IOSR Journal of VLSI and Signal Processing (IOSR-JVSP)
Volume 6, Issue 6, Ver. I (Nov. - Dec. 2016), PP 05-09
e-ISSN: 2319 – 4200, p-ISSN No. : 2319 – 4197
www.iosrjournals.org

Mathematical Principals and Modeling of EEG Signal Exploration

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Abstract: *The electroencephalographic signal is a resultant signal of the action potential of neuron in the brain which inspects the neural functions. The brain signal is so subtle that it cannot be analyzed without amplification and this amplified signal is electroencephalogram (EEG). Electroencephalography is non-invasive appliance which is used in observing brain activities and detection of different disorder relating to the human brain. There are several objections of EEG for instance small signal amplitude, synchronizations, artifacts, temporal variability of signal and its sensitivity to noise. These objections have potential impact ontoreal time electroencephalographic signal measurement and analysis. The EEG analysis plays very significant role in feature extraction of EEG signal in order to diagnosis and predicts different brain disorders. The primary target of this paper is to investigate electroencephalographic signal by using some mathematical schemes like time domain, frequency domain and time-frequency domain.*
Keywords: *Electroencephalogram, EEG,Brain signal, Time domain, Frequency domain.*

I. Introduction

Human brain is the control unit of human body as it controls the muscle activities to move, talk, visualize, feel and it also makes different facial expressions. Two important categories called invasive and non-invasive methods are usually used for EEG signal measurement. The magneto-encephalogram called MEG is a modern technique which observes the magnetic activity generated by brain and is similar to the electrical activity measured by the EEG. Most of the scientist uses EEG for signal investigation over fMRI and MEG. EEG is physiologically and physically most preferable as it is movable, simple, well tested method by different organizations and also economical.EEG measurement may be used as intra-cranial in brain surgeries and also used as extra-cranial in finding some diseases contained in the brain. EEG is a sum of short transient pulse containing significant pathological information, and interference [1].There are mainly two advantages of EEG over other techniques. Firstly since it is non-invasive and easy to examine the typical human objects. Secondly it has excellent time resolution which contributes better temporal dynamics of brain mechanisms [1].Quantitative inquiry of EEG includes measurement of mean spectral magnitude or power for numerous frequency bands. Quantitative inquiry is the process of evaluating distinct properties of raw EEG and transforming it into digitally recorded EEG signal. Then numerical parameters are explored for numerous applications for instance brain disorder detection, emotion analysis etc. Organization of this paper describes part 2 as thebehavior of brain signals and its acquisition and measurement procedure, part 3 as mathematical methods of EEG signal, part 4 as EEG data study and part 5 as summary of this study.

II. Characteristics And Acquisition Procedure Of Brain Signals

To collect EEG raw data different electrodes of EEG are embedded on the scalp from various regions for advanced analysis of functions in several parts of the human brain. There is a system called 10-20 system is the worldwide approved technique that explains the location of electrodes during EEG test or experiment. In this system two numbers 10 and 20 describethe distances between two neighboring electrodes can be 10% or 20% of the total right to left or front to back length of the corresponding scalp [2]. The distances of inter-electrode are equal along any antero-posterior or transverse line, and electrode positioning is symmetrical. Thus the acquired EEG signals can be used to study the nervous system, observe the sleep stages, biofeedback and control and detect several abnormalities like epilepsy [3]. EEG signals expose several patterns of rhythmic or periodic activity. The most frequently used terms for EEG frequency bands are [4]: Delta band 1-4 Hz*;* Theta band 4-8 Hz*;* Alpha band 8-13 Hz*;* Beta band 13-21 Hz. The brain behavior of responding based on its spatial scales can be classified as micro-scopic, meso- scopic and macro-scopic. Small scale spatial behavior is describedbythe micro-scopic, it includes a single or a few brain cells. Macro-scopicbehavior covers whole regions of brain, is observed at large spatial scales (cm). Meso-scopic is the intermediate scale that finds out the behavior of networks of neurons across millimeters rather than centimeters. The brain function also can be represented using other temporal scales. Because of natural condition and neural dynamic response scales like temporal and spatial are correlated. The micro-scopic scale is defines normally with 1000 Hz and above frequencies the mechanismsare very fast in brain single neurons.Besides the meso-scopic scales are defined with frequencies between 10-1000Hz and the macro-scopic level is defined with frequencies in the range of 1-100Hz. The system of measurement of brain can be expressed with simple model as shown in Fig. (1), it comprises of electrode signals as a input to the system. After some delay those input signals are integrated together and then they are applied to EEG measurement and recording system.

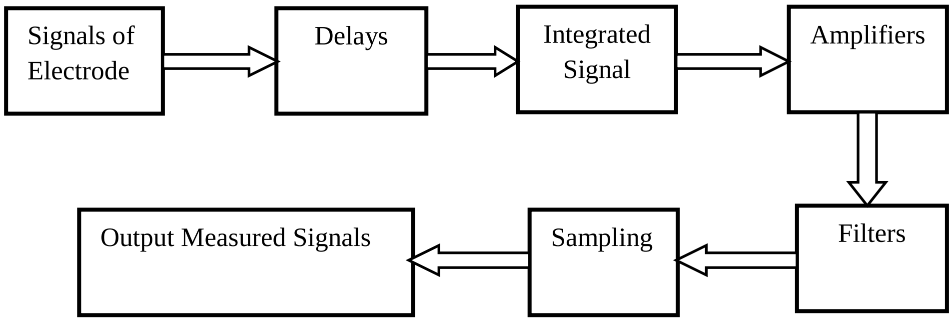

Fig. (1) Measurement and Recording of Brain Signal.

After being filtered and amplified, the signals of EEG are transformed to digital output.EEG has well temporal resolution and it helps to study the quick electrical activity changing but bad spatial resolution because of limited electrodes, their localization and properties of the head.The EEG is generated by potentials in hundred billions of brain neurons and gives the measurements of temporal distribution of the electrical activity which occurs in several sections of head. In the process of EEG signals measurement and recording, synchronization amongst the channels and artifact are some of the challenges. MahdiJalili, ElhamBarzegaran, and Maria G. Knyazeva [5] inspects the state of art in multivariate synchronization analysis and compare several synchronization measures that is applied to the time series gained by numerical simulation and to real EEG.

2.1 EEG types based on electrodes placement:

2.1.1Scalp EEG:

Various electrodes Placing on scalp evenly assists to record brain activity. Maximum 256 electrodes can be placed,localized and synchronized on scalp. This brain signal goes through different layers such as cerebrospinal fluid, skull and scalp. A typical adult human EEG signal is about 10 µV to 100 µV in amplitude when measured from the scalp and is about 10–20 mV when measured from subdural electrodes. This method is cheap and easy and provides enough information.

2.1.2Intra-cranial EEG:

EEG is recorded by placing electrodes on cortex and it is called cortical electrodes, which normally run through the sub cortical systems. The intra-cranial EEG measures small spatial scales but uses the principles of macro-scopic or meso-scopic[6]. This records on intra-cranial are used to various pre-surgical tests.

2.2 Artifacts in electroencephalogram signals observation:

Because of low amplitude of electroencephalogram signals, these signals are affected by outside sources. These “artifacts,” innate of scalp EEG recordings, are generated by head movements, electrocardiogram, and muscle activity. Artifacts are unavoidable electrical signals observed during scalp recording and originated from noncerebral origin. These are because of some extrinsic source like electromagnetic interference and improper electrode-scalp junction or some biological sources such as eye blinks, eye motions and extraocular muscle activity, chewing, muscle movement of scalp and heart beats also.There are different existing strategies, which reduce artifacts, they are

1. Ignoring: Few features of extraction methods regards artifacts effect is negligible.
2. Rejecting: EEG channels which measure impact of artifacts are rejected from future analysis.
3. Removing: Artifacts are filtered from actual EEG signals and true signals are passed to measure and analysis. Loss of data due to elimination is much less.
4. Training: Training based mechanism is developed to identify some common artifacts and their removal. Essential point is that artifacts generally limit the length of EEG signal recordings that can be thought as immobile. The parts in which the principal quality of the signal, as its mean and power spectrum does not change if ascribed as stationary. It is true that the relatively less duration of stationary EEG recordings is a great challenge for study. Hartmann, Schindler, Gebbink, Gritschand Kluge [7] proposes artifact removal algorithm which removes artifacts from EEGs and improves the readable quality of EEGs impaired by artifacts.

III. Mathematical Principles of EEG

Mathematical principle provides static and changing characteristics of electroencephalographic signal with different specifications. Basically EEG signal supports all the features of continuous time signal. Temporal features may be found from EEG signals over regions described by different channels for continuous time t. If the raw EEG signal over the continuous time t is sampled with time interval of$T$, it may be closed to digital EEG as;

$$X[n] = X(nT); \qquad n = 1,2,3 \ldots\ldots\ldots (1)$$

Here $X[n]$ is the discrete time signal that is sampled at time intervals of$t$, where $t = T, 2T, \ldots\ldots, nT$. Moreover X[n] is measured from the single channel behaviour of the brain. Mathematically the summation of the real signal and the noise signal is the resultant behavior;

$$X[n] = X(nT) + \eta(nT); \qquad n = 1, 2, 3 \ldots\ldots\ldots (2)$$

Where $X(nT)$ represents the real signal at $t = nT$ and $\eta(nT)$ represents the noise signal at this time. However $X(nT) \approx X[n]$ if $\eta(nT)$ is so small.The real signals can be detached from the raw EEG data through proper filtering. Jun Lu, KanXie, and Dennis J. McFarland [8] proposed Adaptive Spatio-Temporal Filtering method to avoid these noisy characteristics of EEG signal. It also optimizes temporal information more accurately in lower dimensional spaces.

IV. EEG Data Exploration

It is very difficult to explore EEG data because the volume-conducted EEG experiences from a low spatial resolution, such that the signal collected from each individual channel is an amalgamation of attenuated exercises from more than one brain region, and it frequently suffers interference from various artifacts for instance cardiac, muscular, and ocular. In order to find out these difficulties, it is essential to augment the signal-to-noise ratio (SNR) and also detach the overlapping exercises through proper filtering [9].Multiple electrodes of EEG simultaneously records coordinated brain exercises at multiple sites on the skull at millisecond temporal resolution, which is valuable for cognitive and neural science studies and for clinical purposes [10].

4.1 Time domain analysis of EEG signal:

When a distinct signal is evaluated over a certain range of measurement and it's components are described by time then it is called time domain analysis. In this analysis, Analyzed components are signal amplitude$|X[n]|$, signal power$|X[n]|^2$, and periodicity of signal. If$\mu_x[k]$ is the mean of sequence$X[n]$ of length N starting at time K , $X[n]$ is original signal with its energy $|X[n]|$, and $|X[n]|^2$it's power than the mean behavior of x[n] signal is estimated as;

$$\mu_x[K] = \frac{1}{N}\sum_{n=K+1}^{K+N} X[n] \qquad (3)$$

In order to know how much the signal is regular and spread over window, it is necessary to familiarize with the variance of a signal $X[n]$which is a deviation of average and its mean. If$\sigma_X^2[K]$ is the variance of a sequence $X[n]$of length N and $\mu_x[k]$is the mean then the variance of $X[n]$may be estimated by the equation below;

$$\sigma_X^2[K] = \frac{1}{N-1}\sum_{n=K+1}^{K+N}(X[n] - \mu_x[k])^2 \qquad (4)$$

Since EEG signal is non-stationary it influences $\mu_x[k]$ greatly instead of variance variability should be estimated. A statistic of total variation for window may be calculated as;

$$v_y[K] = \frac{1}{N-1}\frac{\sum_{n=K+2}^{K+N}|X[n]-X[n-1]|}{max_{X[k]} - min_{X[K]}} \qquad (5)$$

It is defined only for non-stationary signals x[n] with maximum value$max_X[k]$ and minimum value $min_X[k]$ over the window as;

$$max_{X[K]} = \max(X[n])$$
$$min_{X[K]} = \min(X[n]) \qquad (6)$$
Where $K + 1 \leq n \leq K + N$

However average, variance and variability estimate variations in EEG signal over time with window length of N. Excellent temporal resolution and short affairs in signal can be analyzed by short window size and longtimebehavior can be calculated by long window.Repeatability and periodicity of signal can be explored very fairly through auto-correlation function. In case of real signal x[n] this function can be calculated over the range from 0 to$N$. Time domain analysis also estimates about the synchronization of various EEG signals measured from various electrodes. A measure of synchronicity gives an idea of how similar signals are to each other. Various methods exist for calculating synchronicity, one of the basic methods is calculating cross correlation between multiple signals. Such as linear cross correlation of two different EEG signal over the same window length$X_1[n]$ and $X_2[n]$define synchronization of these two signals. Thus time domain analysis features are amplitude, variability, regularity and synchronicity.

4.2 Frequency domain analysis of EEG signal:

Frequency is the measure of occurrence of the events in specified time as EEG is non-stationary signal comprises of events at different frequencies. If a signal is represented in its frequency component and estimates all related features in frequency then it is known as frequency domain analysis. Here time domain signal is first transform to frequency domain with Fourier transform. For discrete-time, finite time domain signals the fast Fourier transform of signal $X[n]$ for $n = k + 1;\ k + 2 \ldots\ldots\ldots\ldots k + N$is given by;

$$FFT[\omega, k] = \sum_{n=1}^{N} X[n+k]\, e^{-i\omega n}, = \frac{2\pi m}{N} \qquad (7)$$

Where $\omega = 0\ to\ 2\pi$and$m = 0,1,2 \ldots\ldots\ldots N-1$. The value of the FFT at each ω represents the relative contribution of events that occur at that frequency to x[n]. To scale correct frequency range in Hz, $\omega$is convertedto $\omega = \frac{2\pi f}{F_s}$, where Fs is the sampling rate of the data and f is the frequency in Hz between $Fs\,/\,2$ andFs as per Nyquist rate. Power contributed to overall signal$y[n]$ by single frequency component is defined with normalized power spectral density (PSD) over, which is given as;

$$PSD\ [\omega, k] = |FFT[\omega, k]|^2 \qquad (8)$$

PSD analysis is a tool helps to define the static and dynamic properties of the EEG and these static and dynamic properties describespatio temporal behavior of EEG signal. Signal power for different events at their frequency can be calculated by power spectral density.Thus Time domain and frequency domain representations contain exactly the same information, but the features accentuated in each domain differ.

4.3 Time frequency analysis of EEG signal:

Time domain analysis is not able to give sufficient frequency content information which is required for classifying EEG signal. Analysis of Frequency domain can provide temporal information but only after windowing the function. To acquire optimal result, window size should be perfectly estimated.Time frequency resolves both of these problems and recent research refers wavelet analysis is the best method for time frequency analysis.

A wavelet ψ [n] is a function defines as;

$$\sum_{-\infty}^{\infty} \psi[n] = 0 \qquad (9)$$

And power is defined as;

$$\sum_{-\infty}^{\infty} |\psi[n]|^2 = 0 \qquad (10)$$

If wavelet function is scaled by a and shifted (or translated) by b then;

$$\psi_{ab}[n] = \frac{1}{\sqrt{a}} \psi[\frac{n-b}{a}] \qquad (11)$$

When $a = 1$ and $b = 0$ then $\psi_{10}[n]$ is entitled as mother wavelet. With$0 < a < 1$, the mother wavelet contracts in time and when $a > 1$ then $\psi_{ab}[n]$ stretches in time. The larger the a the longer is wavelet function. Each value of a represents different scales at which the frequency and temporal content is extracted with various resolutions. Thus time frequency analysis has uncertainty principle as resolution can be high for either temporal or frequency information, but not both. These data analysis process discussed in this part are considered as linear systems whereas the EEG signal contains the property of non-linear system. To analyze this signal with less computation, it is considered as linear signal over small window causes linear approximation.

V. Conclusion

EEG is a method which is non-invasive to measure the action of brain; it is simple, inexpensive and has a better temporal resolution. The disadvantages of EEG are that ratio of signal to noise is poor, thus demands extensive data analysis, low spatial resolution and large subject-specific database is required.Frequency analysis of the EEG signal extracts quantitative parameters and comparison between the power at various frequency bands and their topological distribution. Time frequency resolution is used in several applications for instance automatic convulsion detection. Thus EEG is most preferable diagnosis tool for analyzing and detecting several brain disorders.

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