

## Lane keeping control based on preview mechanism in automated highway systems

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**Abstract:** In this paper, the preview control for lane keeping in automated highway systems was studied. Assuming that the information of lateral offset to the lane center line can be obtained by using the onboard sensors, with single point look-ahead method, based on lateral vehicle dynamics model, the mathematical model of vehicle lateral position error and yaw angle error was established, the control law for lane keeping was designed by applying terminal sliding mode method. By using Lyapunov function method, the asymptotic stability of the lateral position error was obtained.

**Keywords** - automated highway systems, lane keeping, sliding mode control, preview mechanism

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### I. Introduction

A lane-keeping system automatically controls the steering to keep the vehicle in its lane and also follow the lane as it curves around[1]. Many achievements have been made in lane-keeping control, such as Ref. [2] research on video-based lane-keeping PID control, Ref.[3] studies the modeling and control of lane-keeping system based on dual magnetic sensors. Because the dynamic behavior of the vehicle has time delay relative to the control input, a preview mechanism is introduced in the lateral control of the vehicle, that is, a preview point is selected along the direction of motion of the vehicle and at an appropriate distance from the center of mass of the vehicle, the deviation of the lateral displacement of the vehicle at this point from the center line of the lane is a preview[4,5]. Due to the interaction of vehicle longitudinal and lateral motion, Ref.[6] studies the coupling control of lane keeping and vehicle following based on preview method. It is well known that the control system design should be based on stability analysis, however, there are few literatures on the stability of lane-keeping system.

In this paper, by applying terminal sliding mode method, the lane-keeping control based on preview method is studied and its stability is analyzed. Firstly, the dynamic equations of vehicle lateral position error and yaw angle error at the preview point are established based on the vehicle dual position sensors, then, based on the Lyapunov stability method, the asymptotic stability of the position error is analyzed. The research results are verified by simulation.

### II. Vehicle Dynamics Model

The vehicle dynamics model adopted in this paper is derived from the ideal model proposed by Ackermann. According to the automobile theory, the vehicle lateral dynamics model can be expressed as

$$\ddot{y} = -\frac{2(C_f + C_r)}{mv_x} \dot{y} - \left[ v_x + \frac{2(C_f l_f - C_r l_r)}{mv_x} \right] \dot{\psi} + \frac{2C_f}{m} \delta \quad (1)$$

$$\ddot{\psi} = -\frac{2(C_f l_f^2 + C_r l_r^2)}{I_z v_x} \dot{\psi} - \frac{2(C_f l_f - C_r l_r)}{I_z v_x} \dot{y} + \frac{2C_f l_f}{I_z} \delta \quad (2)$$

where,  $y$  is the vehicle lateral displacement,  $\psi$  is the vehicle yaw angle,  $v_x$  is the vehicle longitudinal velocity,  $m$  is the mass of vehicle,  $I_z$  is the total vehicle inertia about vertical axis,  $C_f$  and  $C_r$  are the front and rear tire cornering stiffness, respectively,  $l_f$  and  $l_r$  are respectively the distance from center of mass of vehicle to the front axle and the centroid to the rear axle,  $\delta$  is the front wheel steering angle.

### III. Error Model

In the following, Fig.1 shows a preview method of the vehicle's position. In Fig.1, A and B represent the sensors mounted on the rear bumper and front bumper respectively, C represents the position of the center of mass of vehicle, and D represents the preview point,  $d$  represents the preview distance, that is, the distance from the preview point to the center of mass,  $d_f$  and  $d_r$  represent the distance from the front and rear magnetic sensors to the vehicle centroid,  $y_{fs}$  and  $y_{rs}$  represent the lateral position deviation of the front and rear bumper center respectively,  $y_r$  and  $y_s$  represent the lateral position deviation of the vehicle centroid and the preview point, respectively.  $\psi_r$  represents the directional deviation of a vehicle's longitudinal axis from the tangent to the centerline of the lane.

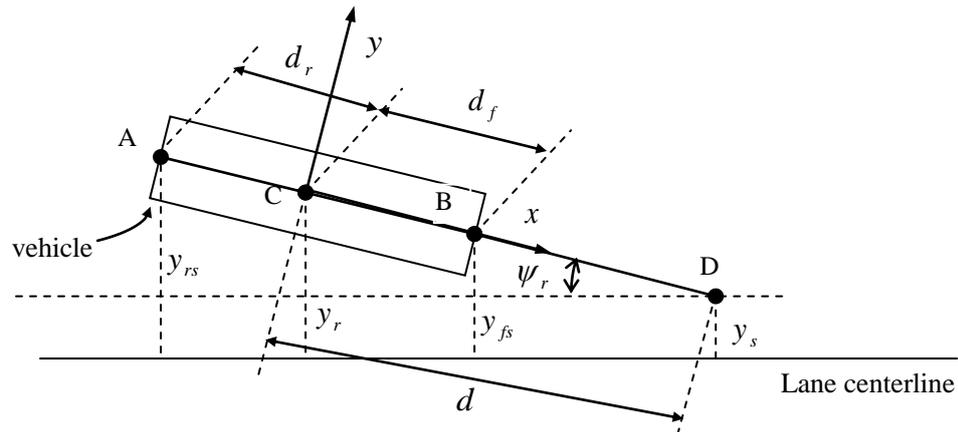


Fig. 1 Vehicle position preview diagram

From Fig.1, we get

$$y_s = y_r + d \sin \psi_r \tag{3}$$

Since the rate of change of lateral position deviation satisfies the following equations,

$$\dot{y}_r = v_y \cos \psi_r + v_x \sin \psi_r \tag{4}$$

so we have

$$\begin{aligned} \dot{y}_s &= \dot{y}_r + d \dot{\psi}_r \cos \psi_r \\ &= v_y \cos \psi_r + v_x \sin \psi_r + d \dot{\psi}_r \cos \psi_r \end{aligned} \tag{5}$$

Take the derivative of the above equation, we get

$$\begin{aligned} \ddot{y}_s &= \dot{v}_y \cos \psi_r - v_y \dot{\psi}_r \sin \psi_r + v_x \dot{\psi}_r \cos \psi_r + d \ddot{\psi}_r \cos \psi_r - d \dot{\psi}_r \sin \psi_r \\ &= -(v_y + d) \dot{\psi}_r \sin \psi_r + (\dot{v}_y + v_x \dot{\psi}_r + d \ddot{\psi}_r) \cos \psi_r \\ &= -(\dot{y} + d) \dot{\psi}_r \sin \psi_r + (\ddot{y} + v_x \dot{\psi}_r + d \ddot{\psi}_r) \cos \psi_r \end{aligned} \tag{6}$$

Due to

$$\ddot{\psi}_r = \ddot{\psi} - \ddot{\psi}_d$$

so, from (1) and (2), we have

$$\begin{aligned} \ddot{y}_s &= -(\dot{y} + d) \dot{\psi}_r \sin \psi_r + (v_x \dot{\psi}_r + d \ddot{\psi}_r) \cos \psi_r \\ &\quad + \cos \psi_r \left\{ -\frac{2(C_f + C_r)}{mv_x} \dot{y} - \left[ v_x + \frac{2(C_f l_f - C_r l_r)}{mv_x} \right] \dot{\psi} + \frac{2C_f}{m} \delta \right\} \end{aligned} \tag{7}$$

$$\ddot{\psi}_r = -\frac{2(C_f l_f^2 + C_r l_r^2)}{I_z v_x} \dot{\psi} - \frac{2(C_f l_f - C_r l_r)}{I_z v_x} \dot{y} + \frac{2C_f l_f}{I_z} \delta - \ddot{\psi}_d \tag{8}$$

Hence, according to

$$\dot{y} = \frac{\dot{y}_s - v_x \tan \psi_r - d\dot{\psi}_r}{\cos \psi_r} = [\cos \psi_r]^{-1} \dot{y}_s - v_x \tan \psi_r - d\dot{\psi}_r$$

$$\dot{\psi} = \dot{\psi}_r + \dot{\psi}_d$$

from (7), (8), the nonlinear dynamic equations of vehicle lateral position error and yaw angle error are obtained as follows.

$$\begin{aligned} \ddot{\psi}_r &= -\frac{2(C_f l_f^2 + C_r l_r^2)}{I_z v_x} (\dot{\psi}_r + \dot{\psi}_d) \\ &\quad - \frac{2(C_f l_f - C_r l_r)}{I_z v_x} [(\cos \psi_r)^{-1} \dot{y}_s - v_x \tan \psi_r - d\dot{\psi}_r] + \frac{2C_f l_f}{I_z} \delta - \ddot{\psi}_d \\ &= \left(\frac{2da_2}{I_z v_x} - \frac{2a_3}{I_z v_x}\right) \dot{\psi}_r - \frac{2a_2}{I_z v_x \cos \psi_r} \dot{y}_s + \frac{2a_2}{I_z} \tan \psi_r + \frac{2C_f l_f}{I_z} \delta - \frac{2a_3}{I_z v_x} \dot{\psi}_d - \ddot{\psi}_d \end{aligned} \quad (9)$$

$$\begin{aligned} \ddot{y}_s &= -[(\cos \psi_r)^{-1} \dot{y}_s - v_x \tan \psi_r - d\dot{\psi}_r + d] \dot{\psi}_r \sin \psi_r + (v_x \dot{\psi}_r + d\dot{\psi}_r) \cos \psi_r \\ &\quad + \cos \psi_r \left\{ -\frac{2(C_f + C_r)}{m v_x} [(\cos \psi_r)^{-1} \dot{y}_s - v_x \tan \psi_r - d\dot{\psi}_r] \right. \\ &\quad \left. - [v_x + \frac{2(C_f l_f - C_r l_r)}{m v_x}] (\dot{\psi}_r + \dot{\psi}_d) + \frac{2C_f}{m} \delta \right\} \\ &= d \sin \psi_r (\dot{\psi}_r)^2 - (\tan \psi_r) \dot{\psi}_r \dot{y}_s + [v_x \sin \psi_r \tan \psi_r - d \sin \psi_r \\ &\quad + \frac{2(a_1 \cos \psi_r d - a_2)}{m v_x} + \left(\frac{2da_2}{I_z v_x} - \frac{2a_3}{I_z v_x}\right) d \cos \psi_r] \dot{\psi}_r \\ &\quad - \left[\frac{2a_1}{m v_x} + \frac{2da_2}{I_z v_x}\right] \dot{y}_s + \left[\frac{2a_1}{m} + \frac{2da_2}{I_z}\right] \sin \psi_r + 2C_f \cos \psi_r \left[\frac{1}{m} + \frac{dl_f}{I_z}\right] \delta \\ &\quad - \cos \psi_r \left[\left(v_x + \frac{2a_2}{m v_x} + \frac{2da_3}{I_z v_x}\right) \dot{\psi}_d + d\ddot{\psi}_d\right] \end{aligned} \quad (10)$$

Let  $\varepsilon_1 = \psi_r$ ,  $\varepsilon_2 = \dot{\psi}_r$ ,  $e_1 = y_s$ , and  $e_2 = \dot{y}_s$ , we get

$$\begin{cases} \dot{\varepsilon}_1 = \varepsilon_2 \\ \dot{\varepsilon}_2 = f_1(\varepsilon_1, \varepsilon_2, e_1, e_2) + g_1(\varepsilon_1, \varepsilon_2, e_1, e_2) \delta + d_1(\varepsilon_1, \varepsilon_2, e_1, e_2) \\ \dot{e}_1 = e_2 \\ \dot{e}_2 = f_2(\varepsilon_1, \varepsilon_2, e_1, e_2) + g_2(\varepsilon_1, \varepsilon_2, e_1, e_2) \delta + d_2(\varepsilon_1, \varepsilon_2, e_1, e_2) \end{cases} \quad (11)$$

$$y = [\varepsilon_1, e_1]^T$$

where

$$\begin{aligned} f_1(\varepsilon_1, \varepsilon_2, e_1, e_2) &= \left(\frac{2da_2}{I_z v_x} - \frac{2a_3}{I_z v_x}\right) \varepsilon_2 - \frac{2a_2}{I_z v_x \cos \varepsilon_1} e_2 + \frac{2a_2}{I_z} \tan \varepsilon_1 \\ g_1(\varepsilon_1, \varepsilon_2, e_1, e_2) &= \frac{2C_f l_f}{I_z}, d_1(\varepsilon_1, \varepsilon_2, e_1, e_2) = -\frac{2a_3}{I_z v_x} \dot{\psi}_d - \ddot{\psi}_d \\ f_2(\varepsilon_1, \varepsilon_2, e_1, e_2) &= d \sin \varepsilon_1 (\varepsilon_2)^2 - (\tan \varepsilon_1) \varepsilon_2 e_2 + [v_x \sin \varepsilon_1 \tan \varepsilon_1 - d \sin \varepsilon_1 \\ &\quad + \frac{2(a_1 \cos \varepsilon_1 d - a_2)}{m v_x} + \left(\frac{2da_2}{I_z v_x} - \frac{2a_3}{I_z v_x}\right) d \cos \varepsilon_1] \varepsilon_2 \\ &\quad - \left[\frac{2a_1}{m v_x} + \frac{2da_2}{I_z v_x}\right] e_2 + \left[\frac{2a_1}{m} + \frac{2da_2}{I_z}\right] \sin \varepsilon_1 \end{aligned}$$

$$g_2(\varepsilon_1, \varepsilon_2, e_1, e_2) = 2C_f \left[ \frac{1}{m} + \frac{dl_f}{I_z} \right] \cos \varepsilon_1, d_2(\varepsilon_1, \varepsilon_2, e_1, e_2) = \left[ -(v_x + \frac{2a_2}{mv_x} + \frac{2da_3}{I_z v_x}) \dot{\psi}_d + d\ddot{\psi}_d \right] \cos \varepsilon_1$$

#### IV. Design of Control Law

Introduce error variable

$$Z = (1 - \eta)e_1 + \eta\varepsilon_1 \tag{12}$$

Where  $\eta \in [0,1]$ ,  $e_1$  and  $\varepsilon_1$  represent the lateral position error and the yaw angle error respectively. Design switching function

$$s = \dot{Z} + \lambda_1 Z + \lambda_2 Z^{q/p} \tag{13}$$

Where  $p$  and  $q$  are positive odd numbers, and  $q < p$ ,  $\lambda_1 > 0$ ,  $\lambda_2 > 0$ . Differentiate equation (13), we have

$$\begin{aligned} \dot{s} &= \ddot{Z} + \lambda_1 \dot{Z} + \lambda_2 \dot{Z}^{q/p} \\ &= \eta \ddot{\varepsilon}_1 + (1 - \eta) \ddot{e}_1 + \left\{ \lambda_1 + \frac{p}{q} \lambda_2 [\eta \varepsilon_1 + (1 - \eta) e_1]^{q/p-1} \right\} [\eta \dot{\varepsilon}_1 + (1 - \eta) \dot{e}_1] \end{aligned}$$

from (11), we get

$$\begin{aligned} \dot{s} &= \eta \dot{\varepsilon}_2 + (1 - \eta) \dot{e}_2 + \left\{ \lambda_1 + \frac{p}{q} \lambda_2 [\eta \varepsilon_1 + (1 - \eta) e_1]^{q/p-1} \right\} [\eta \varepsilon_2 + (1 - \eta) e_2] \\ &= \eta(f_1 + d_1) + (1 - \eta)(f_2 + d_2) + [\eta g_1 + (1 - \eta) g_2] \delta \\ &\quad + \left\{ \lambda_1 + \frac{p}{q} \lambda_2 [\eta \varepsilon_1 + (1 - \eta) e_1]^{q/p-1} \right\} [\eta \varepsilon_2 + (1 - \eta) e_2] \end{aligned} \tag{14}$$

where

$$\begin{aligned} f_1 &= \left( \frac{2da_2}{I_z v_x} - \frac{2a_3}{I_z v_x} \right) \varepsilon_2 - \frac{2a_2}{I_z v_x \cos \varepsilon_1} e_2 + \frac{2a_2}{I_z} \tan \varepsilon_1, g_1 = \frac{2C_f l_f}{I_z}, d_1 = -\frac{2a_3}{I_z v_x} \dot{\psi}_d - \ddot{\psi}_d, \\ f_2 &= d \sin \varepsilon_1 (\varepsilon_2)^2 - (\tan \varepsilon_1) \varepsilon_2 e_2 + [v_x \sin \varepsilon_1 \tan \varepsilon_1 - d \sin \varepsilon_1 + \frac{2(a_1 \cos \varepsilon_1 d - a_2)}{mv_x} \\ &\quad + \left( \frac{2da_2}{I_z v_x} - \frac{2a_3}{I_z v_x} \right) d \cos \varepsilon_1] \varepsilon_2 - \left[ \frac{2a_1}{mv_x} + \frac{2da_2}{I_z v_x} \right] e_2 + \left[ \frac{2a_1}{m} + \frac{2da_2}{I_z} \right] \sin \varepsilon_1, \\ g_2 &= 2C_f \left[ \frac{1}{m} + \frac{dl_f}{I_z} \right] \cos \varepsilon_1, d_2 = \left[ -(v_x + \frac{2a_2}{mv_x} + \frac{2da_3}{I_z v_x}) \dot{\psi}_d + d\ddot{\psi}_d \right] \cos \varepsilon_1 \end{aligned}$$

From  $\dot{s} = 0$ , the equivalent control is obtained as follows.

$$\begin{aligned} \delta_{equ} &= -[\eta g_1 + (1 - \eta) g_2]^{-1} \{ \eta(f_1 + d_1) + (1 - \eta)(f_2 + d_2) \\ &\quad + [\lambda_1 + \frac{p}{q} \lambda_2 (\eta \varepsilon_1 + (1 - \eta) e_1)^{q/p-1}] [\eta \varepsilon_2 + (1 - \eta) e_2] \} \end{aligned} \tag{15}$$

Design the nonlinear control as

$$\delta_s = -[\eta g_1 + (1 - \eta) g_2]^{-1} (\phi s + \rho s^{q/p}) \tag{16}$$

Where  $\phi > 0$  and  $\rho > 0$ . Take the control law as

$$\delta = \delta_{equ} + \delta_s \tag{17}$$

Define the Lyapunov function

$$V = \frac{1}{2} s^2 \tag{18}$$

Take the derivative of equation (18), connect formula (14), (15), (16) and (17), we get

$$\dot{V} = s\dot{s} = -\phi s^2 - \rho s^{(p+q)/p} \tag{19}$$

For  $p$  and  $q$  are positive odd numbers, so we get  $\dot{V} < 0$  ( $s \neq 0$ ), hence the sliding mode is asymptotically reachable.

According to the nonlinear sliding surface

$$s = \dot{Z} + \lambda_1 Z + \lambda_2 Z^{q/p} = 0$$

the system state can reach the equilibrium point in a finite time.

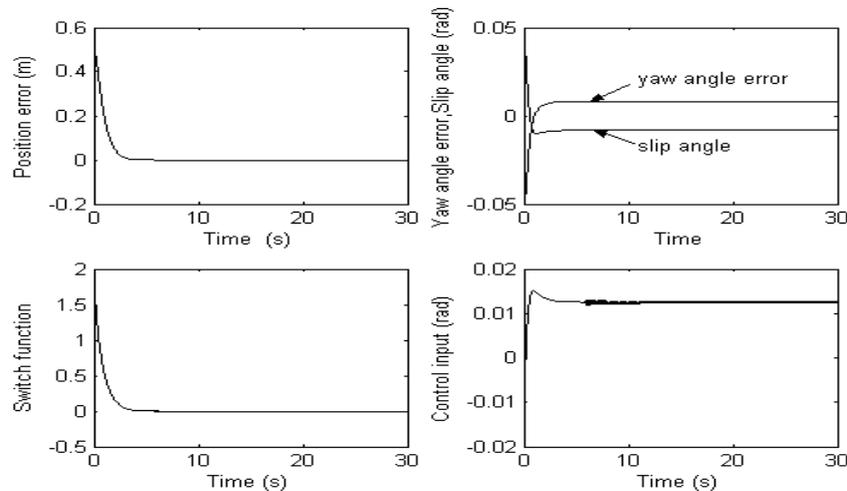
### V. Simulation Results

Control law adopts the form as (17), the parameters of vehicle as shown in table 1. It is assumed that the vehicle velocity  $v_x$  is 25m/s, the preview distance  $d$  is 3.5m. The initial values of lateral displacement error  $y_s$  and yaw angle error  $\psi_r$  are 0.5m and -0.05rad, respectively, the initial value of slip angle is 0.05 rad, the initial value of sideslip velocity and yaw rate are all 0. Fig.2 shows the simulation results with the radius of curvature of the road is 500m.

**Table1** Parameters of vehicle

Parameters	$m$ (kg)	$I_z$ (kgm <sup>2</sup> )	$l_f$ (m)	$l_r$ (m)	$C_f$ (kN/rad)	$C_r$ (kN/rad)
	2100	3150	1.35	1.28	72	82

As can be seen from the figure, the position error approach 0 with the control method presented in the paper, the yaw angle error and slip angle all tend to steady values. It indicates that the control system has good stability.



**Fig.2** Simulation results

### VI. Conclusion

In this paper, based on the position preview strategy, by using the lateral dynamics model of vehicle, the control method for lane-keeping is studied. Firstly, the dynamic equations of vehicle lateral position error and yaw angle error at the preview point are established. Next, the lane-keeping control law is designed by applying terminal sliding mode control method. Finally, the stability of the control system is analyzed by using Lyapunov function method. The simulation results show that the vehicle lateral position error, yaw angle error have fast convergence rate.

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