivastava, Nirmal. (2016). Oseen's correction to Stokes drag on axially symmetric bodies in micro polar fluid, IJAAMM, Vol. 3, No. 3, pp. 41-78.

Deepak Kumar Srivastava. "A note on Stokes drags in the perspective of Newtonian and Micro-polar fluid." *IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE)*, 17(3), 2020, pp. 48-54.