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Exact Approach to Buckling Analysis of SSSS and CCCC Thin Rectangular Plates under Vibration Using Split-Deflection Method.

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Abstract: *This paper presents exact approach to buckling analysis of SSSS and CCCC thin rectangular plates under vibration using split-deflection method. In this method, deflection function, w is split into w_x and w_y . Total potential energy functional for a thin rectangular plate subjected to both buckling load and vibration was formulated from principles of theory of elasticity. By direct variation, the total potential energy functional was minimized with respect to deflection coefficient to obtain the equation for critical buckling load under vibration. The deflection function is of trigonometric-polynomial family. Numerical analyses for plate with all edges simply supported (SSSS) and a plate with all edges clamped (CCCC) were performed. The non-dimensional critical buckling load results from the present work under no vibration at aspect ratios ($0.1 \leq P \leq 1.0$) were compared with ones from previous scholars. The average percentage differences from the previous works and the present study for SSSS and CCCC plates when compared stood at 0.013%; 0.066% and 1.654%; 3.538% respectively. This shows that this study gave very close values to the exact solution. Also, the non-dimensional critical buckling loads for SSSS and CCCC thin rectangular plates under vibration at aspect ratios ($0.1 \leq P \leq 1.0$) and the corresponding resonating frequency ratios ($0 \leq n \leq 1.0$) were determined.*

Keywords: *Split-deflection, total potential energy functional, direct variation, trigonometric-polynomial function.*

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I. Introduction

Relatively thin structures with a wide flat planar surface are normally called plates [20]. Plates are structural components extensively used to model aircraft wings, spacecraft panels, ship hulls and decks, building floors, bridge decks, roof slabs and offshore platforms. In the field of mechanics of material, the study of plates consists of three main analyses: bending, buckling and vibration analysis [20]. Though most plates perform satisfactorily under tensile forces when they are subjected to in-plane compressive forces which maybe uniform or non-uniform, they are prone to elastic buckling failures [23,24,25,3,18,4,32,9,29,31,5,1,17]. Usually buckling of plates due to in-plane compressive forces which may be uniform or non-uniform is a nonlinear phenomenon which may occur suddenly leading to catastrophic structural failures. It is important to determine the load buckling capacities of plates as part of their design and analysis in order to avert premature failures. Many theories have been proposed to model plates based on the simplifications gained from the relative smallness of their thickness. The simplest one is the classical plate theory (CPT) developed in 1881 [27], which describe thinner plates. Classical plate theory (CPT) buckling analysis has dominated by energy methods such as Releigh, Ritz, Galerkin, Minimum potential energy, Work-error, etc [26,27,13]. Most of these energy methods applied single orthogonal deflection (unseperated) function. Thin plates as two dimensional structures when subjected to in-plane loading (loading parallel to their fibre direction), they transit from their stable state of equilibrium to the unstable one just like columns. Such transition is normally referred to as buckling or structural stability. During this transition, a critical point exists where an infinitesimal increase in load can cause the plate surface to buckle. The load at this critical point defines the buckling strength of the plate or the critical buckling load. Increase in load beyond the load at the initiation of buckling increase the buckling deformations until collapse occur [2]. Most academic works on CPT analysis of rectangular plates as seen from the literature rely on this orthogonal deflection function [10,30,26,27,28,21,22,19,6,7,12]. For instance, a typical work-error function is (Ibearugbulem et al., 2014) that applied single orthogonal deflection function:

$$\Pi = \frac{D}{2}\int_0^a \int_0^b \left(\frac{\partial^4 w}{\partial x^4}w + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}w\right)\partial x \partial y - \frac{m\,.\lambda^2}{2}\int_0^a \int_0^b w^2\,\partial x \partial y$$

This makes it difficult to some analysts in obtaining orthogonal function for a plate of a particular boundary condition. Recently, some scholars have applied split-deflection method in buckling analysis of some plates of particular boundary conditions but did not consider the effect of vibration on it. Ibearugbulem et al. [14] applied split-deflection method in buckling analysis of CCSS and CCCS thin rectangular plates under vibration. From the existing literatures, none has considered buckling analysis of SSSS and CCCC thin rectangular plates under vibration. Thus, this paper used trigonometric-polynomial deflection function in split form to solve the buckling problems of SSSS and CCCC thin rectangular plates under vibration caused by lateral loads subjected on both sides at x-axis shown in Figures 1 & 2.

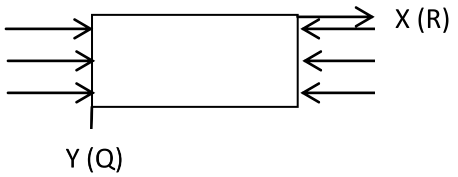

Figure 1: SSSS plate under lateral loads.

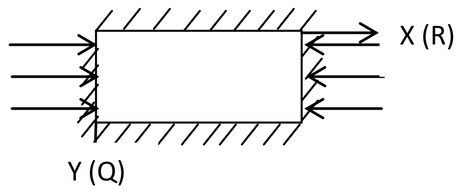

Figure 2: CCCC plate under lateral loads.

The boundary conditions for SSSS plate at x-axis are:

$w = (R = 0) = \frac{\partial^2 w(R=0)}{\partial R^2} = 0$

$w = (R = 1) = \frac{\partial^2 w(R=1)}{\partial R^2} = 0$

The boundary conditions for CCCC plate at x-axis are:

$w = (R = 0) = \frac{\partial w(R=0)}{\partial R} = 0$

$w = (R = 1) = \frac{\partial w(R=1)}{\partial R} = 0$

II. Split-Deflection

The general deflection, w is split into $w_x \; and \; w_y$. The split-deflection function is given as:

$$w = w_x . w_y \quad (1)$$

Where the $w_x \; and \; w_y$ components of the deflection are defined:

$$w = w_x . w_y = Ah_x . Ah_y \quad (2)$$

III. IN – Plane Displacement

From the hypothesis that vertical shear strains are zero for classical plates and making use of split-deflection approach we obtain;

$$u = -Z\frac{dw}{dx} = -Z\frac{dw_x}{dx}w_y \quad (3)$$

$$v = -Z\frac{dw}{dy} = -Z\frac{dw_y}{dy}w_x \quad (4)$$

IV. Strain Deflection Relationship

The three in-plane strains of classical plate are obtained using Equations (3) and (4):

$$\varepsilon_x = \frac{du}{dx} = -Z\frac{d^2w_x}{dx^2}w_y \quad (5)$$

$$\varepsilon_y = \frac{dv}{dy} = -Z\frac{d^2w_y}{dy^2}w_x \quad (6)$$

$$\Upsilon_{xy} = \frac{du}{dy} + \frac{dv}{dx} = -2Z\frac{dw_x}{dx}\frac{\mathrm{dw_y}}{\mathrm{dw_y}} \quad (7)$$

V. STRESS – STRAIN RELATIONSHIP

The in-plane constitutive equations for plates are:

$$\sigma_x = \frac{E}{1-\mu^2}[\varepsilon_x + \mu\varepsilon_y]$$
(8)

$\sigma_y = \frac{E}{1-\mu^2}[\mu\varepsilon_x + \varepsilon_y]$
(9)
$\tau_{xy} = \frac{E(1-\mu)}{2(1-\mu^2)}\gamma_{xy}$
(10)

VI. Stress – Deflection Relationship

Substituting Equations (5), (6) and (7) into Equations (8), (9) and (10) as appropriate give:
$\sigma_x = -\frac{EZ}{1-\mu^2}[\frac{d^2w_x}{dx^2}w_y + \mu\frac{d^2w_y}{dy^2}w_x]$
(11)
$\sigma_y = -\frac{EZ}{1-\mu^2}[\mu\frac{d^2w_x}{dx^2}w_y + \frac{d^2w_y}{dy^2}w_x]$
(12)
$\tau_{xy} = -\frac{EZ(1-\mu)}{(1-\mu^2)}\frac{dw_x}{dx}\frac{dw_y}{dy}$
(13)

VII. Total Potential Energy

The strain energy is defined as:
$$U = \frac{1}{2}\int_x\int_y[\int_{-\frac{t}{2}}^{\frac{t}{2}}[\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \tau_x\gamma_{xy}]dz]dxdy$$
(14)
For buckling and vibration analysis, the external work is given as:
$V = \frac{N_x}{2}\int_x\frac{d^2w_x}{dx^2}w_x dx\int_y w_y^2 dy + \frac{m\lambda^2}{2}\int_x w_x^2 dx\int_y w_y^2 dy$
(15)
Substituting Equations (8) to (13) into Equation (14) gives strain energy-deflection relationship as:
$U = \frac{D}{2}\int_x\int_y[(\frac{d^2w_x}{dx^2})^2 w_y^2 + 2(\frac{dw_x}{dx})^2\left(\frac{dw_y}{dy}\right)^2 + \left(\frac{d^2w_y}{dy^2}\right)^2 w_x^2]dxdy$
That is;
$U = \frac{D}{2}[\int_x(\frac{d^2w_x}{dx^2})^2 dx\int_y w_y^2 dy] + \frac{2D}{2}[\int_x(\frac{dw_x}{dx})^2 dx\left(\frac{dw_y}{dy}\right)^2 dy] + \frac{D}{2}[\int_x w_x^2 dx\int_y\left(\frac{d^2w_y}{dy^2}\right)^2 dy]$
(16)
Where; $D = \frac{Eh^3}{12(1-\mu^2)}$
"E", "h" and "μ" are Young's modulus of elasticity, thickness and poison's ratio of plate respectively.
Subtracting Equation (15) from Equation (16) gives the total potential energy functional as:
$$\Pi = \frac{D}{2}[\int_x(\frac{d^2w_x}{dx^2})^2 dx\int_y w_y^2 dy] + \frac{2D}{2}[\int_x(\frac{dw_x}{dx})^2 dx\left(\frac{dw_y}{dy}\right)^2 dy] + \frac{D}{2}[\int_x w_x^2 dx\int_y\left(\frac{d^2w_y}{dy^2}\right)^2 dy] - \frac{1}{2}[N_x\int_x\frac{d^2w_x}{dx^2}w_x dx\int_y w_y^2 dy + m\lambda^2\int_x w_x^2 dx\int_y w_y^2 dy] \quad (17)$$
Substituting Equation (2) into Equation (17) gives:
$$\Pi = \frac{A^2D}{2}[\int_x(\frac{d^2h_x}{dx^2})^2 dx\int_y h_y^2 dy] + \frac{2A^2D}{2}[\int_x(\frac{dh_x}{dx})^2 dx\left(\frac{dh_y}{dy}\right)^2 dy] + \frac{A^2D}{2}[\int_x h_y^2 dx\int_y\left(\frac{d^2h_y}{dy^2}\right)^2 dy] - \frac{A^2}{2}[N_x\int_x\frac{d^2h_x}{dx^2}h_x dx\int_y h_y^2 dy + m\lambda^2\int_x h_x^2 dx\int_y h_y^2 dy] \quad (18)$$
Using non–dimensional form of axes R and Q, Equation (18) can be written as:
$x = aR$
(19)
$y = bQ$
(20)
$P = \frac{a}{b}$
(21)
Where a, b and P are the plate lengths in x and y axes and long span – short span aspect ratio respectively. Substituting Equations (19), (20) and (21) into Equation (18) gives:

$$\Pi = \frac{abA^2D}{2b^4P^4}\left[\int_0^1 \frac{d^4h_x}{dR^4}h_x\,dR\int_0^1 h_y^2dQ\right] + \frac{2abA^2D}{2b^4P^2}\left[\int_0^1 \frac{d^2h_x}{dR^2}h_xdR\int_0^1 \frac{d^2h_y}{dQ^2}h_ydQ\right] + \frac{abA^2D}{2b^4}\left[\int_0^1 h_x^2dR\int_0^1 \frac{d^4h_y}{dQ^4}h_y\,dQ\right] - \frac{abA^2}{2b^2P^2}\left[N_x\int_0^1 \frac{d^2h_x}{dR^2}h_x\,dR\int_0^1 h_y^2dQ + m\lambda^2\int_0^1 h_x^2dR\int_0^1 h_y^2dQ\right]$$

(22)

VIII. Direct Variation of Total Potential Energy

Equation (22) shall be differentiated with respect to the deflection coefficient, A and the outcome is:

$$\frac{d\Pi}{dA} = \frac{AD}{b^4P^4}\left[\int_0^1 \frac{d^4h_x}{dR^4}h_x\,dR\int_0^1 h_y^2dQ\right] + \frac{2AD}{b^4P^2}\left[\int_0^1 \frac{d^2h_x}{dR^2}h_xdR\int_0^1 \frac{d^2h_y}{dQ^2}h_ydQ\right] + \frac{AD}{b^4}\left[\int_0^1 h_x^2dR\int_0^1 \frac{d^4h_y}{dQ^4}h_y\,dQ\right] - \frac{A}{b^2P^2}\left[N_x\int_0^1 \frac{d^2h_x}{dR^2}h_x\,dR\int_0^1 h_y^2dQ + m\lambda^2\int_0^1 h_x^2dR\int_0^1 h_y^2dQ\right] = 0$$

That is;

$$\frac{D}{b^2P^4}\left[\int_0^1 \frac{d^4h_x}{dR^4}h_x\,dR\int_0^1 h_y^2dQ\right] + \frac{2D}{b^2P^2}\left[\int_0^1 \frac{d^2h_x}{dR^2}h_xdR\int_0^1 \frac{d^2h_y}{dQ^2}h_ydQ\right] + \frac{D}{b^2}\left[\int_0^1 h_x^2dR\int_0^1 \frac{d^4h_y}{dQ^4}h_y\,dQ\right] = \frac{1}{P^2}\left[N_x\int_0^1 \frac{d^2h_x}{dR^2}h_x\,dR\int_0^1 h_y^2dQ + m\lambda^2\int_0^1 h_x^2dR\int_0^1 h_y^2dQ\right]$$

(23)

Re-arranging Equation (23) gives:

$$\frac{D}{b^2P^2}\left[\frac{K_x}{P^2} + 2K_{xy} + K_yP^2\right] = \frac{1}{P^2}\left[N_xK_{Nx} + m\lambda^2K_\lambda\right]$$

(24)

Where the stiffness coefficients;

$$K_x = \int_0^1 \frac{d^4h_x}{dR^4}h_x\,dR\int_0^1 h_y^2dQ$$

(25)

$$K_{xy} = \int_0^1 \frac{d^2h_x}{dR^2}h_xdR\int_0^1 \frac{d^2h_y}{dQ^2}h_ydQ$$

(26)

$$K_y = \int_0^1 h_{x^2}dR\int_0^1 \frac{d^4h_y}{dQ^4}h_y\,dQ$$

(27)

$$K_{Nx} = \int_0^1 \frac{d^2h_x}{dR^2}h_x\,dR\int_0^1 h_y^2dQ$$

(28)

Under vibration only, Equation (24) becomes:

$$\frac{D}{b^2}\left[\frac{K_x}{P^2} + 2K_{xy} + K_yP^2\right] = m\lambda^2K_\lambda$$

(29)

The buckling load, N_x using Equation (24) is given as:

$$N_x = \frac{\frac{D}{b^2}\left[\frac{K_x}{P^2}+2K_{xy}+K_yP^2\right] - m\theta^2K_\lambda}{K_{Nx}}$$

(30)

Where θ is the forcing frequency ($0 \leq \theta \leq \boldsymbol{\lambda}$), $\boldsymbol{\lambda}$ is the resonating frequency. Substituting Equation (29) into Equation (30) gives:

$$N_x = \frac{m\lambda^2K_\lambda - m\theta^2K_\lambda}{K_{Nx}}$$

(31)

Re-arranging Equation (31) gives:

$$N_x = \frac{m\lambda^2K_\lambda}{K_{Nx}}\left(1 - \frac{\theta^2}{\lambda^2}\right)$$

(32)

Substituting Equation (29) into Equation (32) gives:

$$N_x = \frac{\frac{D}{b^2}\left[\frac{K_x}{P^2}+2K_{xy}+K_yP^2\right]}{K_{Nx}}\left(1 - \frac{\theta^2}{\lambda^2}\right)$$

(33)

Let the resonating frequency ratio (n) = $\frac{\theta}{\lambda}$, $\frac{D}{b^2}$ = dimensional constants, aspect ratio $(P) = \frac{a}{b}$ where $(0.1 \leq P \leq 1)$.

Equation (33) becomes:

$$N_x = \frac{\frac{D}{b^2}\left[\frac{K_x}{P^2}+2K_{xy}+K_yP^2\right]}{K_{Nx}}\left(1 - n^2\right)$$

(34)

The elastic buckling equation of plate can be expressed in the form;

$$N_x = \frac{\frac{D\pi^2}{b^2}\left[\frac{K_x}{P^2}+2K_{xy}+K_y P^2\right]}{K_{Nx}} (1 - \mathrm{n}^2)$$
(35)

IX. Numerical Examples

Analyze a classical rectangular thin isotropic plate with:

- ❖ Four edges simply supported (SSSS) using polynomial and trigonometric functions respectively for w_x and w_y .
- ❖ Four edges clamped (CCCC) using polynomial and trigonometric functions respectively for w_x and w_y .

FOR SSSS PLATE

The trigonometric deflection equation, w_x for SSSS plates is given by Ibearugbulem et al. (2019) as:

$$w_x = A_x \sin \pi R \quad (36)$$

The polynomial deflection equation, w_y for SSSS plates is given by Ibearugbulem (2012) as:

$$w_y = A_y(Q - 2Q^3 + Q^4) \quad (37)$$

From Equations (36) and (37) the shape functions, h_x and h_y are:

$$h_x = \sin \pi R \quad (38)$$

$$h_y = Q - 2\mathrm{Q}^3 + \mathrm{Q}^4 \quad (39)$$

With these we obtain:

$$\int_0^1 h_x^2 dR = 0.5$$

$$\int_0^1 h_y^2 dQ = 0.049206349$$

$$\int_0^1 \frac{d^2h_x}{dR^2} h_x \, dR = 0.5\pi^2$$

$$\int_0^1 \frac{d^2h_y}{dQ^2} h_y \, dQ = 0.485714285$$

$$\int_0^1 \frac{d^4h_x}{dR^2} h_x \, dR = 0.5\pi^4$$

$$\int_0^1 \frac{d^4h_y}{dQ^2} h_y \, dQ = 4.8$$

$$K_x = 0.5\pi^4 \times 0.049206349 = 2.396572865 \quad (40)$$

$$K_{xy} = 0.5\pi^2 \times 0.485714285 = 2.396903922 \quad (41)$$

$$K_y = 0.5 \times 4.8 = 2.4 \quad (42)$$

$$K_{Nx} = 0.5\pi^2 \times 0.049206349 = 0.242823599 \quad (43)$$

Substituting Equations (40) to (43) into Equation (35) gives:

$$\tilde{\mathrm{N}}_x = \left[\frac{0.99974}{P^2} + 1.99976 + 1.00117P^2\right] (1 - \mathrm{n}^2) \quad (44)$$

Equation (44) is the solution for determining the non-dimensional buckling loads of SSSS thin rectangular plate under vibration.

FOR CCCC PLATE

The trigonometric deflection equation, w_x for CCCC plates is given by Ibearugbulem et al. (2019) as:

$$w_x = A_x(1 - \cos 2\pi R) \quad (45)$$

The polynomial deflection equation, w_y for CCCC plates is given by Ibearugbulem (2012) as:

$$w_y = A_y(Q^2 - 2Q^3 + Q^4) \quad (46)$$

From Equations (45) and (46) the shape functions, h_x and h_y are:

$$h_x = 1 - \cos 2\pi R \quad (47)$$

$$h_y = Q^2 - 2Q^3 + Q^4 \quad (48)$$

With these we obtain:

$$\int_0^1 h_x^2 dR = 1.5$$

$$\int_0^1 h_y^2 dQ = 0.001587301587$$

$$\int_0^1 \frac{d^2h_x}{dR^2} h_x \, dR = 2\pi^2$$

$$\int_0^1 \frac{d^2h_y}{dQ^2} h_y \, dQ = 0.019047619$$

$$\int_0^1 \frac{d^4h_x}{dR^2} h_x \, dR = 8\pi^4$$

$$\int_0^1 \frac{d^4h_y}{dQ^2} h_y \, dQ = 0.8$$

$K_x = 8\pi^4 \times 0.001587301587 = 1.236940838$ (49)
$K_{xy} = 2\pi^2 \times 0.019047619 = 0.375984928$ (50)
$K_y = 1.5 \times 0.8 = 1.2$ (51)
$K_{Nx} = 2\pi^2 \times 0.01587301587 = 0.031332077$ (52)
Substituting Equations (49) to (52) into Equation (35) gives:

$$\tilde{N}_x = \left[\frac{3.99896}{P^2} + 2.43108 + 3.87954P^2\right](1 - n^2) \quad (53)$$

Equation (53) is the solution for determining the non-dimensional buckling loads of CCCC thin rectangular plate under vibration.
The comparison of these results and those from earlier scholars was by simple percentage error. The percentage difference (error) equation used is:

$$\text{Percentage Error} = \left(\frac{value\ obtained\ by\ a\ theory}{corresponding\ value\ by\ exact\ theory} - 1\right) \times 100 \quad (54)$$

X. Results And Discussion

The non-dimensional critical buckling load results of this present work at zero vibration ($n = 0$) for aspect ratios ($0.1 \leq P \leq 1.0$) were compared with those from earlier scholars. These comparisons were presented on Table 1 and Table 2.

Table 1: Non-dimensional critical buckling loads for SSSS rectangular plate under uniform unilateral stress ($n = 0$)

Aspect ratio, P	[18]	[11]	Present study	% Error between [18] and present study	% Error between [18] and [11]
0.1	102.010	102.110	101.984	- 0.025	0.098
0.2	27.040	27.065	27.033	- 0.026	0.092
0.3	13.201	13.212	13.198	- 0.023	0.083
0.4	8.410	8.416	8.408	- 0.024	0.071
0.5	6.250	6.254	6.249	-0.016	0.064
0.6	5.138	5.141	5.137	-0.019	0.058
0.7	4.531	4.533	4.531	0.000	0.044
0.8	4.203	4.205	4.203	0.000	0.048
0.9	4.045	4.047	4.045	0.000	0.050
1.0	4.000	4.002	4.000	0.000	0.050
Average % Error				-0.013	0.066

Table 2: Non-dimensional critical buckling loads for CCCC rectangular plate under uniform unilateral stress ($n = 0$)

Aspect ratio, P	[18]	[11]	Present study	% Error between [18] and present study	% Error between [18] and [11]
0.1	402.707	424.970	402.366	- 0.085	5.528
0.2	102.827	108.222	102.560	- 0.260	5.247
0.3	47.471	49.753	47.213	- 0.543	4.806
0.4	28.307	29.510	28.046	- 0.922	4.251
0.5	19.667	20.384	19.397	-1.373	3.647
0.6	15.218	15.685	14.936	-1.853	3.069
0.7	12.790	13.121	12.493	-2.322	2.583
0.8	11.477	11.734	11.162	-2.745	2.235
0.9	10.845	11.066	10.510	-3.089	2.038
1.0	10.667	10.878	10.310	-3.347	1.978
Average % Error				-1.654	3.538

The absolute average percentage difference between the results of the exact solution by Iyengar (1988) and this present study; Iyengar (1988) and Ibearugbulem (2012) for SSSS plate are 0.013% and 0.066%. Also the absolute average percentage difference between the results of the exact solution by Iyengar (1988) and this present study; Iyengar (1988) and Ibearugbulem (2012) for CCCC plate are 1.654% and 3.538%. Comparing the results, it is seen that this present study gave very close values to the exact solution by Iyengar (1988) than the approximate solution by Ibearugbulem (2012). This paper presents the non-dimensional critical buckling load results of SSSS and CCCC plates when inertia loads were applied ($n \neq 0$) for aspect ratios ($0.1 \leq P \leq 1.0$) on Table 3 and Table 4.

Table 3: Non-dimensional critical buckling loads for rectangular SSSS plate under vibration.

		Non – dimensional critical buckling load ($\tilde{N}_x$)									
		Resonating frequency ratio (n)									
Aspect ratio, P	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.1	101.984	100.964	97.905	92.805	85.667	76.488	65.270	52.012	36.714	19.377	0
0.2	27.033	26.763	25.952	24.600	22.708	20.275	17.301	13.787	9.732	5.136	0
0.3	13.198	13.066	12.670	12.010	11.086	9.899	8.447	6.731	4.751	2.508	0
0.4	8.408	8.324	8.072	7.651	7.063	6.306	5.381	4.288	3.027	1.598	0
0.5	6.249	6.187	5.999	5.687	5.249	4.687	3.999	3.187	2.250	1.187	0
0.6	5.137	5.086	4.932	4.675	4.315	3.853	3.288	2.620	1.849	0.976	0
0.7	4.531	4.486	4.350	4.123	3.806	3.398	2.900	2.311	1.631	0.861	0
0.8	4.203	4.161	4.035	3.825	3.531	3.152	2.690	2.144	1.513	0.799	0
0.9	4.045	4.005	3.883	3.681	3.398	3.034	2.589	2.063	1.456	0.769	0
1.0	4.000	3.960	3.840	3.640	3.360	3.000	2.560	2.040	1.440	0.760	0

Table 4: Non-dimensional critical buckling loads for rectangular CCCC plate under vibration.

		Non – dimensional critical buckling load ($\tilde{N}_x$)									
		Resonating frequency ratio (n)									
Aspect ratio, P	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.1	402.366	398.342	386.271	366.153	337.987	301.775	257.514	205.207	144.852	76.450	0
0.2	102.560	101.534	98.458	93.330	86.150	76.920	65.638	52.306	36.922	19.486	0
0.3	47.213	46.741	45.324	42.964	39.659	35.410	30.216	24.079	16.997	8.970	0
0.4	28.046	27.766	26.924	25.522	23.559	21.035	17.949	14.303	10.097	5.329	0
0.5	19.397	19.203	18.621	17.651	16.293	14.548	12.414	9.892	6.983	3.685	0
0.6	14.936	14.787	14.339	13.592	12.546	11.202	9.559	7.617	5.377	2.838	0
0.7	12.493	12.368	11.993	11.369	10.494	9.370	7.996	6.371	4.497	2.374	0
0.8	11.162	11.050	10.715	10.157	9.376	8.372	7.144	5.693	4.018	2.121	0
0.9	10.510	10.405	10.090	9.564	8.828	7.883	6.726	5.360	3.784	1.997	0
1.0	10.310	10.207	9.898	9.382	8.660	7.733	6.598	5.258	3.712	0.760	0

Table 3 and Table 4 shown that as aspect ratio ($P = \frac{a}{b}$) increases from 0.1 to 1.0 at each resonating frequency ratio (n), the non-dimensional critical buckling load ($\tilde{N}_x$) decreases thereby causing the plate to get more slender. Furthermore, as the resonating frequency ratio increases from 0 to 0.9 at each aspect ratio, the non-dimensional critical buckling load decreases thereby weakening the strength of the plate and hence requires less effort to cause it to buckle. At resonating frequency ratio, $(n = 1)$ at all aspect ratio$(0.1 \le P \le 1.0)$, the non-dimensional critical buckling load equals zero$(\tilde{N} = 0)$. At this stage, the plate buckles without buckling load.

XI. Conclusions

This research work presented the buckling analysis of thin rectangular isotropic plates under vibration using split-deflection method. The boundary conditions of plates are SSSS and CCCC. A unique governing equation for critical buckling loads under vibration was obtained. The deflection function is of trigonometric – polynomial family, different from Iyengar (1988) and Ibearugbulem (2012) that applied trigonometric and polynomial function respectively. This equation is reliable and is recommended for use in classical plate analysis for given insignificant percentage differences between the present and past results at zero vibration. It follows that the results of non-dimensional critical buckling load for n −values and their corresponding aspect ratios presented in Table 3 and Table 4 for which there are no other existing results to compare with in literature are also correct. The authors believe that the presented exact solutions for buckling of SSSS and CCCC plates under vibration are very valuable as they serve as benchmark results for future researchers in this area.

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