IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE)
e-ISSN: 2278-1684, p-ISSN: 2320-334X, Volume 22, Issue 1 Ser. 1 (Jan. – Feb. 2025), PP 08-15
www.iosrjournals.org

A Numerical Analysis Of Molten Metal And Nano Fluid Flow Through A Rectangular Three-Dimensional Geometry With A Free Surface

Rajat Kabiraj[1], Ranajit Midya[2], Snehamoy Majumder[3]
[1, 2] *Research Scholar, Department of Mechanical Engineering, Jadavpur University, Kolkata, India*
[3] *Professor, Department of Mechanical Engineering, Jadavpur University, Kolkata, India*

ABSTRACT: *In the present study the molten hot metal at the fluid state as well as nano fluid of CuO-water combination respectively have been transported through a 3-D rectangular open channel that has to be explored numerically. The governing Laminar Navier-Stokes equation as well as nano fluid flow equations have been solved along with the solution of the energy equation respectively. The detail heat transfer analysis along with the determination of the pressure and velocity field as well as heat transfer coefficient and friction factor etc. will be the major investigations. The pressure and velocity distributions along the radial directions at different axial locations will be furnished to visualize the velocity field*
Keywords: *3-d rectangular geometry, Centre line velocity, Velocity field, Bulk fluid temperature, Nusselt Number and CuO- water nano fluid.*

Date of Submission: 26-12-2024 Date of Acceptance: 06-01-2025

NOMENCLATURE

Re	Reynolds number
Nu	Nusselt number
Pr	Prandtl number
υ	Velocity (m/s)
p	Pressure (Pa)
K	Thermal conductivity (J/m.K)
C_p	Specific heat (J/kg.K)
T_w	Temperature at wall (K)
T_∞	Temperature of fluid (K)
h	Heat transfer coefficient(J/K. m^2)

Greek Symbols

μ	Coefficient of viscosity (Pa-s)
σ	Shear Stress (N/m^2)
ρ	Density of fluid (kg/m^3)
δ	Velocity boundary thickness (m)
δ_t	Thermal boundary thickness (m)
α	Thermal diffusivity W/(m.K)

Subscripts

p	Pressure
w	wall
t	thermal
∞	fluid

I. INTRODUCTION

The fluid molten metal and nano fluid flow is very important in research and industrial activities. Inman [1] analysed the molten metal flow and heat transfer in a rectangular geometry in the sixties. There are lot of research work [2]–[19] considering rectangular geometry with laminar and turbulent flows. The nanofluid flow and heat transfer analysis in rectangular geometry have also been done by several researchers [21]-[22]. In the present work, molten metal (liquid iron) as well as nano fluid (CuO-water) as a Newtonian fluid, so the Newton's law of viscosity is applicable to these categories of fluid, which makes the problem easier and amenable within

the laminar regime in spite of three-dimensional complicacies. However a numerical experimentation of 3-D geometry requires a huge platform of computer peripherals, which is not always possible to get. Hence the present study is an honest endeavor to cater a complex flow with the aid of limited resource of computational strength.
The objectives of the present work are:
1. Analysis of the molten metal and nano fluid flow numerically considering steady, laminar and incompressible flow in a 3-D geometry.
2. To investigate the variation of the bulk fluid temperature as well as the Nusselt number along the stream-wise direction.
3. To estimate the pressure, velocity and temperature distributions along the radial as well as axial directions.
4. To estimate the variation of friction factor pertaining to different Reynolds's numbers.
5. Also an effort has been made to find the variation of heat transfer coefficient along the stream-wise direction

II. GOVERNING EQUATIONS AND SOLUTION METHODOLOGY

problem description

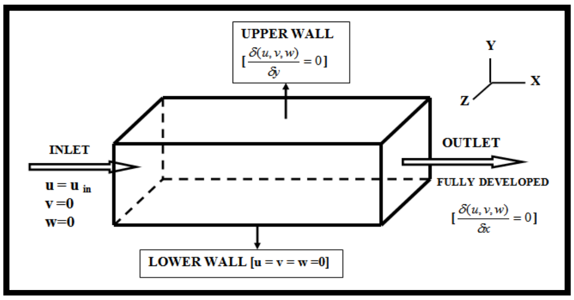

Fig. 1, The geometry of the problem

2.1 Mathematical Modelling:

The working fluids are molten metal and nanofluids with different concentrations of CuO nanoparticle including 0.0%, 0.01%, 0.02%, 0.05% and 0.1% volume fractions in distilled water were used to study heat transfer characteristics in laminar flow. The following are relevant equations to govern the physical process.

Continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \vec{V}) = 0 \qquad (1)$$

Navier Stokes equation

X-direction:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right] + g_x \qquad (2)$$

Y-direction:

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\mu}{\rho}\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right] + g_y \qquad (3)$$

Z-direction:

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\mu}{\rho}\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right] + g_z \qquad (4)$$

Energy equation

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \frac{\mu}{\rho C_p}\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] + \frac{\mu \varphi_v}{\rho C_p} \qquad (5)$$

Where the viscous dissipation term is given below,

$$\varphi_V = \left[2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + 2\left(\frac{\partial w}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)^2\right]$$

The density of a nanofluid

$\rho = \rho_{nf} = \varphi.\rho_s + (1 - \varphi)\rho_w$

Viscosity of the nanofluid

$\mu_{nf} = \mu_w.(1 + 2.5\varphi)$

The equation of specific heat of the nanofluid

$$C_{p_{nf}} = \frac{\varphi.\left(\rho_s.C_{p_s}\right) + (1-\varphi).\left(\rho_w.C_{p_w}\right)}{\rho_{nf}}$$

Effective thermal conductivity of the nanofluid

$$k_{nf} = [\frac{k_s + 2k_w + 2(k_s - k_w)(1+\beta)^3\varphi}{k_s + 2k_w - (k_s - k_w)(1+\beta)^3\varphi}]k_w$$

Convective heat transfer coefficient

$$\overline{h_{nf}} = \frac{q}{A.(T_w - T_b)_{nf}}$$

Where T_b is the bulk mean temperature at a cross section.

Nusselt Number of the nanofluid

$$\overline{Nu_{nf}} = \frac{\overline{h_{nf}}.D_h}{k_{nf}}$$

For pure metal, $\rho=\rho_{s,\ \mu=\mu s, k=ks,\ h=hs}$ etc.

2.2 Boundary Conditions:

INLET

U= Uin, V = W = 0, T = Tin (At the entrance, z=0)

EXIT/OUTLET

$\frac{\partial(u,v,w,T)}{\partial x} = 0$, **Fully developed condition**

WALLS

U = V = W = 0, , q = $-k\frac{\partial T}{\partial y}$

FREE SURFACE

$\frac{\partial(u,v,w)}{\partial y} = 0$, **Neumann boundary condition meaning Shear Stress = 0**

$T_{free\ surface}$ = constant temperature or may be constant heat flux

The dimensionless forms are interpreted as follows:

X = x / D_h; Y = y / D_h;Z = z / D_h;U = u / U_{in}; V = v/U_{in}; W = w/$U_{in}$$\theta$ = (T-T_{in}) / (T_w- Tin), $P = p / \rho_f U_{in}^2$

$Re = U_{in} D_h \rho_f / \mu_f$ $Pr = \mu_f / \rho_f \alpha_f$

2.3 SOLUTION METHODOLOGY:

A fully staggered grid system of S. V. Patankar has been adopted for the velocity components and the scalar variables and these equations were discredited using a control volume formulation. The numerical solution in the present work is accomplished by using SIMPLER algorithm and the power-law scheme proposed by Patankar [20].

NUMERICAL SOLUTION

The numerical solution in the present work is accomplished by using Semi implicit method for pressure linked equation revised (SIMPLER) and the power-law scheme proposed by Patankar.

III. RESULTS AND DISUSSION

3.1 Grid Independence Study

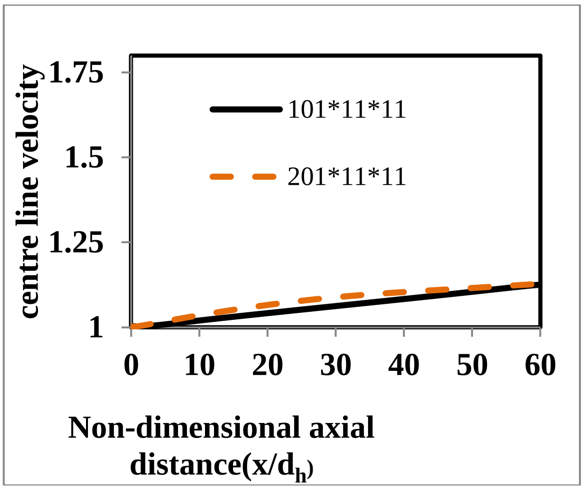

Fig. 2, Grid independent study

In the figure 2 variations of the results are almost negligible for the grid systems of 101X11X11 and 201X11X11. This means the results collapse for these systems. However, to cater a 3-D flow and considering other complexities the higher one is used for the present results. Hence unless otherwise stated the grid system is 201 X 11 X 11.

3.2 CENTRE LINE VELOCITY

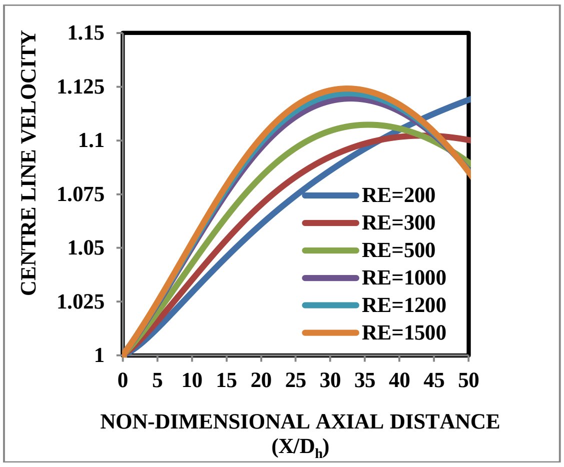

Fig. 3, Variation of centre line velocity

From the graph in fig. 3 it is observed that the centre line velocity is increasing with Reynolds number and the increment is non-linear in the downstream direction. However there is a tendency of drooping for the centre line velocity in the downstream particularly at the exit section. This result indicates that the liquid iron flow is divided in three regimes in which the first zone representing the initial part of the axial flow, where the flow exhibit a linear character so far as centre line velocity is concerned. The exit part has a drooping character very much depicted in the figure. But the intermediate portion is non-linear, clearly showing the three regimes of fluid flow.

3.3 PRESSURE VARIATIONS

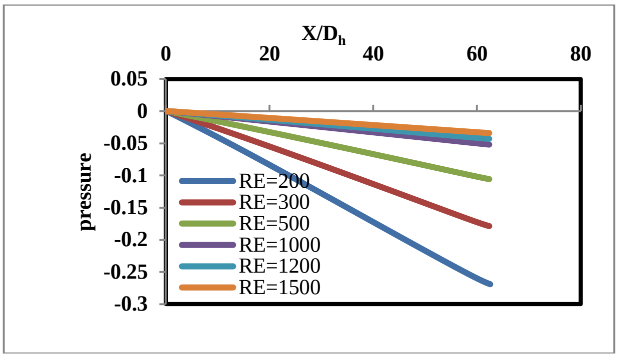

Fig. 4, Pressure variation

The variations of pressure corresponding to different Re have been shown in the fig. 4. Here it is noticed that the slope of pressure drop to be decreasing with respect to the Re.

3.4 VELOCITY DIAGRAM

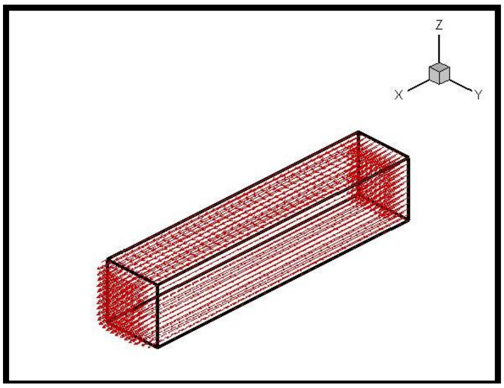

Fig. 5a, velocity vector in 3_D

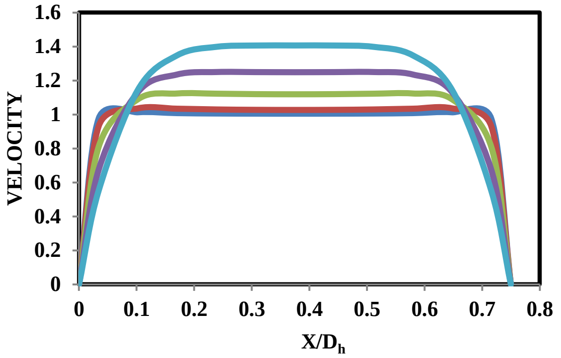

Fig. 5b, Velocity variations at 0%(red), 10% (blue), 25% (light green), 50% (light blue) and 100% (sky blue).

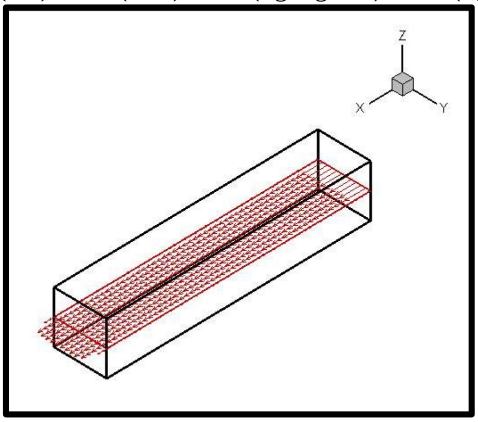

Fig. 5c, Velocity vector in a horizontal plane.

3.5 **FRICTOR FACTOR**

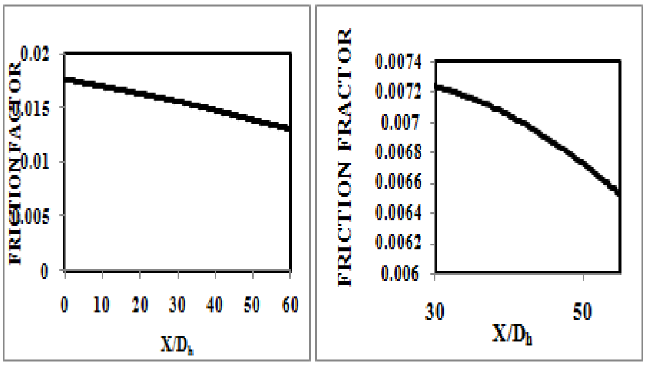

Fig. 6, Friction factor variations

In fig. 6, the friction factor variation has been shown. Though the variation is showing apparently linear but in actual case it is nonlinear which is visible in scale up adjacent figure for non-dimensional position of 30 to 60.

3.6 **THERMAL ANALYSIS**

From the figure it is observed that the Nusselt number is increasing along the axial direction with rise in Reynolds number and this rise is absolutely linear as shown in figures 6 . The same variation i.e. Nu vs. non-dimensional axial length has been observed to be independent of the changes in the heat fluxes in fig. 7.

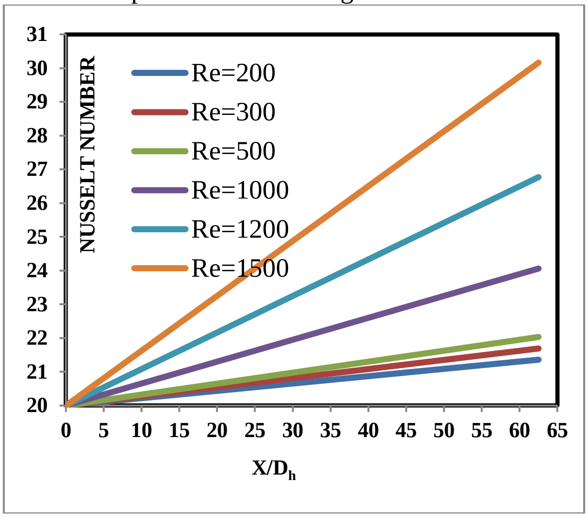

Fig. 6, Nusselt number variation with Re, constant temperature

3.7 **HEAT TRANSFER COEFFIENT**

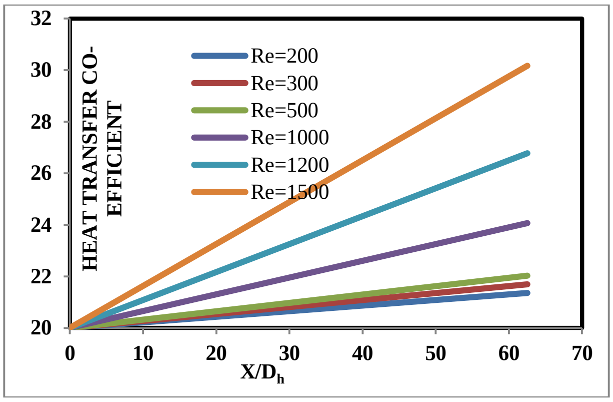

Fig. 7, Variation of Heat transfer coefficient with Re, constant heat flux.

3.8 TEMPERATURE DIAGRAM

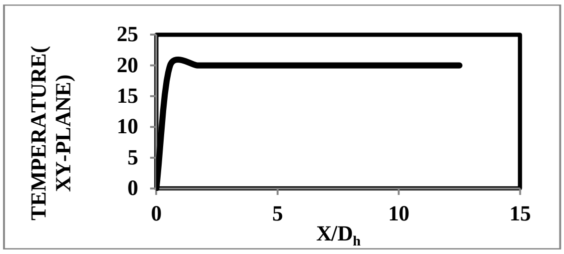

Fig. 8, Temperature variation in middle x-y plane.

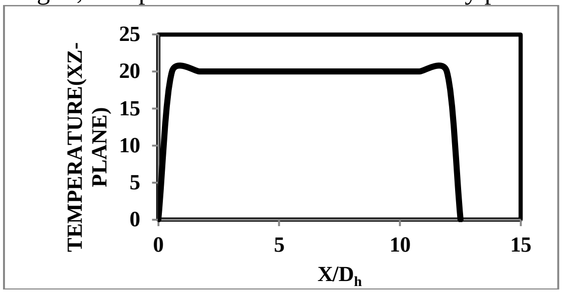

Fig. 9, Temperature in z-plane

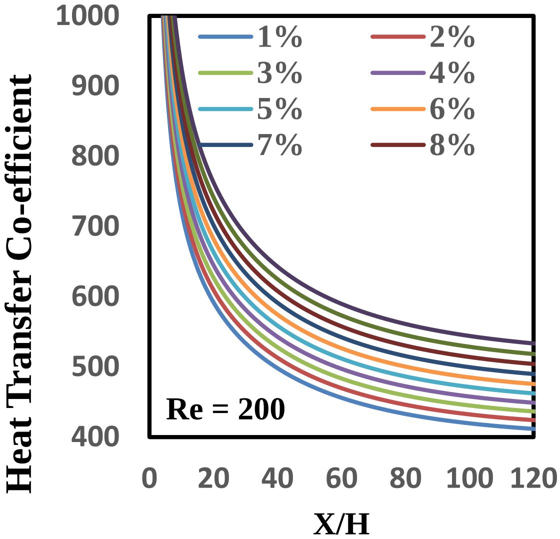

Fig. 10 Heat transfer coefficients for different % of CuO-water

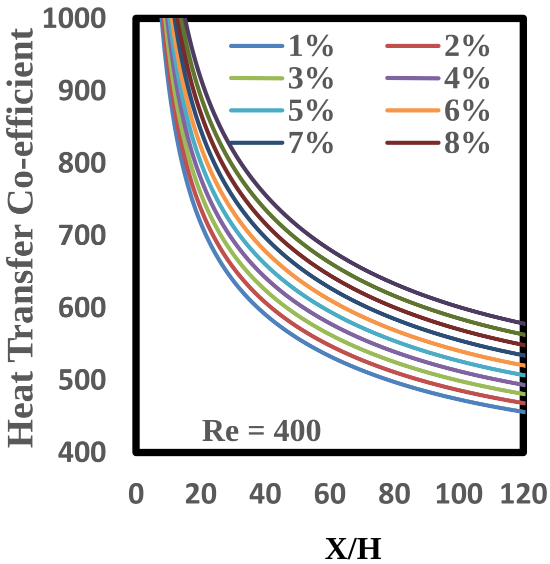

Fig. 11 Heat Transfer co-efficient for different % of CuO-water

Similarly, in figures 8 and 9 it is noticed that the variation of temperature in different plane of the geometry while figures 10 and 11 explain the variation of heat transfer coefficients for Re 200 and 400 respectively. From the figures 10 and 11, indicate that the heat transfer coefficient increases with the increase in percentage of nano particles.

IV. CONCLUSION

The analysis of fluid flow and heat transfer in a rectangular geometry with free surface has been doneeffectively. The results show heat transfer mode is very effectively captured with the variation of Reynolds number. The nanofluid flow analysis shows that heat transfer is considerably increased. The molten metal flow is well captured but further study needed.

REFERENCES

[1]. Robert M. Inman, 1967 'Heat-transfer analysis for liquid-metal flow in rectangular channels with heat sources in the fluid'*National aeronautics and space administration*, February 1967.

[2]. B. E. Launder And W. M. Ying, 1972, 'Secondary flows in ducts of square cross-section' *J .Fluid Mech., vol.* 54, *part* 2, *pp.* 289-295.

[3]. A. Melling And J. H. Whitelaw, 1976, 'Turbulent flow in a rectangular duct, *J. Fluid Mech., vol.* 78, *part* 2, *pp.* 289-315

[4]. A. O. Demuren And W. Rodi, 1983, 'Calculation of turbulence-driven secondary motion in non-circular ducts', *Fluid Mech., vol.* 140, *pp.* 189-222.

[5]. J. P. Van Doormaal and G. D. Raithby**,**1984,'Enhancements of the SIMPLE method for predicting incompressible fluid flows', *Numerical Heat Transfer,* vol. 7, pp. 147-163.

[6]. M. Molki and A. R. Mostoufizadeh, 1988, 'Turbulent heat transfers in rectangular ducts with repeated-baffle blockages', *Int. J. Heat Mass Transfer.*, Vol. 32, No. 8, pp. 1491-1499.

[7]. Ravi K. Madabhushi and S. P. Vanka, 1991 'large eddy simulation on turbulence-driven secondary flow in square duct', *Department of Mechanical and Industrial Engineering, University of Illinois at Urbana-Champaign, Urbana, Illinois,* 6180I.

[8]. Z.Yang and T. H. Shiht,1993,'New Time Scale Based $\kappa - \varepsilon$ Model for Near-Wall Turbulence', *AIAA JOURNAL*, 1993, Vol. 31, No. 7, July.

[9]. Asmund Husert And Sedat Biringen, 1993, 'Direct numerical simulation of turbulent flow in a square duct', *J. Fluid Mech., vol.* 257, *pp.* 65-95 .

[10]. David C. Wilcox, 1993, 'Comparison of Two-Equation Turbulence Models for Boundary Layers with Pressure Gradient' *AIAA JOURNAL,* August 1993, Vol. 31, No. 8.

[11]. Fan Sixin and Lakshminarayanat Budugur, 1993 'Low-Reynolds-Number k-ϵ Model for Unsteady TurbulentBoundary-Layer Flows', *AIAA JOURNAL,* October 1993,Vol. 31, No.10.

[12]. Menter F. R., 1994, 'Two-Equation Eddy-Viscosity Turbulence Models for Engineering Applications' *AIAA JOURNAL,* August 1994Vol. 32, No. 8.

[13]. Tsan-Hsing Shih, William W. Liou, Aamir Shabbir, Zhigang Yang and Jiang Zhu, 1994, 'A new k-ϵ eddy viscosity model for high reynolds number turbulent flows',*Computers Fluids,* 1995, Vol. 24, No. 3, pp. 227-238.

[14]. Andrew T. Thies and Christopher K. W. Tam, 1996, 'Computation of Turbulent Axisymmetric and Non axisymmetric Jet Flows Using the k-ϵ Model'*AIAA JOURNAL*, Vol. February 1996, 34, No. 2.

[15]. Srinath V. Ekkadand J. E-Chin Han, 1996, 'Detailed heat transfer distributions in two-pass square channels with rib turbulators', *Int. J. Heat Mass Transfer,* 1997, Vol. 40, No. 11, pp. 2527-2537.

[16]. Akira Murataa, Sadanari Mochizuki, Tatsuji Takahashi, 1999, 'Local heat transfer measurements of an orthogonally rotating square duct with angled rib turbulators', *Columbia International Publishing American Journal of Heat and Mass Transfer,* 1999, 42, 3047-3056

[17]. M. A. Leschziner and W. Rodi, *Calculation of Annular and Twin Parallel Jets Using Various Discretization Schemes and Turbulence-Model Variations,* Trans. ASME, J. Fluids Engg. 103(1981), pp. 352-360.

[18]. S. Majumder, and D.Sanyal, *Destabilization of Laminar Wall Jet Flow and Re- Laminarization of the Turbulent Confined Jet Flow in Axially Rotating Circular Pipe*,Trans. ASME, Journal of Fluids Engg.130 (2008),pp. 011203-1 – 011203-8.

[19]. A. Ali, M Asif Memon, and A. Majed Albugami, 2021, 'Numerical analysis of laminar flow and heat transfer through a rectangular channel containing perforated plate at different angles. Energy Reports 8(1):539-550

[20]. Mohd. Imran Ansari and Dr. D.K. Singh, On the Analysis of Molten Metal Flow through Sprue in Casting Process, International Journal of Engineering Research & Technology (IJERT), Vol. 1 Issue 6, August – 2012 ISSN: 2278-0181.

[21]. V.I. Odinokov, A.I. Evstigneev, E.A. Dmitriev, S.Yu. Aleksandrov and G.I. Usanov, *Modeling of Molten Metal Flows in a Continuous-Casting Machine Mold at Free Rotation of the Floating Closed-Bottom Nozzle*, Key Engineering Materials Submitted: 2021-07-06ISSN: 1662-9795, Vol. 910, pp 24-29 Accepted: 2021-09-11doi:10.4028/p-soh58m Online: 2022-02-15© 2022 Trans Tech Publications Ltd, Switzerland

[22]. Casey Bate, Philip King, Jay Sim and Guha Manogharan, A Novel Approach to Visualize Liquid Aluminum Flow to Advance Casting Science, Materials 2023, 16, 756

[23]. Siva P 1 , Bharathikanna R 2 Amitkumar M 3, Numerical ANALYSIS OF MOULD FILLING AND GATING DESIGN FOR ALUMINIUM MOULD CASTINGS, Journal of Cardiovascular Disease Research ISSN:0975-3583,0976-2833 VOL12,ISSUE07,2021

[24]. Patankar, S.V., "Numerical Heat Transfer and Fluid Flow", Hemisphere Publishing Corporation, New York, USA.

[25]. Vasefi, I., Alizadeh, M., 2013, "A Numerical Investigation of Cuo-water Nano fluid in Different Geometries by Two-Phase Euler Lagrange Method", World Applied Sciences Journal, 26 (10), pp. 1323-1329.

[26]. Choi and Eastman, 1995, "Enhancing Thermal Conductivity of Fluids with Nanoparticles " , Energy Technology Division and Materials Science Division, Argonne National Laboratory, Argonne.