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The influence of the fluid flow speed on the axisymmetric wave propagationvelocity in the cylinder containing this fluid

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Abstract: *The present work investigates the influence of the fluid flow velocity and flow direction on the velocity of axisymmetric waves propagating in a hollow cylinder containing this fluid. In the context of this study, the motion of the cylinder is described by the exact equations and relations of linear elastodynamics, but the flow of the fluid is described by the linearized Euler equations for compressible barotropic inviscid fluids. Analytical expressions for the sought values containing unknown constants are obtained, and with the help of contact and of compatibility conditions, the system of homogeneous algebraic equations with respect to these unknown constants is obtained.Using the known procedures, the corresponding dispersion equation is attained. This equation is solved numerically, whereupon the dispersion curves are obtained for different values of the problem parameters, in particular for the different flow velocities under different flow directions. These dispersion curves are constructed for the zeroth and first modes and it is made corresponding analysis of these curves, as well as, it is formulated corresponding conclusions.*
Key Word*: Cylinder containing flowing fluid, compressible inviscid barotropic fluid, hollow cylinder, axisymmetric waves dispersion, fluid flow, elastodynamics*

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I. Introduction

The dynamic pressure of fluids flowing at high velocity in hollow cylinders is used in manyareas of modern industry and mining. In addition, the flow of liquid during transportationthrough the pipes (hollow cylinders) can sometimes occur at high velocities. In such cases,when applying the ultrasonic wave propagation method for nondestructive defect detection inthese pipes, it is necessary to have available theoretical results on the influence of the fluidflow velocity on the propagation velocities of the waves in these pipes. In fact, the presentwork is devoted to these questions and the influence of the fluid flow velocity on the velocityof axisymmetric waves propagating in the hollow cylinder containing this fluid isinvestigated. The investigations are carried out within the framework of the exact equationsand relations of elastodynamics, which describe the motion of the cylinder, and within theframework of the linearized Euler equations for the inviscid compressible barotropic fluids inthis cylinder which describe the flow of this fluid.

Note that the mathematical modeling of the corresponding more general problems for the casewhen the cylinder has inhomogeneous initial stresses induced by the hydrostatic pressureacting on the inner surface of the hollow cylinder is described in the paper [1] using the three-dimensional linearized equations and relations of the theory of elastic waves in bodies withinitial stresses.[2, 3, 4]. A brief overview of the related research was given in the paper [1],therefore, we do not repeat that overview here and readers interested in that overview may usethe paper [1].However, in the paper [1] the concrete numerical results are presented and discussed for thecase when the fluid is at rest in the cylinder, i.e., for the case when there is no fluid flow in thecylinder. In order to answer the questions formulated above, the present work is an attempt tostudy the influence of the fluid flow velocity on the velocity of axisymmetric wavespropagating in this cylinder. At the same time, in the present work, unlike the work [1], weassume that there are no initial stresses in the cylinder in the initial state.

II. Mathematical formulation of the problem

Consider the hydro-elastic system consisting of an infinite hollow cylinder and of acompressible barotropic inviscid fluid contained in this cylinder. We associate the cylindrical$Or\theta z$ and Cartesian$Ox_1x_2x_3(x_3 = z)$(Fig. 1) systems of coordinates with the central axis of the cylinder. Like the rules, we use the Lagrange and Euler coordinates for describing the motion of the cylinder and fluid respectively. We distinguish two states, namely the initial state and the disturbed state of the hydroelastic system under consideration, and assume that in the initial state the quantities characterizing the stress-strain state in the cylinder are zero.Let us also assume that the fluid in the initial state flows inside the cylinder with constant velocityV_0along the cylinder axis (in theOzaxis or in the opposite direction to this axis), so that the components of the velocity vector of the fluid in the initial state are as follows:

$$V_r^0 = 0, V_\theta^0 = 0, V_z^0 = V_0 = const \quad (1)$$

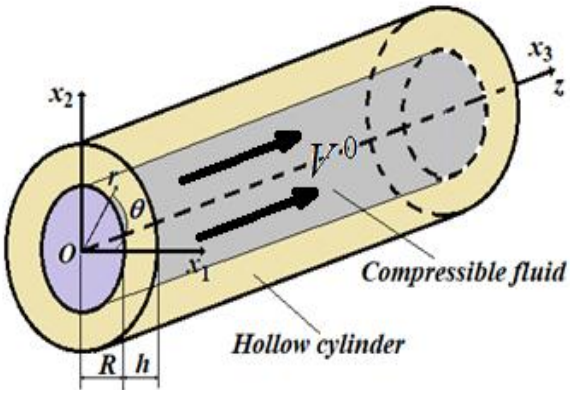

Fig. 1 The sketch of the hydro-elastic system under consideration: (a) cylinder containing flowing fluid; (b) initial pressure and density of the fluid.

The direction of the fluid flow in the initial state is determined by the sign of the values of the velocity V_0, i.e. in the cases when$V_0 > 0 (V_0 < 0)$, the fluid flows in the direction of the Oz axis (opposite to the Oz axis).

Thus, we determine the quantities related to the initial state of the hydro-elastic system under consideration by the expressions in (1) and assume that after the occurrence of this initial state, the hydro-elastic system undergoes a certain dynamical perturbation, as a result of which the axisymmetric waves propagate. It is necessary to investigate how this initial state, i.e., the flow velocity V_0, affects the propagation of said waves. For this investigation, we use the exact equations and relations of linear elastodynamics and the linearized Euler equations to describe the flow of the inviscid compressible barotropic fluid.

Now we write the corresponding field equations and relations for the cylinder in the cylindrical coordinate system $Or\theta z$ under the axisymmetric stress-strain case [2, 3, 4].

The equations of motion:

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial \sigma_{zr}}{\partial z} + \frac{1}{r}(\sigma_{rr} - \sigma_{\theta\theta}) = \rho \frac{\partial^2 u_r}{\partial t^2}, \frac{\partial \sigma_{rz}}{\partial r} + \frac{1}{r}\sigma_{rz} + \frac{\partial \sigma_{zz}}{\partial z} = \rho \frac{\partial^2 u_z}{\partial t^2} \quad (2)$$

The elasticity relations:

$$\sigma_{(jj)} = \lambda (\varepsilon_{rr} + \varepsilon_{\theta\theta} + \varepsilon_{zz}) + 2\mu \varepsilon_{(jj)}, (jj) = rr; \theta\theta; zz, \sigma_{rz} = 2\mu \varepsilon_{rz} \quad (3)$$

The strain-displacement relations:

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r}, \varepsilon_{\theta\theta} = \frac{u_r}{r}, \varepsilon_{zz} = \frac{\partial u_z}{\partial z}, \varepsilon_{rz} = \frac{1}{2}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right) \quad (4)$$

In (2) – (4) the conventional notation is used.

For describing the flow of the fluid, according to [5], we use the following linearized field (or linearized Euler) equations for barotropic compressible inviscid fluids.

The linearized continuity equation:

$$\frac{\partial \rho'}{\partial t} + \rho_0 \left(\frac{\partial V_r}{\partial r} + \frac{V_r}{r} + \frac{\partial V_z}{\partial z} \right) + V_z^0 \frac{\partial \rho'}{\partial z} = 0 \tag{5}$$

Linearized equations of the fluid flow:

$$\frac{\partial V_r}{\partial t} + V_z^0 \frac{\partial V_r}{\partial z} = -\frac{1}{\rho_0} \frac{\partial p'}{\partial r}, \quad \frac{\partial V_z}{\partial t} + V_z^0 \frac{\partial V_z}{\partial z} = -\frac{1}{\rho_0} \frac{\partial p'}{\partial z}. \tag{6}$$

The state equation:

$$a_0^2 = \frac{\partial p'}{\partial \rho'}. \tag{7}$$

where a_0 is the sound speed in the fluid.

Note that equations (5) – (7) compose the complete system of equations within the scope of which the flow of the fluid in the perturbed state is described.

Now we add to the foregoing equations corresponding boundary and compatibility conditions.

The boundary conditions on the external surface of the cylinder are:

$$\sigma_{rr}\Big|_{r=R+h} = 0, \quad \sigma_{rz}\Big|_{r=R+h} = 0 \tag{8}$$

The compatibility conditions on the interface surface between the fluid and cylinder, i.e. on the internal surface of the cylinder are:

$$\sigma_{rr}\Big|_{r=R} = -p', \quad \sigma_{rz}\Big|_{r=R} = 0 \quad \frac{\partial u_r}{\partial t}\Big|_{r=R} = V_r\big|_{r=R} \tag{9}$$

Finally, we write the condition on boundedness of the quantities related to the fluid at the central axis of the cylinder.

$$\left\{ |p'|, |\rho'|, |V_r|, |V_z| \right\}\Big|_{r=0} < \infty \tag{10}$$

Since the perturbations are assumed to be sufficiently small, if the above compatibility conditions are satisfied, the difference between the Lagrangian and Euler coordinates is not considered.

This completes the mathematical formulation of the problem under consideration.

III. Method of solution of the formulated problem

For the solution of the system of equations (2) - (4) we use the classical Lame decomposition (see, e.g., the monograph [4]), which can be written for the axisymmetric problems as follows.

$$u_r = \frac{\partial \Phi}{\partial r} + \frac{\partial^2 \Psi}{\partial r \partial z} \quad u_z = \frac{\partial \Phi}{\partial z} - \frac{\partial^2 \Psi}{\partial r^2} - \frac{\partial \Psi}{r \partial r} \tag{11}$$

where the functions Φ and Ψmust satisfy the following equations:

$$\frac{\partial^2 \Phi^n}{\partial r^2} + \frac{\partial \Phi^n}{r\partial r} + \frac{\partial^2 \Phi^n}{\partial z^2} = \frac{1}{(c_1)^2}\frac{\partial^2 \Phi^n}{\partial t^2}, \frac{\partial^2 \Psi^n}{\partial r^2} + \frac{\partial \Psi^n}{r\partial r} + \frac{\partial^2 \Psi^n}{\partial z^2} = \frac{1}{(c_2)^2}\frac{\partial^2 \Psi^n}{\partial t^2} \tag{12}$$

In (12) the notation $c_1 = \sqrt{(\lambda + 2\mu)/\rho}$ and $c_2 = \sqrt{\mu/\rho}$ is used.

Representing the functions Φ, u_r, σ_{rr}, $\sigma_{\theta\theta}$ and σ_{zz} with the multiplying $sin(kz - \omega t)$ and the functions Ψ, u_z and σ_{rz} with the multiplying $cos(kz - \omega t)$, and denoting the amplitudes of the corresponding quantities with the same symbols, we obtain the following equations for the amplitudes of the potentials Φ and Ψ.

$$\frac{d^2\Phi}{d(r_2)^2} + \frac{1}{r_2}\frac{d\Phi}{dr_2} + \Phi = 0, \frac{d^2\Psi}{d(r_1)^2} + \frac{1}{r_1}\frac{d\Psi}{dr_1} + \Psi^n = 0 \tag{13}$$

where

$$r_1 = kr\sqrt{\frac{c^2}{(c_2)^2} - 1}, \; r_2 = kr\sqrt{\frac{c^2}{(c_1)^2} - 1} \tag{14}$$

It is know that the solution of the equations in (13) can be presented as follows:

$$\Phi = A_1 E_0(r_2) + A_2 F_0(r_2), \Psi = B_1 E_0(r_1) + B_2 F_0(r_1) \tag{15}$$

where A_1, A_2, B_1 and B_2 are unknown constants and

$$E_0(r_m) = \begin{cases} J_0(r_m) \text{ if } (r_m)^2 > 0 \\ I_0(r_m) \text{ if } (r_m)^2 < 0 \end{cases}, F_0(r_m) = \begin{cases} Y_0(r_m) \text{ if } (r_m)^2 > 0 \\ K_0(r_m) \text{ if } (r_m)^2 < 0 \end{cases}, \; m = 1, 2. \tag{16}$$

In (16), $J_0(x)$ and $I_0(x)$ are the Bessel and modified Bessel functions of the first kind in the zeroth order, however, $Y_0(x)$ and $K_0(x)$ are also the Bessel and Modified Bessel functions of the second kind in the zeroth order.

Thus, substituting the solutions in (15) and (16) into the representations in (11), we determine the expressions for the displacements and then, using the relations in (4) and (3), we obtain the expressions for the stresses. These expressions are:

$$u_r(r) = A_1 \frac{dr_2}{dr}\frac{dE_0(r_2)}{dr_2} + A_2 \frac{dr_2}{dr}\frac{dF_0(r_2)}{dr_2} \quad u_r(r) = A_1 \frac{dr_2}{dr}\frac{dE_0(r_2)}{dr_2} + A_2 \frac{dr_2}{dr}\frac{dF_0(r_2)}{dr_2}$$

$$u_z^n(r)=A_1 E_0(r_2)+A_2F_0(r_2)-B_1\left[\frac{dr_1}{dr}\frac{dE_0(r_1)}{d(r_1)}+\left(\frac{dr_1}{dr}\right)^2\frac{d^2E_0(r_1)}{d(r_1)^2}\right]-B_2\frac{1}{r}$$

$$\left[\frac{dr_1}{dr}\frac{dF_0(r_1)}{d(r_1)}+\left(\frac{dr_1}{dr}\right)^2\frac{d^2F_0(r_1)}{d(r_1)^2}\right],$$

$$\frac{\sigma_{rr}(r)}{\mu}=A_1\left\{\left(\frac{dr_2}{dr}\right)^2 2\left(1+\frac{\lambda}{2\mu}\right)\frac{d^2E_0(r_2)}{d(r_2)^2}\right]+\frac{\lambda}{\mu}\left(\frac{dr_2}{dr}\right)^2\frac{dE_0(r_2)}{d(r_2)}+\frac{\lambda}{\mu}E_0(r_2)\right\}+$$

$$A_2\left\{\left(\frac{dr_2}{dr}\right)^2 2\left(1+\frac{\lambda}{2\mu}\right)\frac{d^2F_0(r_2)}{d(r_2)^2}\right]+\frac{\lambda}{\mu}\left(\frac{dr_2^n}{dr}\right)^2\frac{dF_0(r_2)}{d(r_2)}+\frac{\lambda}{\mu}F_0(r_2)\right\}+$$

$$B_1\left\{\left(\frac{dr_1}{dr}\right)^2 2\left(1+\frac{\lambda}{2\mu}\right)\frac{d^2E_0(r_1)}{d(r_1)^2}\right]+\frac{\lambda}{\mu}\left(\frac{dr_1}{dr}\right)^2\frac{dE_0(r_1)}{d(r_1)}+\frac{\lambda}{\mu}E_0(r_1)\right\}+$$

$$B_2\left\{\left(\frac{dr_1}{dr}\right)^2 2\left(1+\frac{\lambda}{2\mu}\right)\frac{d^2F_0(r_1)}{d(r_1)^2}\right]+\frac{\lambda}{\mu}\left(\frac{dr_1}{dr}\right)^2\frac{dF_0(r_1)}{d(r_1)}+\frac{\lambda}{\mu}F_0(r_1)\right\},$$

$$\frac{\sigma_{rz}(r)}{\mu}=A_1 2\frac{dr_2}{dr}\frac{dE_0(r_2)}{d(r_2)}+A_2 2\frac{dr_2}{dr}\frac{dF_0(r_2)}{d(r_2)}+B_1\left[\left(\frac{dr_1}{dr}\right)^3\frac{d^3E_0(r_1)}{d(r_1)^3}+\frac{1}{r_1}\left(\frac{dr_1}{dr}\right)^3\frac{d^2E_0(r_1)}{d(r_1)^2}+\right.$$

$$\left.\frac{dr_1}{dr}(r_1)^{-1}\frac{dE_0(r_1)}{d(r_1)}\right]+B_2\left[\left(\frac{dr_1}{dr}\right)^3\frac{d^3F_0(r_1)}{d(r_1)^3}+\frac{1}{r_1}\left(\frac{dr_1}{dr}\right)^3\frac{d^2F_0(r_1)}{d(r_1)^2}+\frac{dr_1}{dr}(r_1)^{-1}\frac{dF_0(r_1)}{d(r_1)}\right] \quad (17)$$

In this way, we determine the displacement and stress field in the cylinder that is in it when waves propagate.

Now we consider the determination of the quotients related to the fluid flow, which also occurs during wave propagation in the hydroelastic system under consideration. For this purpose, according to [5], we use following representations.

$$\rho' = a_0^{-2}\rho_0\left(-V_z^0\frac{\partial}{\partial z}-\frac{\partial}{\partial t}\right)\Phi_f,\ p' = \rho_0\left(-V_z^0\frac{\partial}{\partial z}-\frac{\partial}{\partial t}\right)\Phi_f \quad ,V_r=\frac{\partial}{\partial r}\Phi_f, V_z=\frac{\partial}{\partial z}\Phi_f \quad ,$$

(18)

where

$$\left[\Delta-\frac{1}{a_0^2}\left(\frac{\partial}{\partial t}+V_z^0\frac{\partial}{\partial z}\right)^2\right]\Phi_f=0,\ \Delta=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2} \quad (19)$$

Representing the functions V_z, p' and ρ' by multiplying $sin(kz-\omega t)$, and the functions Φ_f and V_r by multiplying $cos(kz-\omega t)$, we obtain the following equation from (18) for Φ_{f1} (where $\Phi=\Phi_{f1}(r)\,cos(kz-\omega t)$).

$$\left(\frac{d^2}{dr_3^2}+\frac{1}{r_3}\frac{d}{dr_3}+1\right)\Phi_{f1}(r)=0, r_3=kr\sqrt{\left(\frac{c}{a_0}\right)^2+2\frac{c}{a_0}\frac{V_z^0}{a_0}+\left(\frac{V_z^0}{a_0}\right)^2-1} \quad (20)$$

According to the conditions in (10), the solution to equation (20) is found as follows.

$$\Phi_{f1}(r) = \begin{cases} FJ_0(r_3) \text{ if } r_3^2 > 0 \\ FI_0(r_3) \text{ if } r_3^2 < 0 \end{cases} \tag{21}$$

where $J_0(r_3)$ ($I_0(r_3)$) is the first kind Bessel (modified Bessel) function of the zeroth order and $F$is a unknown constant.

Using the expression (21) and substituting $\Phi = \Phi_{f1}(r)\, cos(kz - \omega t)$ into the equations in (18) we obtain the following expressions for the sought values related to the fluid.

$$p' = \rho_0 \left(V_z^0 k + \omega \right) \sin(kz - \omega t) \begin{cases} FJ_0(r_3) \text{ if } r_3^2 > 0 \\ FI_0(r_3) \text{ if } r_3^2 < 0 \end{cases}$$

$$\rho' = a_0^{-2} \rho_0 \left(V_z^0 k + \omega \right) \sin(kz - \omega t) \begin{cases} FJ_0(r_3) \text{ if } r_3^2 > 0 \\ FI_0(r_3) \text{ if } r_3^2 < 0 \end{cases}$$

$$V_r = k \frac{dr_3}{dr} \cos(kz - \omega t) \begin{cases} -FJ_1(r_3) \text{ if } r_3^2 > 0 \\ FI_1(r_3) \text{ if } r_3^2 < 0 \end{cases}, V_z = -k \sin(kz - \omega t) \begin{cases} FJ_0(r_3) \text{ if } r_3^2 > 0 \\ FI_0(r_3) \text{ if } r_3^2 < 0 \end{cases} \tag{22}$$

Note that in (22) ρ_0 shows the density of the fluid in the initial state.

This completes the determination of the quantities related to the fluid flow in the perturbed state.

Thus, after the previous preparations, if we substitute the expressions (17) and (22) into the boundary conditions (8) and the compatibility conditions (9), we obtain the system of homogeneous linear algebraic equations for the unknown constants A_1, A_2, B_1, B_2, and F. If we set the determinant of the coefficient matrix of this system equal to zero, we obtain the following dispersion equation.

$$\det(a_{nm}(c/c_2,\ kR, V_0/a_0,\ \rho/\rho_0,\ h/R,\ a_0/c_2)) = 0,\ n;m = 1,2,3,4,5 \tag{23}$$

The explicit expressions of the components a_{nm} in (23) can be easily determined from formulas (17) and (22) and are therefore not given here.

IV. Numerical results and discussions

Under obtaining numerical results the dispersion equation (23) is solved numerically by employing the “bi-section” method. Moreover, these results are obtained for the case where the material of the cylinder is steel with the Lame constants $\lambda = 1.075 \times 10^{11} Pa$, $\mu = 0.77 \times 10^{11} Pa$ and with the material density $\rho = 7910 \frac{kg}{m^3}$and the fluid is the water with the sound speed $a_0 = 1495 \frac{m}{sec}$ and with the density $\rho_0 = 1000 \frac{kg}{m^3}$. The main purpose of these numerical results is to investigate how the ratio V_0/a_0 affects the dispersion curves obtained for different values of h/R, the meaning of which is shown in Fig. 1.Note that these investigations are done for the zeroth and first modes, that is, for the first two lowest modes. Sometimes the mentioned zeroth mode is also called the quasi-Scholte mode. Note also that the dispersion

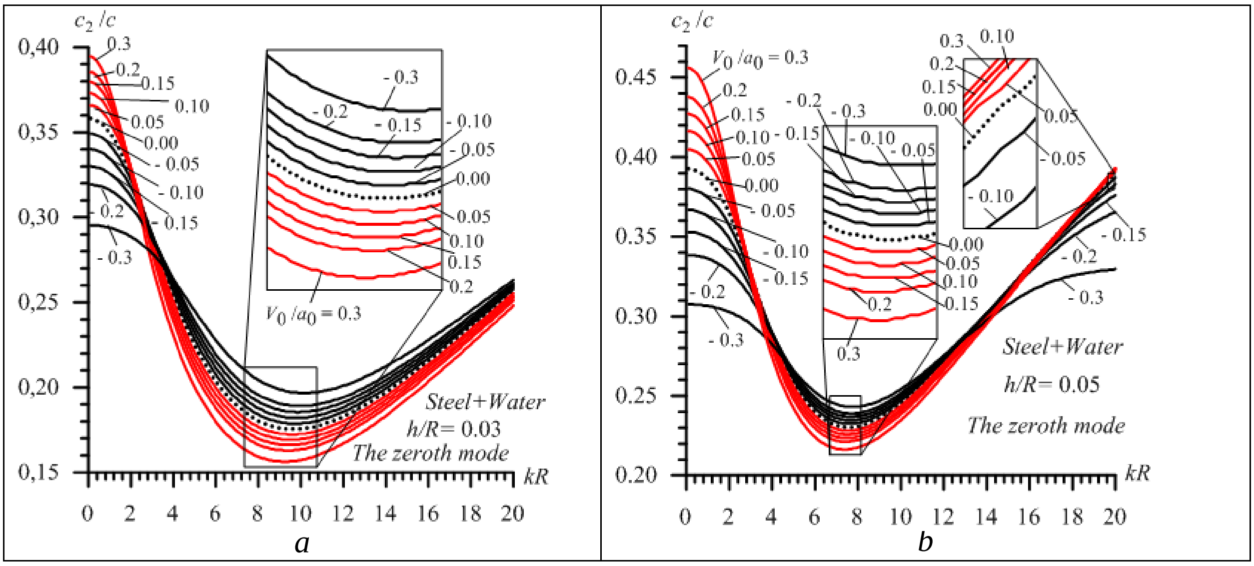

Fig (1)

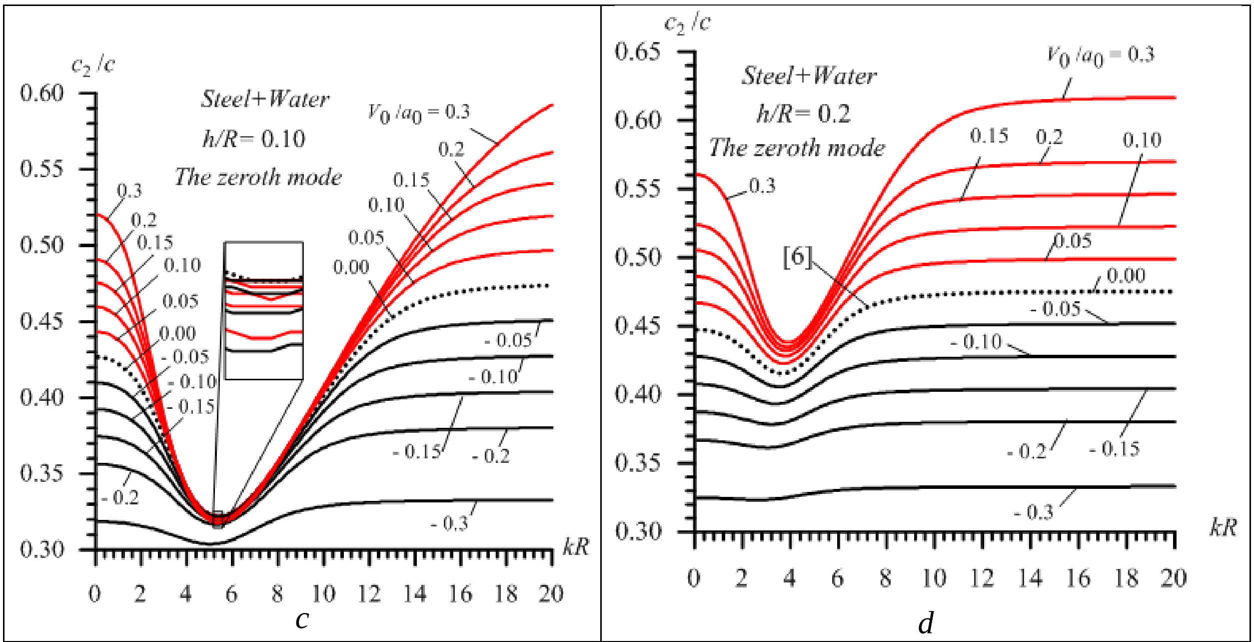

Fig. 2. The influence of the fluid flow velocity and direction on the propagation velocity of axisymmetric longitudinal quasi-Scholte waves propagating in a hollow cylinder containing this fluid in the cases h/R=0.03 (a); 0.05 (b); 0.10 (c) and 0.20 (d)

curves mentioned were constructed for the following two cases: *Case 1* assumes that $V_0/a_0 > 0$ (0.05; 0.10; 0.15; 0.20; 0.30), i.e., the flow direction of the fluid coincides with the wave propagation direction, but *Case 2* assumes that $V_0/a_0 < 0$ (-0.05; -0.10; -0.15; -0.20; -0.30), i.e., the flow direction of the fluid is opposite to the wave propagation direction

Thus, first, we consider the dispersion curves related to the zeroth mode which are obtained in the cases $h/R = 0.03$; 0, 0.05, 0.1, and 0.2 and shown in Figs. 2a, 2b, 2c, and 2d, respectively.

Thus, first, we consider the dispersion curves related to the zeroth mode which are obtained in the cases $h/R = 0.03$; 0, 0.05, 0.1, and 0.2 and shown in Figs. 2a, 2b, 2c, and 2d, respectively.

From the analysis of these curves, it appears that the character of the influence of the flow velocity V_0/a_0 depends not only on the direction of this flow but also on the ratio h/R and on the dimensionless wavenumber kR. Nevertheless, in all considered cases, at the lower wavenumbers (or at long wavelengths), which are close to

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the corresponding limits in the above-mentioned Case1 (Case 2), the wave propagation velocity c/c_2 of the quasi-Scholte wave increases (decreases) monotonically with the absolute values of the flow velocity. At the same time, from the observations of the graphs shown in Figs. 2a, 2b, and 2c, it follows that as the dimensionless wavenumber kRincreases, the character of the influence of the fluid flow on the wave propagation velocity changes.To be more precise, in the relatively small values of the ratio h/R (for example, in the cases h/R =0.03, 0.05, 0.1) there is such an interval for the kR (denote this interval as $[kR_1, kR_2]$) in which, conversely, in Case 1 (Case 2) the propagation velocity c/c_2 of the quasi-Scholte wave decreases (increases) monotonically with the absolute values of the fluid flow velocity. This means that in the cases $kR = kR_1$ and $kR = kR_2$ the fluid flow has no effect of the fluid flow on the wave propagation velocity. Note that in Fig. 2a, kR_2 is not observed because in the case $h/R = 0.03$, kR_2 appears at kR, which is greater than 20, which is considered a high threshold value for kR in the present study.The results also show that the values of kR_1 and kR_2 also depend on the flow velocity and the difference $(kR_2 - kR_1)$decreases with h/R. At the same time, it is clear from the results that after a certain value of h/R (for example, at $h/R = 0.2$ (Fig. 2d) and $h/R > 0.2$) the mentioned interval $[kR_1, kR_2]$disappears. In other words, after a certain value of h/R the character of the influence of V_0/a_0 does not depend on the ratio h/R and in such cases the

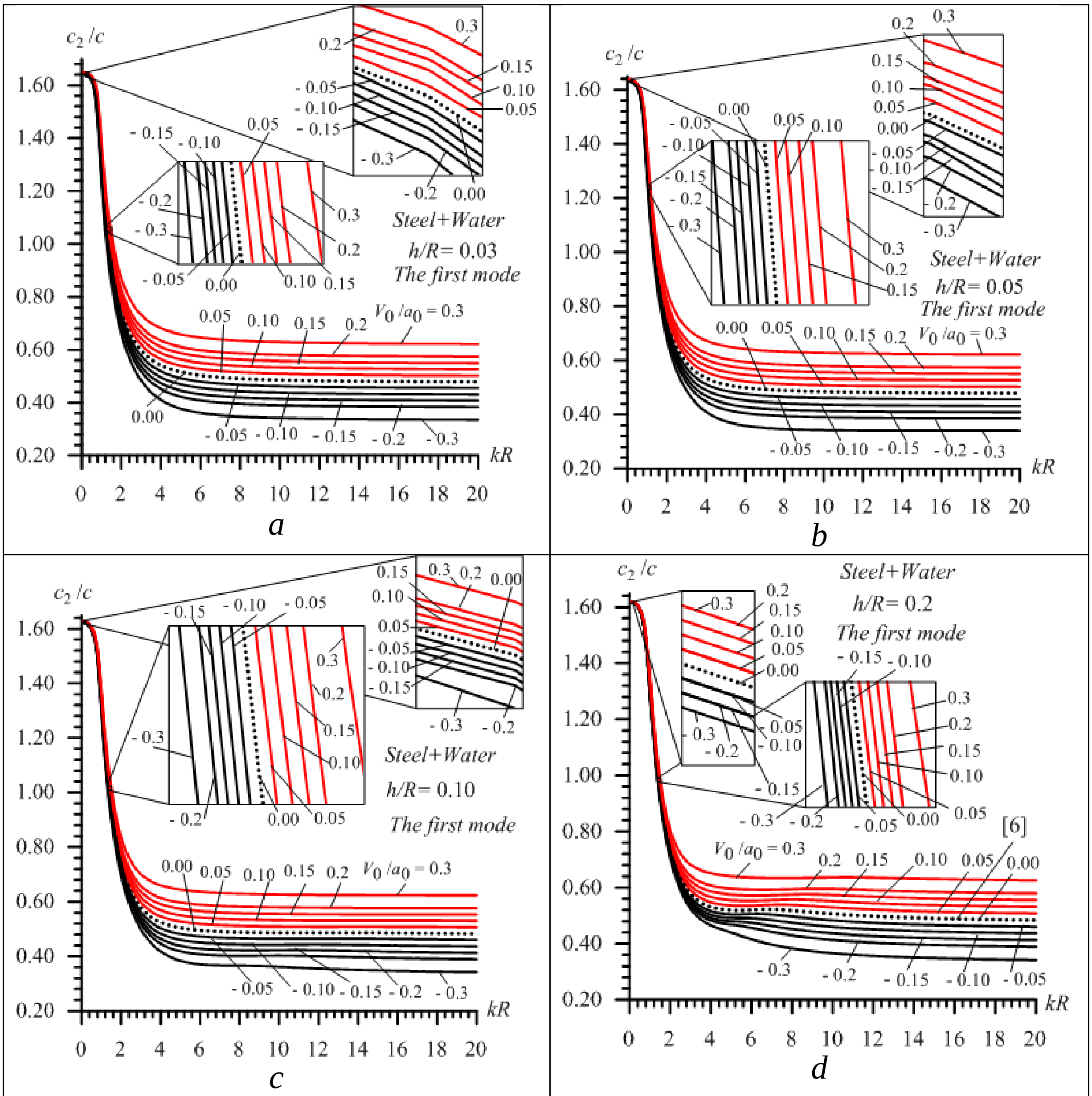

Fig. 3. The influence of the fluid flow velocity and direction on the propagation velocity of axisymmetric longitudinal waves for the first mode propagating in a hollow cylinder with this fluid in the cases *h/R*=0.03 (*a*); 0.05 (*b*); 0.10 (*c*) and 0.20 (*d*)

speed of propagation of the quasi-Scholte wave in Case 1 (Case 2) increases (decreases) monotonically with the absolute values of V_0/a_0 for all values of kR (Fig. 2d).

We also note that the dispersion curve represented by a dashed line in Fig. 2d for the case $h/R = 0.2$ and V_0/a_0 coincides with the corresponding curve in [6] and with that in [1]. This situation gives some guarantee of the reliability of the numerical results obtained and of the computational algorithm and PC programs used in obtaining these results

In the literature, we have not found related investigations for the case when $V_0/a_0 \neq 0$ has been carried out in the framework of the exact equations of elastodynamics and the linearized Euler equations for compressible fluids, in order to compare the present results with them. Note that so far in the corresponding investigations the motion of the cylinder has been described by means of the approximate shell theories, in the framework of which it is not possible to study the quasi-Scholte waves and the influence of the fluid flow velocity of these waves. An example of such investigations can be used the work [7], in which the wave propagation in a buried pipe carrying a flowing fluid is studied.

Let us now consider the results for the first mode obtained in the cases h/R =0.03, 0.05, 0.1, and 0.2, shown in Figs. 3a, 3b, 3c, and 3d, respectively. Note that the dispersion curves shown in these figures were also obtained in the two cases mentioned above (Case 1 and Case 2) for the various values of V_0/a_0 given above. The analysis of these results shows that in the first mode the character of the influence of the flow velocity on the curves does not depend on the ratio h/R and on the dimensionless wavenumber kR and that in Case 1 (in Case 2) under $V_0/a_0 > 0$ ($V_0/a_0 < 0$) the flow leads to an increase (decrease) of the wave propagation velocity in the first mode. At the same time, the magnitude of this increase (decrease) grows with the absolute values of the flow velocity. Note that this result is consistent in a qualitative sense with the corresponding results in the paper [7] and recall that only Case 1 is considered in the paper [7].
This completes the consideration of the numerical results.

V. Conclusion

Thus, in the present work, the influence of the flow velocity and the flow direction of the fluid on the velocity of axisymmetric waves propagating in a hollow cylinder containing this fluid is studied. In the context of this study, the motion of the cylinder is described by the exact equations and relations of linear elastodynamics, but the flow of the fluid is described by the linearized Euler equations for compressible barotropic inviscid fluids. Analytical expressions for the sought values containing unknown constants are obtained, and with the help of contact and appropriate compatibility conditions, the system of homogeneous algebraic equations with respect to these unknown constants is obtained. By equating with the zero determinant of the coefficient matrix of this system of equations, the dispersion equation is obtained, from which, by applying the numerical solution method for this equation, the dispersion curves are constructed for different values of the problem parameters, in particular for the different flow velocities under different flow directions.

The dispersion curves are presented and analyzed for the zeroth and first modes. As a result of these analyzes, it is found that in the zeroth mode (i.e., the mode associated with quasi-Scholte waves) the character of the influence of the fluid velocity on the wave propagation velocity depends not only on the magnitude of the fluid flow velocity, but also on the ratio h/R,on the dimensionless wave number kR, and on the fluid flow direction.

It is also found that in the first mode the influence of the fluid velocity on the wave propagation velocity depends only on the magnitude and sign of the flow velocity V_0/a_0. In particular, it is found that in the case $V_0/a_0 > 0$ (i.e., in the case where the direction of the flow velocity coincides with the wave propagation direction), an increase in the values of V_0/a_0 leads to an increase in the wave propagation velocity. However, in the case $V_0/a_0 < 0$ (i.e. in the case when the direction of the fluid flow velocity is opposite to the wave propagation direction), the increase of the absolute values of V_0/a_0 leads to a decrease of the wave propagation velocity
The detailed analysis of the mentioned numerical results is described in the text of the paper.

References

[1]. Akbarov S.D., Sevdimaliyev Y.M. and Valiyev G.J. (2021), "Mathematical modeling of the dynamics of a hydroelastic system—A hollow cylinder with inhomogeneous initial stresses and compressible fluid." *Math Meth Appl Sci.***44(9)**: 7858 – 7872. https://doi.org/10.1002/mma.7329

[2]. Guz A.N. (2004), "Elastic waves in bodies with initial (residual) stresses. "A.C.K. Kyiv (in Russian).

[3]. Akbarov S.D. (2015)," Dynamics of pre-strained bi-material elastic systems: Linearized three-dimensional approach." Springer, Heidelberg, New York, USA.

[4]. Eringen A.C. and Suhubi E.S. (1975), "Elastodynamics, Finite motion, vol. I; Linear theory, vol. II," Academic Press, New York.

[5]. Guz A.N. (2009), "Dynamics of compressible viscous fluid." Cambridge Scientific Publishers. Cambridge.

[6]. Sinha B.K., Plona T.J., Kostek S. and Chong S-K. (1992), "Axisymmetric wave propagation in fluid loaded cylindrical shells. I: Theory." *J. Acoust. Soc. Am.***92**(2), 1132 – 1143.
[7]. Deng Qing-Tian, Yang Zhi-Chun, (2013). Wave propagation analysis in buried pipe conveying fluid, *Applied Mathematical Modelling* 37,6225–6233. http://dx.doi.org/10.1016/j.apm.2013.01.014

Gurbanali J. Veliyev. "The influence of the fluid flow speed on the axisymmetric wave propagationvelocity in the cylinder containing this fluid". *IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE)*, 19(6), 2022, pp. 52-61.